Physics Point

Place To Discover, Learn & Succeed

www.physicspoint.in

8368406646

INDEX

Chapter 0

Basic Mathematics for Physics

INTRODUCTION

If you want to excel in science field, it is very important to have good mathematical aptitude. This chapter is presenting Basic Mathematics that is very useful for developing problem solving strategies whenever required. It includes application in physics section that will develop your scientific temper. Go through it twice or thrice before start learning physics. Take your teachers help wherever you feel any doubt or require more illustrations. "Mathematics is the language of physics".

TRIGONOMETRIC IDENTITIES

1. $\sin(-\theta) = -\sin\theta$
2. $\cos(-\theta) = \cos\theta$
3. $\tan(-\theta) = -\tan\theta$
4. $(\sin\theta) / (\cos\theta) = \tan\theta$
5. $\sin 2\theta = 2\sin\theta\cos\theta$
6. $\cos 2\theta = 2\cos^2\theta - 1 = \cos^2\theta - \sin^2\theta$
7. $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$
8. $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$; If $\alpha = 90^\circ$, $\sin(90^\circ \pm \beta) = \cos\beta$; If $\alpha = \beta$, $\sin 2\beta = 2\sin\beta\cos\beta$
9. $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$;

   If $\alpha = 90^\circ$, $\cos(90^\circ \pm \beta) = \mp \sin$; If $\alpha = \beta$, $\cos 2\beta = \cos^2\beta - \sin^2\beta = 1 - 2\sin^2\beta$.
10. $\sin(90^\circ - \theta) = \cos\theta$; $\cos(90^\circ - \theta) = \sin\theta$; $\tan(90^\circ - \theta) = \cot\theta$
11. $\sin(90^\circ + \theta) = \cos\theta$; $\cos(90^\circ + \theta) = -\sin\theta$; $\tan(90^\circ + \theta) = -\cot\theta$
12. $\sin(180^\circ - \theta) = \sin\theta$; $\cos(180^\circ - \theta) = -\cos\theta$; $\tan(180^\circ - \theta) = -\tan\theta$
13. $\sin(180^\circ + \theta) = -\sin\theta$; $\cos(180^\circ + \theta) = -\cos\theta$; $\tan(180^\circ + \theta) = \tan\theta$
14. $\sin(270^\circ - \theta) = -\cos\theta$; $\cos(270^\circ - \theta) = -\sin\theta$; $\tan(270^\circ - \theta) = \cot\theta$
15. $\sin(270^\circ + \theta) = -\cos\theta$; $\cos(270^\circ + \theta) = \sin\theta$; $\tan(270^\circ + \theta) = -\cot\theta$
16. $\sin(360^\circ - \theta) = -\sin\theta$; $\cos(360^\circ - \theta) = \cos\theta$; $\tan(360^\circ - \theta) = -\tan\theta$
17. $\sin^2\theta + \cos^2\theta = 1$; $\sec^2\theta - \tan^2\theta = 1$; $\mathrm{cosec}^2\theta - \cot^2\theta = 1$

18. $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$

19. $\sin A + \sin B = 2\sin\frac{(A+B)}{2}\cos\frac{(A-B)}{2}$

20. $\cos A - \cos B = 2\cos\frac{(A+B)}{2}\cos\frac{(A-B)}{2}$

21. $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$

22. $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \pm \tan A \tan B}$

23. $\sin A - \sin B = 2\cos\frac{(A \quad B)}{2}\cos\frac{(A-B)}{2}$

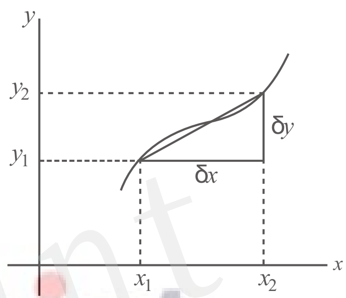
24. $\cos A - \cos B = -2\sin\frac{(A+B)}{2}\sin\frac{(A-B)}{2}$

25. $1 - \cos\theta = 2\sin^2\frac{\theta}{2}$

DIFFERENTIAL CALCULUS

The derivative y with respect to x is defined as the limit of the slopes of chords drawn between two points on the y versus x curve as Δx approaches zero. Mathematically, we write this defintion as

$$\frac{dy}{dx} = \lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0}\frac{y(x+\Delta x) - y(x)}{\Delta x} \qquad ...(1)$$



where Δy and Δx are defined as $\Delta x = x_2 - x_1$ and $\Delta y = y_2 - y_1$ (See figure)

A useful expression to remember when $y(x) = ax^n$, where a is a constant and n is any positive or negative number (integer or fraction), is

$$\frac{dy}{dx} = nax^{n-1} \qquad ...(2)$$

If $y(x)$ is a polynomial or algebraic function of x, we apply above equation to each term in the polynomial and take $\frac{da}{dx} = 0$. It is important to note that $\frac{dy}{dx}$ does not mean dy divided by dx, but is simply a notation of the limiting process of the derivative. In Examples, we evaluate the derivatives of several well-behaved functions.

ILLUSTRATION : 1

Solve $y(x) = 8x^5 + 4x^3 + 2x + 7$

SOLUTION:

Applying equation 2 to each term independently, and remembring that $\frac{d}{dx}$ (constant) = 0, we have

$$\frac{dy}{dx} = 8(5)x^4 + 4(3)x^2 + 2(1)x^0 + 0$$

$$\Rightarrow \frac{dy}{dx} = 40x^4 + 12x^2 + 2$$

Special Properties of the Derivative

(i) **Derivative of the product of two functions:** If a function y is given by the product of two functions, say $g(x)$ and $h(x)$, then the derivative of y is defined as

$$\frac{d}{dx}[f(x)] = \frac{d}{dx}[g(x)h(x)] = g\frac{dh}{dx} + h\frac{dg}{dx} \qquad ...(3)$$

(ii) **Derivative of the sum of two functions:** If a function y is equal to the sum of two functions, then the derivative of the sum is equal to the sum of the derivatives.

$$\frac{d}{dx}[f(x)] = \frac{d}{dx}[g(x) + h(x)] = \frac{dg}{dx} + \frac{dh}{dx} \qquad ...(4)$$

(iii) **Chain rule of differential calculus:** If $y = f(x)$ and x is a function of some other variable z, then $\frac{dy}{dx}$ can be written as the product of two derivatives.

$$\frac{dy}{dx} = \frac{dy}{dz}\frac{dz}{dx} \qquad ...(5)$$

(iv) **The second derivative:** The second derivative of y with respect to x is defined as the derivative of the function $\frac{dy}{dx}$ (or the derivative of the derivative). It is usually written

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) \qquad ...(6)$$

Derivatives for Several Functions

(i)	$\frac{d}{dx}(a) = 0$ where 'a' is a constant	(vi)	$\frac{d}{dx}(\tan ax) = a\sec^2 ax$
(ii)	$\frac{d}{dx}(ax^n) = nax^{n-1}$	(vii)	$\frac{d}{dx}(\cot ax) = -a\,\text{cosec}^2 ax$
(iii)	$\frac{d}{dx}(e^{ax}) = ae^{ax}$	(viii)	$\frac{d}{dx}(\sec x) = \tan x \sec x$
(iv)	$\frac{d}{dx}(\sin ax) = a\cos ax$	(ix)	$\frac{d}{dx}(\text{cosec}\, x) = -\cot x\, \text{cosec}\, x$
(v)	$\frac{d}{dx}(\cos ax) = -a\sin ax$	(x)	$\frac{d}{dx}(\ln ax) = \frac{1}{x}$

Maxima and Minima

To find the maxima and minima for a function or extreme values of the function $f(x)$ we proceed as follows.

Step-I : Find $f'(x) = \frac{d}{dx}[f(x)]$

Step-II : Put $f'(x) = 0$ and solve the obtained equations for values of x or simply saying, find the zero of the function $f'(x)$. Let $x = a_1, a_2, \ldots a_n$ be the zeros of the function $f'(x)$ also called the stationary values of $f'(x)$.

Step-III : Find $f''(x) = \frac{d^2}{dx^2}[f(x)] = \frac{d}{dx}[f'(x)]$

Step-IV : Find the values of $f''(x)$ at $x = a_1, a_2, \ldots a_n$.

Step-V : If $f''(x) > 0$ then we get MINIMA
If $f''(x) < 0$ then we get MAXIMA

Step-VI : If $f''(x) = 0$ then we get the point of INFLEXION i.e. a point which is neither MINIMA nor MAXIMA.

INTEGRAL CALCULUS

Integration is the inverse of differentiation.

Some indefinite integrals

(i) $\int x^n dx = \frac{x^{n+1}}{n+1}$ (provided $n \neq -1$)

(ii) $\int \frac{dx}{x} = \int x^{-1}dx = \ln x$

(iii) $\int \frac{dx}{a+bx} = \frac{1}{b}\ln(a+bx)$

(iv) $\int \frac{dx}{(a+bx)^2} = \frac{1}{b(a+bx)}$

(v) $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$

(vi) $\int e^{ax}dx = \frac{1}{a}e^{ax}$

(vii) $\int \sin ax dx = -\frac{1}{a}\cos ax$

(viii) $\int \cos ax dx = \frac{1}{a}\sin ax$

ILLUSRATION : 2

Integrate the following functions with respect to x

(a) $\int (5x^2 + 3x - 2)dx$ (b) $\int \left(4\sin x - \frac{2}{x}\right)dx$ (c) $\int \frac{dx}{4x+5}$ (d) $\int (6x+2)^3 dx$

SOLUTION :

(a) $\int (5x^2 + 3x - 2)dx = 5\int x^2 dx + 3\int x\, dx - 2\int dx = \frac{5x^2}{3} + \frac{3x^2}{2} - 2x + c$

(b) $\int \left(4\sin x - \frac{2}{x}\right)dx = 4\int \sin x\, dx - 2\int \frac{dx}{x} = -4\cos x - 2\ln x + c$

(c) $\int \frac{dx}{4x+5} = \frac{1}{4}\int \frac{dX}{X}$, where $X = 4x + 5$

$= \frac{1}{4}\ln X + c_1 = \frac{1}{4}\ln(4x+5) + c_2$

(d) $\int (6x+2)^3 dx = \frac{1}{6}\int X^3 dX$, where $X = 6x + 2$

$= \frac{1}{6}\left(\frac{X^4}{4}\right) + c_1 = \frac{(6x+2)^4}{24} + c_2$

LOGARITHMS

(i) $e \approx 2.7183$

(ii) If $e^x = y$, then $x = \log_e y = \ln y$

(iii) If $10^x = y$, then $x = \log_{10} y$

(iv) $\log_{10} y = 0.4343 \log_e y = 0.4343 \ln y$

(v) $\log(ab) = \log(a) + \log(b)$

(vi) $\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$

(vii) $\log a^n = n \log(a)$

VECTORS

Physical quantities which need both magnitude and direction to express them and obey the triangle or parallelogram law of vector addition are called vectors.

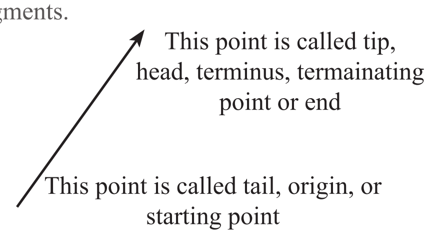
Examples: Displacement, velocity, acceleration, momentum, force etc.

Representation of Vector

Vectors are represented either graphically or by symbols.

Graphical representation of vectors

Vectors are represented by directed line segments.



The length of the line segment represents the magnitude of the vector and its arrow-head depicts its direction. The arrow head can be put on the extreme end of the line or at any other point of the line.

 represents a vector whose direction is perpendicular to the plane of the paper, coming out of the paper i.e. towards you. Similarly, ⊗ represents a vector which is perpendicular to the plane of the paper but going inside it.

Symbolic Representation of Vectors

1. Vectors are represented in either of the following two ways.
Either as $\vec{r}, \overrightarrow{PQ}$, etc.i.e., arrow on the head.
or as **r, PQ**, etc. i.e., bold letters.

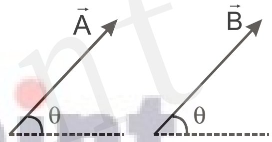
2. Magnitude (also called modulus or mod values) of vectors are represented by normal alphabets.
e.g. magnitude of $\vec{r}$ or $\overrightarrow{PQ}$ = r or $|\vec{r}|$, PQ or $|\overrightarrow{PQ}|$

3. Unit vectors (detail study in types of vector) (i.e. whose magnitude is one unit) are represented as $\hat{r}, \widehat{PQ}$ etc. (read as r-cap, PQ-cap, r-hat, PQ-hat or as r-caret, PQ-caret etc.)

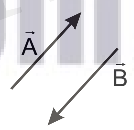
Types of Vector

1. Equal vectors : Two vectors $\vec{A}$ and $\vec{B}$ are said to be equal when they have equal magnitudes and same direction.

2. Parallel vectors : Two vectors $\vec{A}$ and $\vec{B}$ are said to be parallel when
(i) both have same direction.
(ii) one vector is scalar (positive non-zero) multiple of another vector.
Use : If a vector is shifted parallel to itself there will be no change in it.



3. Anti-parallel vectors : Two vectors $\vec{A}$ and $\vec{B}$ are said to be anti-parallel when
(i) both have opposite direction.
(ii) one vector is scalar (non-zero negative) multiple of another vector.
Use : In subtraction of vector.



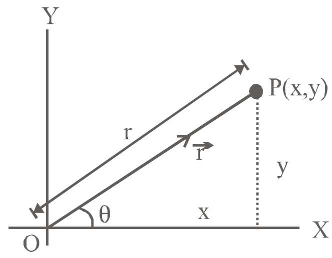
4. Collinear vectors : When the vectors under consideration can share the same support or have a common support then the considered vectors are collinear vectors.

5. Zero or null vector $(\vec{0})$ **:** A vector having zero magnitude and arbitrary direction (not known to us) is a zero vector.
Properties of zero vector :
(i) The sum of a finite vector $\vec{A}$ and the zero vector is equal to the finite vector.
i.e., $\vec{A} + \vec{0} = \vec{A}$
(ii) The multiplication of a zero vector by a finite number n is equal to the zero vector.
i.e., $n\,\vec{0} = \vec{0}$
(iii) The multiplication of a finite vector $\vec{A}$ by zero is equal to zero vector.
i.e., $0\,\vec{A} = \vec{0}$
Examples : The position vector of the origin in a system of coordinates is a zero vector. If an object is stationary, then its displacement in a finite time is a zero vector.

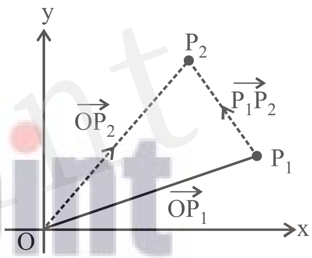
6. Unit vector : A vector divided by its magnitude (= 1)is a unit vector. Unit vector for $\vec{A}$ is $\hat{A}$ (read as A cap or A hat)
Since, $\hat{A} = \frac{\vec{A}}{A} \Rightarrow \vec{A} = A\hat{A}$ thus we can say that unit vector gives us the direction.
Orthogonal unit vectors : $\hat{i}, \hat{j}$ and $\hat{k}$ are called orthogonal unit vectors. These vectors must form a right handed triad (It is a coordinate system such that when we curl the fingers of right hand from x to y then we must get the direction of z along thumb). Thus $\hat{i} = \frac{\vec{x}}{x},\ \hat{j} = \frac{\vec{y}}{y},\ \hat{k} = \frac{\vec{z}}{z} \quad \therefore \quad \vec{x} = x\hat{i},\ \vec{y} = y\hat{j}$ and, $\vec{z} = z\hat{k}$
Use : To convert any magnitude into vector form multiply it by unit vector in that direction.

7. Coplanar vector : Three or more vectors are called coplanar vector if they lie in the same plane. Two (free) vectors are always coplanar.

8. **Free and localised vectors :** A vector whose value depends only on its length and direction and is independent of its position in space, is called a free vector. Such a vector may be shifted, at pleasure, parallel to itself. However, if a vector is associated (bound) with a given line or point, it is said to be localised in that line or at that point.
9. **Coterminous vectors :** Vectors having the same terminating point are called coterminous vectors.
10. **Area vector :** The area-vector of a plane figure is a vector whose magnitude is equal to the area of the plane figure and whose direction is perpendicular to the plane area, as given by R.H. Screw rule. (We will learn rule in vector product)
11. **Position vector $(\vec{r})$:** It specifies the position of a point with reference to the given set of coordinate axes. If we join the origin O with the given point P, then $\overrightarrow{OP}$ is called position vector of P.
 It is generally represented by $\vec{r}\,.\,\vec{r}$ means the magnitude as well as θ i.e., the direction of the point P.
 $\therefore \quad r = |\vec{r}| = \sqrt{x^2 + y^2} \quad$ and $\quad \tan\theta = \dfrac{y}{x}$



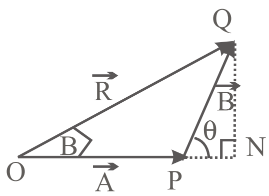
12. **Displacement vector $(\vec{s})$:** If a particle moves from position P_1 to position P_2, we call $\overrightarrow{P_1P_2}$ as displacement vector. Suppose the particle was originally at the origin O. It first moves to P_1, thereafter to P_2, then $\overrightarrow{OP_2}$ is called final displacement vector.
 Symbolically, we state it as $\overrightarrow{OP_1} + \overrightarrow{P_1P_2} = \overrightarrow{OP_2}$ or $\overrightarrow{P_1P_2} = \overrightarrow{OP_2} - \overrightarrow{OP_1}$
 or, Displacement vector = final position of vector – initial position of vector.
 or, Displacement = final position – initial position



LAWS OF VECTOR ADDITION

Triangle Law of Vector Addition

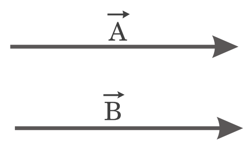
It states that if two vectors acting on a particle at the same time are represented in magnitude and direction by the two sides of a triangle taken in one order, their resultant vector is represented in magnitude and direction by the third side of the triangle taken in opposite order.



Magnitude of $\vec{R}$ is given by $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$
where θ is the angle between $\vec{A}$ and $\vec{B}$.

Direction of $\vec{R}$: Let the resultant $\vec{R}$ make an angle β with the direction of $\vec{A}$, then from right angled triangle QNO,

$$\tan\beta = \frac{QN}{ON} = \frac{QN}{OP + PN} = \frac{B\sin\theta}{A + B\cos\theta}$$



(i) $|\vec{R}|$ is maximum, if $\cos\theta = 1$ i.e., θ = 0º (parallel vectors)

$$R_{max} = \sqrt{A^2 + B^2 + 2AB} = A + B$$

(ii) $|\vec{R}|$ is minimum, if $\cos\theta = 1$ i.e., θ = 180º (opposite vectors)

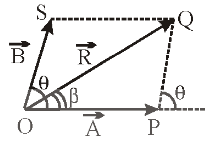
$$R_{min} = \sqrt{A^2 + B^2 - 2AB} = A - B$$

(iii) If the vectors A and B are orthogonal, i.e. $\theta = 90°$, then, $R = \sqrt{A^2 + B^2}$

Parallelogram Law of Vector Addition

It states, that if two vectors are represented in magnitude and direction by the two adjacent sides of a parallelogram then their resultant is represented in magnitude and direction by the diagonal of the parallelogram.

Let the two vectors $\vec{A}$ and $\vec{B}$, inclined at angle θ are represented by sides $\overrightarrow{OP}$ and $\overrightarrow{OS}$ of parallelogram OPQS, then resultant vector $\vec{R}$ is represents by diagonal $\overrightarrow{OQ}$ of the parallelogram.



$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$; and $\tan\beta = \dfrac{B\sin\theta}{A + B\cos\theta}$

If θ < 90º i.e., acute angle then, $\vec{R} = \vec{A} + \vec{B}$ and $\vec{R}$ is called major or main diagonal of parallelogram.

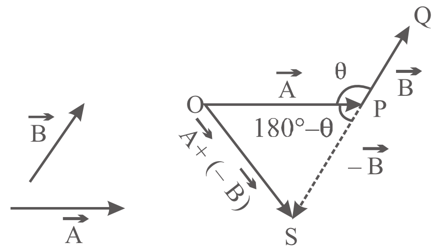
If θ < 90º i.e., obtuse angle then, $\vec{R} = \vec{A} + \vec{B}$ and $\vec{R}$ is called minor diagonal of the parallelogram.

SUBTRACTION OF VECTOR

Subtraction of a vector $\vec{B}$ from a vector $\vec{B}$ is defined as the addition of vector $-\vec{B}$ (negative of vector $\vec{B}$) to vector $\vec{A}$.

Thus $\quad \vec{A} + (-\vec{B}) = \vec{A} - \vec{B}$

If θ is the angle between $\vec{A}$ an $\vec{B}$, then angle between $\vec{A}$ and $-\vec{B}$ is (180º – θ).



(a) (b)

(a) Addition of a vector to its own negative vector or null vector $\vec{A} + (-\vec{A}) = \vec{O}$ i.e., a vector with zero magnitude and an arbitrary direction.

(b) Subtraction is not commutative, i.e., $\vec{A} - \vec{B} \neq \vec{B} - \vec{A}$

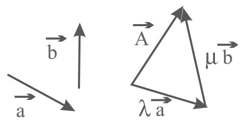
(c) Change in a vector physical quantity means subtraction of initial vector from the final vector.

COMPONENTS OF A VECTOR AND RESOLUTION OF VECTORS

Components of a Vector

If $\vec{a}$ and $\vec{b}$ be any two non-zero vectors in a plane with different directions and be another vector in the same plane then, $\vec{A}$ can be expressed as a sum of two vectors-one obtained by multiplying $\vec{a}$ by a real number and the other obtained by multiplying $\vec{b}$ by another real number.

$\vec{A} = \lambda\vec{a} + \mu\vec{b}$ (where λ and μ are real numbers)



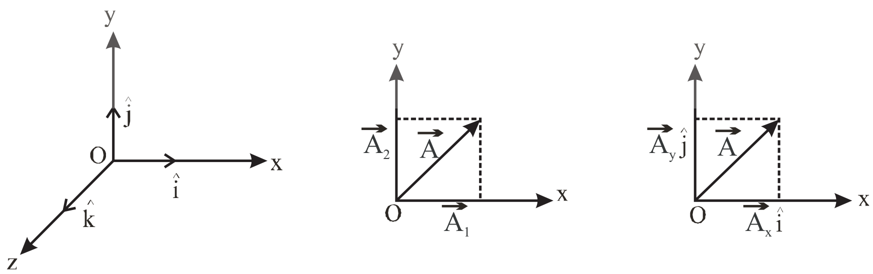
We say that $\vec{A}$ has been resolved into two component vectors $\lambda\vec{a}$ and $\mu\vec{b}$ along $\vec{a}$ and $\vec{b}$ respectively. Hence one can resolve a given vector into two component vectors along a set of two vectors-all the three lie in the same plane.

Resolution of Vectors

The process of splitting a single vector into two or more vectors, which together produce the same effect as is produced by single vector alone, is called resolution of vectors.

The vectors into which the given single vector is splitted are called components of a vector. (Resolution of a vector is just opposite to composition of vectors).

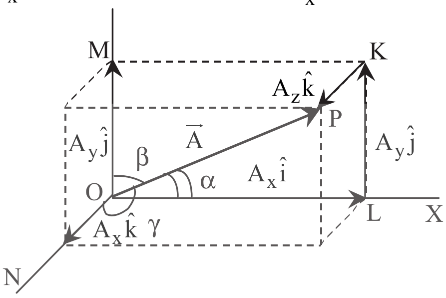
Consider a vector $\vec{A}$ that lies in xy plane, $\vec{A} = \vec{A}_1 + \vec{A}_2$ as shown.



$$\vec{A}_1 = A_x\hat{i},\quad \vec{A}_2 = A_y\hat{j} \quad \Rightarrow \quad \vec{A} = A_x\hat{i} + A_y\hat{j}$$

The quantities A_x and A_y are called x- and y-components of the vector $\vec{A}$. A_x is itself not a vector but $A_x\hat{i}$ is a vector and so is $A_y\hat{j}$.

$$A_x = A\cos\theta \quad \text{and} \quad A_y = A\sin\theta$$



It is clear from above equation that **a component of a vector can be positive, negative or zero depending on the value of θ.**

A vector $\vec{A}$ can be specified in a plane by two ways :

(a) Its magnitude A and the direction θ it makes with the x-axis, or

(b) Its components A_x and A_y.

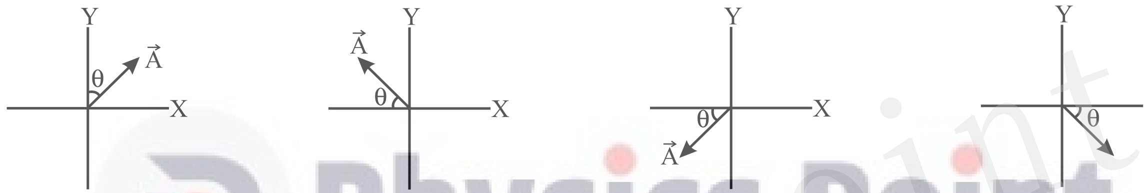
$$A = \sqrt{A_x^2 + A_y^2}, \quad \text{and} \quad \theta = \tan^{-1}\frac{A_y}{A_x}s$$

If $A = A_x \Rightarrow A_y = 0$ and if $A = A_y \Rightarrow A_x = 0$ i.e. components of a vector perpendicular to itself is always zero.

In three dimensions, a vector $\vec{A}$ in components along x, y and z-axis can be written as :

$$\overrightarrow{OP} = \overrightarrow{OB} + \overrightarrow{BP} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BP} \Rightarrow \vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z = \hat{i}A_x + \hat{j}A_y + \hat{k}A_z$$

See following diagram carefully :



X Comp→ +A sinθ	X Comp→ –A sinθ	X Comp→ –A sinθ	X Comp→ +A sinθ
Y Comp→ +A cosθ	Y Comp→ +A cosθ	Y Comp→ –A cosθ	Y Comp→ –A cosθ

ILLUSTRATION : 3

If $0.3\hat{i} + 0.4\hat{j} + c\hat{k}$ is a unit vector then find the value of c.

SOLUTION:

$$\sqrt{0.3^2 + 0.4^2 + c^2} = 1 \Rightarrow 0.09 + 0.16 + c^2 = 1 \Rightarrow c^2 = 1 - 0.25 = 0.75 \Rightarrow c = \sqrt{0.75}$$

ILLUSTRATION : 4

What is the magnitude of the vector (i) $3\hat{i} - 4\hat{j}$ and (ii) $2\hat{i} + 3\hat{j} - 6\hat{k}$?

SOLUTION:

We know that the magnitude of a vector $x\hat{i} + y\hat{j} + z\hat{k}$ is given by $\sqrt{x^2 + y^2 + z^2}$ therefore,

(i) $|3\hat{i} - 4\hat{j}| = \sqrt{(3)^2 + (-4)^2 + (0)^2} = 5$ units

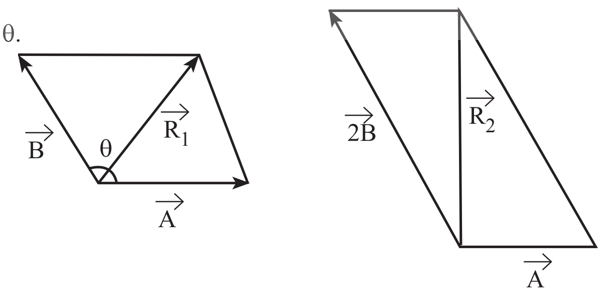
(ii) $|2\hat{i} + 3\hat{j} - 6\hat{k}| = \sqrt{(2)^2 + (3)^2 + (-6)^2} = 7$ units

ILLUSTRATION : 5

Two forces $\vec{A}$ and $\vec{B}$ have resultant $\vec{R}_1$. If $\vec{B}$ is doubled, the new resultant $\vec{R}_2$ is at right angle to $\vec{A}$. Find value of R_1 and R_2 in terms of A and B.

SOLUTION:

Let angle between $\vec{A}$ and $\vec{B}$ be θ, so angle between $\vec{A}$ and $2\vec{B}$ is also θ.



$\Rightarrow \quad R_1^2 = A^2 + B^2 + 2AB\cos\theta$ .... (1)

and $\quad \tan 90° = \dfrac{2B\sin\theta}{A + 2B\cos\theta} \to \infty$

$\Rightarrow \quad A + 2B\cos\theta = 0 \Rightarrow \cos\theta = -\dfrac{A}{2B}$ .... (2)

Substituting value of cos θ in eq. (1), we get, $R_1 = B$

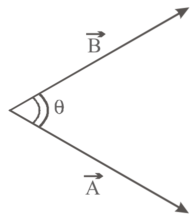
Also, $R_2^2 + A^2 = (2B)^2 = 4B^2 \Rightarrow R_2 = \sqrt{4B^2 - A^2}$

MULTIPLICATION OF VECTORS-SCALAR OR DOT PRODUCT

Scalar or Dot Product

The scalar product or dot product of any two vectors $\vec{A}$ and $\vec{B}$, denoted as $\vec{A}.\vec{B}$ (read as $\vec{A}$ dot $\vec{B}$) is defined as *the product of their magnitude with cosine of angle between them.* Thus,

$\vec{A}.\vec{B} = AB\cos\theta$ (here θ is the angle between the vectors)



Properties of Scalar or Dot Product

(a) It is always a scalar.

(b) It is commutative, i.e., $\vec{A}.\vec{B} = \vec{B}.\vec{A}$

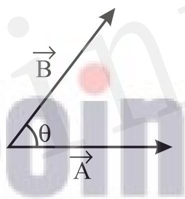
(c) It is distributive, i.e. $\vec{A}.(\vec{B}+\vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$

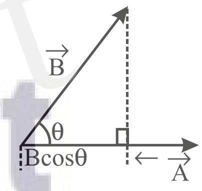
(d) As $\vec{A}.\vec{B} = AB\cos\theta$ therefore,

Angle between the vectors A and B, $\theta = \cos^{-1}\left[\dfrac{\vec{A}.\vec{B}}{AB}\right]$

(e) $\vec{A}.\vec{B} = A\,(B\cos\theta) = B\,(A\cos\theta)$

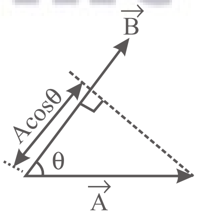
Geometrically, B cos θ is the projection of $\vec{B}$ onto $\vec{A}$ and A cos θ is the projection of $\vec{A}$ onto $\vec{B}$ as shown. So $\vec{A}.\vec{B}$ is the product of the magnitude of $\vec{A}$ and the component of $\vec{B}$ along $\vec{A}$ and vice versa.





Component of $\vec{B}$ along $\vec{A}$ = $B\cos\theta = \dfrac{\vec{A}.\vec{B}}{A} = \hat{A}.\vec{B}$

Component of $\vec{A}$ along $\vec{B}$ = $A\cos\theta = \dfrac{\vec{A}.\vec{B}}{B} = \vec{A}.\hat{B}$



(f) Scalar product of two vectors will be maximum when cos θ = max = 1 = 1, i.e. θ = 0°,

i.e., vectors are parallel $\Rightarrow (\vec{A}.\vec{B})_{max} = AB$

(g) If the scalar product of two non-zero vectors vanishes then the vectors are orthogonal.

(h) The scalar product of a vector by itself is termed as self dot product and is given by

$$(\vec{A})^2 = \vec{A}.\vec{A} = AA\cos\theta = A^2 \Rightarrow A = \sqrt{\vec{A}.\vec{A}}$$

(i) In case of unit vector $\hat{n}$, $\hat{n}.\hat{n} = 1 \times 1 \times \cos 0° = 1$

$\Rightarrow \hat{\mathbf{n}}.\hat{\mathbf{n}} = \hat{\mathbf{i}}.\hat{\mathbf{i}} = \hat{\mathbf{j}}.\hat{\mathbf{j}} = \hat{\mathbf{k}}.\hat{\mathbf{k}} = 1$

(j) In case of orthogonal unit vectors, $\hat{i}$, $\hat{j}$ and $\hat{k}$; $\hat{\mathbf{i}}.\hat{\mathbf{j}} = \hat{\mathbf{j}}.\hat{\mathbf{k}} = \hat{\mathbf{k}}.\hat{\mathbf{i}} = 0$

(k) Scalar product in cartesian coordinates

$\vec{A}.\vec{B} = (\hat{i}A_x + \hat{j}A_y + \hat{k}A_z).(\hat{i}B_x + \hat{j}B_y + \hat{k}B_z) = [A_xB_x + A_yB_y + A_zB_z]$

Example:

(i) Work (W) : In physics for constant force work is defined as, $W = Fs\cos\theta$ ... (1)

But by definition of scalar product of two vectors, $\vec{F}.\vec{s} = Fs\cos\theta$ ... (2)

So from eq. (1) and (2) $W = \vec{F}.\vec{s}$ i.e., work is the scalar product of force and displacement.

(ii) Power (P) : As $W = \vec{F}.\vec{s}$ or $\dfrac{dW}{dt} = \vec{F}.\dfrac{d\vec{s}}{dt}$ (as $\vec{F}$ is constant)

or $P = \vec{F}.\vec{v}$ i.e., power is the scalar product of force with velocity. [as $\dfrac{dW}{dt} = P$ and $\dfrac{d\vec{s}}{dt} = \vec{v}$]

Uses of Scalar Product

It is used to find
1. angle between two vectors.
2. condition for two vectors to be perpendicular to each other.
3. physical quantities based on scalar product.

ILLUSTRATION : 6

Prove that vectors $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{B} = 2\hat{i} - \hat{j}$ are perpendicular to each other.

SOLUTION:

Here, $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$, and $\vec{B} = 2\hat{i} - \hat{j}$

Two vectors are perpendicular to each other if, $\vec{A}.\vec{B} = 0$

Now $\vec{A}.\vec{B} = (\hat{i} + 2\hat{j} + 3\hat{k}).(2\hat{i} - \hat{j})$

$= 1 \times 2 + 2 \times (-1) + 3 \times (0) = 2 - 2 + 0 = 0$

Since $\vec{A}.\vec{B} = 0$, so vectors $\vec{A}$ and $\vec{B}$ are perpendicular to each other.

ILLUSTRATION : 7

Find the angle between the vectors $\vec{A} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{B} = -\hat{i} + 2\hat{j} - \hat{k}$.

SOLUTION:

Here $\vec{A} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{B} = -\hat{i} + 2\hat{j} - \hat{k}$

We know that $\vec{A}.\vec{B} = AB\cos\theta$ or $\cos\theta = \dfrac{\vec{A}.\vec{B}}{AB}$

Now $A = \sqrt{1^2 + 1^2 + (-2)^2} = \sqrt{6}$, $B = \sqrt{(-1)^2 + 2^2 + (-1)^2} = \sqrt{6}$

$\vec{A}.\vec{B} = (\hat{i} + \hat{j} - 2\hat{k}).(-\hat{i} + 2\hat{j} - \hat{k}) = -1 + 2 + 2 = 3$

$\therefore \cos\theta = \dfrac{\vec{A}.\vec{B}}{AB} = \dfrac{3}{\sqrt{6} \times \sqrt{6}} = \dfrac{3}{6} = \dfrac{1}{2} \Rightarrow \theta = 60°$

MULTIPLICATION OF VECTORS-VECTOR OR CROSS PRODUCT

Vector or Cross Product

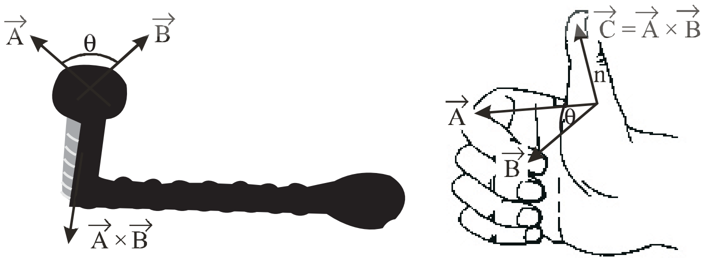
The vector product or cross product of two vectors is defined as a vector having a magnitude equal to the product of the magnitudes of two vectors with the sine of angle between them, and direction perpendicular to the plane containing the two vectors in accordance with right hand screw rule. $\vec{C} = \vec{A} \times \vec{B}$

Thus, if $\vec{A}$ and $\vec{B}$ are two vectors, then their vector product written as $\vec{A} \times \vec{B}$ is a vector $\vec{C}$ defined by

$$\vec{C} = \vec{A} \times \vec{B} = AB\sin\theta\hat{n}$$

Direction of Vector or Cross Product

The direction of $\vec{A} \times \vec{B}$, i.e. $\vec{C}$ is perpendicular to the plane containing vectors $\vec{A}$ and $\vec{B}$ in the sense of advance of a right handed screw rotated from $\vec{A}$ (first vector) to $\vec{B}$ (second vector) through the small angle between them.



Thus, if a right handed screw whose axis is perpendicular to the plane framed by $\vec{A}$ and $\vec{B}$ is rotated from $\vec{A}$ to $\vec{B}$ through the smaller angle between them, then the direction of advancement of the screw gives the direction of $\vec{A}\times\vec{B}$ i.e. $\vec{C}$.

$C \perp A, C \perp B$ Hence, $\vec{C}.\vec{A} = 0$, $\vec{C}.\vec{B} = 0$

Properties of Vector or Cross Product

(i) Vector product of any two vectors is always a vector perpendicular to the plane containing these two vectors, i.e., orthogonal to both the vectors $\vec{A}$ and $\vec{B}$, though the vectors $\vec{A}$ and $\vec{B}$ may or may not be orthogonal.

(ii) Vector product of two vectors is not *commutative*, i.e. $\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$ [but = $-\vec{B}\times\vec{A}$]

Here it is worthnoting that $|\vec{A}\times\vec{B}| = |\vec{B}\times\vec{A}| = AB\sin\theta$, i.e. in case of vector $\vec{A}\times\vec{B}$ and $\vec{B}\times\vec{A}$ magnitudes are equal but directions are opposite.

(iii) The vector product is *distributive* when the order of the vectors is strictly maintained, i.e. $\vec{A}\times(\vec{B}+\vec{C}) = \vec{A}\times\vec{B}+\vec{A}\times\vec{C}$

(iv) The vector product of two vectors will be maximum when $\sin\theta = \max = 1$, i.e., $\theta = 90°$

$[\vec{A}\times\vec{B}]_{max} = AB\sin\theta\ \hat{n}$, i.e., vector product is maximum if the vectors are orthogonal.

(v) The vector product of two non-zero vectors will be minimum when

$$|\sin\theta| = \text{minimum} = 0, \text{ i.e., } \theta = 0° \text{ or } 180° \quad [\vec{A}\times\vec{B}]_{min} = 0$$

i.e., if the vector product of two non-zero vectors vanishes, the vectors are collinear.

(vi) The self cross product, i.e. product of a vector by itself vanishes, i.e., null vector, $\vec{A}\times\vec{A} = AA\sin 0°\ \hat{n} = \vec{0}$.

(vii) In case of unit vector $\hat{n}\times\hat{n} = \vec{0}$ so that $\hat{\mathbf{i}}\times\hat{\mathbf{i}} = \hat{\mathbf{j}}\times\hat{\mathbf{j}} = \hat{\mathbf{k}}\times\hat{\mathbf{k}} = \vec{\mathbf{0}}$

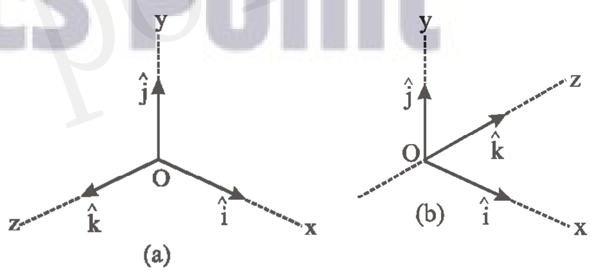
(viii) Vector or cross product of orthogonal unit vectors.

(a) A right-handed coordinate system, in which

$\hat{\mathbf{i}}\times\hat{\mathbf{j}} = \hat{\mathbf{k}}$, $\hat{\mathbf{j}}\times\hat{\mathbf{k}} = \hat{\mathbf{i}}$ and $\hat{\mathbf{k}}\times\hat{\mathbf{i}} = \hat{\mathbf{j}}$.

(b) A left-handed coordinate system, in which

$\hat{i}\times\hat{j} = -\hat{k}$, and so on. We'll use only right-handed systems.



(ix) Vector or cross product in cartesian coordinates

$$A\times B = (A_x\hat{i} + A_y\hat{J} + A_z\hat{k})\times(B_x\hat{i} + B_y\hat{J} + B_z\hat{k})$$

$$= (A_zB_z - A_zB_y)\hat{i} - (A_xB_z - B_xA_z)\hat{J} + (A_xB_y - A_yB_x)\hat{k}$$

Since vector product of two vectors is a vector, vector physical quantities (particularly representing rotational effects) like torque, angular momentum, velocity and force on a moving charge in a magnetic field can be expressed as the vector product of two vectors. It is well-established in physics.

Example:

(i) Torque $\vec{\tau} = \vec{r}\times\vec{F}$

(ii) Angular momentum $\vec{L} = \vec{r}\times\vec{p}$

(iii) Linear velocity $\vec{v} = \vec{\omega}\times\vec{r}$

(iv) Force on a charged particle q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is given by $\vec{F} = q\,(\vec{v}\times\vec{B})$

(v) Torque on a dipole in electric and magnetic field, $\vec{\tau}_E = \vec{p}\times\vec{E}$ and $\vec{\tau}_B = \vec{M}\times\vec{B}$.

(vi) Force on current carrying conductor in magnetic field $\vec{F} = i\,(\vec{\ell}\times\vec{B})$.

Uses of Vector or Cross Product

Vector product or cross product is used to find

1. condition for two vectors to be parallel to each other $\vec{A} \times \vec{B} = 0$
2. physical quantities based on cross product.
3. area of parallelogram/triangle.
   (i) The area of a parallelogram with adjacent sides $\vec{a}$ and $\vec{b}$ is $|\vec{a} \times \vec{b}|$.
   (ii) The area of a parallelogram or a plane quadrilateral with diagonals $\vec{d}_1$ and $\vec{d}_2$ is $\frac{1}{2}|\vec{d}_1 \times \vec{d}_2|$.
   (iii) The area of a triangle with adjacent sides $\vec{a}$ and $\vec{b}$ is $\frac{1}{2}|\vec{a} \times \vec{b}|$.

(iv) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are position vectors of vertices of a Δ ABC, then its area = $\frac{1}{2}|(\vec{a} \times \vec{b}) + (\vec{b} \times \vec{c}) + (\vec{c} \times \vec{a})|$

(v) Three points with position vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are collinear, if $(\vec{a} \times \vec{b}) + (\vec{b} \times \vec{c}) + (\vec{c} \times \vec{a}) = 0$

LLUSTRATION: 8

Considering two vectors, $F = (4\vec{i} - 10\vec{j})$ N and $\vec{r} = (-5\vec{i} - 3\vec{j})$ m. Compute $\vec{r} \times \vec{F}$ and state what physical quantity it represents ?

SOLUTION:

As $\vec{F} = (4\vec{i} - 10\vec{j} + 0\vec{k})$ and $r = (-5\vec{i} - 3\vec{j} + 0\vec{k})$

$$\vec{r} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -5 & -3 & 0 \\ 4 & -10 & 0 \end{vmatrix} \quad \text{i.e.,} \quad \vec{r} \times \vec{F} = \vec{k}\,(50 + 12) = 62\,\vec{k}\ \text{N m}$$

The given physical quantity $\vec{r} \times \vec{F}$ represents torque (i.e., moment of force) if $\vec{F}$ represents force and $\vec{r}$ is the position vector.

ILLUSTRATION: 9

Find the area of the triangle formed by the tips of the vectors $\vec{a} = \hat{i} - \hat{j} - 3\hat{k}$, $\vec{b} = 4\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$.

SOLUTION:

Let *ABC* is the triangle formed by the tips of the given vectors, then

$\overrightarrow{AB} = \vec{b} - \vec{a}$

$= (4\hat{i} - 3\hat{j} + \hat{k}) - (\hat{i} - \hat{j} - 3\hat{k}) = 3\hat{i} - 2\hat{j} + 4\hat{k}$

And $\overrightarrow{AC} = \vec{c} - \vec{a}$

$= (3\hat{i} - \hat{j} + 2\hat{k}) - (\hat{i} - \hat{j} - 3\hat{k}) = 2\hat{i} + 5\hat{k}$

Now $\overrightarrow{AB} \times \overrightarrow{AC} = (3\hat{i} - 2\hat{j} + 4\hat{k}) \times (2\hat{i} + 5\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 4 \\ 2 & 0 & 5 \end{vmatrix} = \hat{i}(-10 - 0) + \hat{j}(8 - 15) + \hat{k}(0 + 4) = -10\hat{i} - 7\hat{j} + 4\hat{k}$$

and $|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(-10)^2 + (-7)^2 + (4)^2} = \sqrt{165} = 12.8$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2} \times 12.8 = 6.4\ m^2$

Chapter 1

MOTION

INTRODUCTION

Motion is everywhere. It is fundamental to our human existence. We need motion for growing, for learning, and for enjoying life.

Like all animals, we rely on motion to get food and to survive dangers. Like all living beings, we need motion to reproduce, to breathe and to digest; like all objects, motion keeps us warm.

From everyday experience we recognise that motion represents continuous change in the position of an object when compared to a non-moving object i.e., refrence point.

Motion in an object can take place in different ways and style. When an object moves in one direction it is called motion in a straight line or motion in one dimension. When an object moves along two directions at the same time, it is called motion in a plane or motion in two dimensions and when an object moves on a circular path, the motion is called circular motion. This chapter deals with the different aspects of the motion.

REST AND MOTION

Rest : An object is said to be at rest if it does not change its position with respect to its surroundings with the passage of time.

Motion : A body is said to be in motion if its position changes continuously with respect to the surroundings (or with respect to an observer) with the passage of time.

We know that earth is rotating about its axis and revolving around the sun. The stationary objects like your class-room, a tree and the lamp posts etc. do not change their position with respect to each other i.e. they are at rest. Although earth is in motion. To an observer situated outside the earth say in a space ship, your classroom, trees etc. would appear to be in motion. Therefore, all motions are relative. There is nothing like absolute motion. If you move with book in your hand, book is not moving with respect to you.

Rest and Motion are Relative Terms

Rest and motion are relative terms. A particle at rest with respect to an observer can be in motion with respect to another observer. To the passengers in a moving bus or train, trees, buildings and people on the roadsides observe that the bus or the train and its passengers are moving in the forward direction. At the same time, each passenger in a moving bus or train finds that fellow passengers are not moving, as the distance between them is not changing.

If you will observe the man moving on moving flat car from ground your observation will be different from what man himself will observe. Similarly, if you will observe pendulum in moving car from ground your observation will be different from what person inside car will observe.

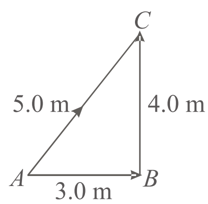
Frame of Reference

To locate the position of object we need a frame of reference. A convenient way to set up a frame of reference is to choose three mutually perpendicular axes and name them *x-y-z* axes. The co-ordinates (*x, y, z*) of the particle then specify the position of object w.r.t. that frame. If any one or more co-ordinates change with time, then we say that the object is moving w.r.t. this frame.

DISTANCE AND DISPLACEMENT

Motion is related to change of position. The length traveled in changing position may be expressed in terms of distance, the actual path length between two points. Distance is a scalar quantity, which has only a magnitude with no direction.

The direct straight line pointing from the initial point to the final point is called displacement (change in position). Displacement only measures the change in position, not the details involved in the change in position. Displacement is a vector quantity, which has both magnitude and direction.



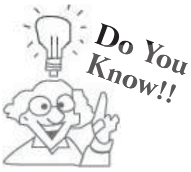
In the figure shown, an object goes from point *A* to point *C* by following paths *AB* and *BC*. The distance travelled is 3.0m + 4.0m = 7.0m, and the displacement is 5.0m in the direction of the arrow A→C.

If one states 'the car has travelled 200m', it means that the distance travelled by the car is 200m. But if one states 'the car has travelled 200m due east' it means that the displacement of the car is 200m towards east.

Distance $\geq$ Displacement

The **displacement can be zero even if the distance is not zero**. For example when a body is thrown vertically upwards from a point on the ground, after sometime it returns back to the same point, then the displacement of the body is zero but the distance travelled by the body is not zero, it is 2*h* if *h* is the maximum height attained by the body.

Similarly, if a body is moving in a circular or closed path and reaches its original position after one rotation, then the displacement in one rotation is zero, but the distance travelled is equal to the circumference of the circular path = $2\pi r$ if r is the radius of the circular path.



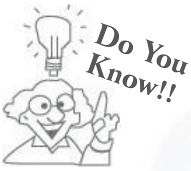
The actual distance travelled by an object in a given time interval can be equal to or greater than sthe magnitude of displacement. It can never be less than the magnitude of displacement.
The displacement of an object in a given time interval can be positive, zero or negative. However, distance covered by the object in a given time interval is always positive.

UNIFORM MOTION AND NON-UNIFORM MOTION

Uniform Motion

It is a motion in which a material point moves in a straight line (rectilinear) and covers equal distances in equal intervals of time.
The path length of a body in a uniform rectilinear motion is equal to the magnitude of the displacement. Consequently, the path length (s) in the motion is equal to the magnitude of the velocity (v) multiplied by the time (t) i.e., $s = vt$.

$$x = x_0 + s = x_0 + vt$$



No force is required to keep an object in uniform motion. When an object has uniform motion along a straight line in a given direction, the magnitude of displacement is equal to actual distance covered.

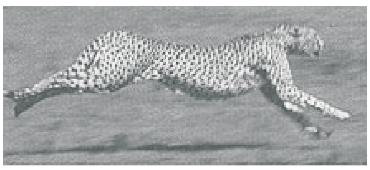
Non-uniform motion

If a body covers unequal distances in equal intervals of time, it is said to be moving with a non-uniform motion.
It is a motion in which the velocity varies with time.
The change in the velocity of a material point in nonuniform motion is characterized by acceleration.
Uniformly variable motion is a motion with a constant acceleration.
Uniformly variable motion can be curvilinear like circular motion.
If a uniformly variable motion is rectilinear, i.e., the velocity v changes only in magnitude, it is convenient to take the straight line in which a material point moves as one of the coordinate axes (say, the x-axis).

MEASURING THE RATE OF MOTION

Average Speed

Average speed is defined as *the total distance traveled divided by the time interval to travel that distance.*
Average speed $V_{av} = \frac{d}{t}$, d is distance travelled, and t is time interval (change in time).
It is a scalar quantity. The average speed of Cheetah is 70 m/s for 30 seconds



Instantaneous Speed

Instantaneous speed is *the speed at a particular time instant* (t is infinitesimal small or close to zero). It is also a scalar quantity.

$$V_{inst} = \underset{dt \to 0}{Lt} \frac{dx}{dt}$$

Knowledge ENHANCER

(i) If a particle covers two consecutive equal distances with speeds v_1 and v_2 then,

$$\text{Average speed} = \frac{2v_1v_2}{v_1 + v_2}$$

(ii) If a particle covers three consecutive equal distances with speeds v_1, v_2 and v_3 then,

$$\text{Average speed} = \frac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}$$

(iii) If a particle has speed v_1 for time t_1 and speed v_2 for time t_2 then,

$$\text{Average speed} = \frac{v_1t_1 + v_2t_2}{t_1 + t_2}$$

Uniform and Non-uniform Speed

A body is said to be moving with uniform speed if it covers equal distances in equal time intervals and with non-uniform or variable speed if covers unequal distances in the same time intervals.

SPEED WITH DIRECTION (VELOCITY)

Average Velocity

Average velocity is defined as *the ratio of change in position or displacement to the time taken.*

$$\bar{v} = v_{av} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

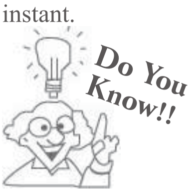
Here x_1 and x_2 are the positions of the particle at time t_1 and t_2 respectively.
Also, $\Delta x = x_2 - x_1$ = change in position and $\Delta t = t_2 - t_1$ = change in time
It is a vector quantity, its unit is ms^{-1}, cms^{-1} or km h^{-1}.

Instantaneous Velocity

Velocity of a body at a particular instant or moment of time is called instantaneous velocity.

Instantaneous velocity $v_{inst} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$ or $v_{inst} = \frac{dx}{dt}$

Instantaneous velocity is a vector quantity, as it has direction as well as magnitude, its unit is m/s, cm/s or km/h.
Remember that the magnitude of instantaneous velocity at an instant would always be equal to the instantaneous speed at that instant.



The velocity of an object may be positive, zero or negative, but the speed of an object can never be negative.

CHECK Point

- **Under what condition is the average velocity equal to instantaneous velocity?**

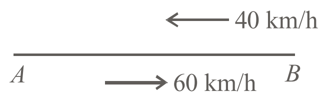
Solution

- When the displacement-time curve is a straight line, *i.e.*, when the body is moving with uniform velocity.

ILLUSTRATION : 1

A particle moved from point A to point B, travelling some distance with speed 60 kmh^{-1}. Then it moves back from B to A with speed 40 km/h. Find displacement, distance covered, average velocity and average speed for the entire journey.

SOLUTION :



Let us take, AB = s

As particle comes back to the same point, **displacement** = 0

Total distance covered = length of path = $2s = 2AB$

Average velocity $\dfrac{displacement}{time} = \dfrac{0}{\Delta t} = 0$ (As displacement is zero)

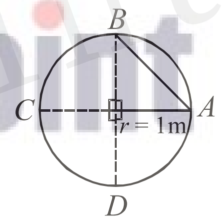
For average speed, total time = $\dfrac{s}{60} + \dfrac{s}{40}$ and total distance = $s + s = 2s$

So, average speed = $\dfrac{2s}{\dfrac{s}{40} + \dfrac{s}{40}} = 48 \text{ km h}^{-1}$

ILLUSTRATION : 2

A particle moves in a circular path of radius 1 m with uniform speed and takes 4 seconds to complete the circular path. Find distance, displacement, average speed and average velocity for : (i) A to B (ii) A to C (iii) A to D and (iv) A to A.

SOLUTION :

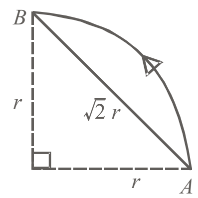


(i) **For A to B :** Distance covered = $\dfrac{2\pi r}{4} = \dfrac{\pi r}{2} = \dfrac{\pi}{2}$ m = 1.57 m

Displacement = shortest distance between A and B

$= \sqrt{2}\, r = \sqrt{2}$ m = 1.41 m

Average velocity = $\dfrac{\text{displacement}}{\text{time}} = \dfrac{\sqrt{2}\text{ m}}{1\text{s}} = \sqrt{2}\ \dfrac{\text{m}}{\text{s}} = 1.41\text{ms}^{-1}$



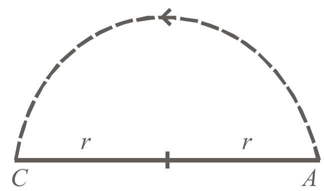
Average speed = $\dfrac{\text{total distance}}{\text{total time}} = \dfrac{\pi/2}{1} = 1.57\text{ms}^{-1}$

(ii) **For A to C :** Distance covered = $\pi r = 3.14 \times 1 = 3.14$ m

Displacement = $2r = 2 \times 1 = 2$ m

Average velocity = $\dfrac{2}{2} = 1\text{ms}^{-1}$

Average speed = $\dfrac{3.14}{2} = 1.57\text{ms}^{-1}$

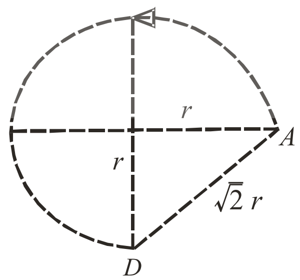


(iii) **For A to D :** Distance covered = $\dfrac{3\pi r}{2} = 4.71$ m

Displacement = $\sqrt{2}\ r = 1.41$ m

Average velocity = $\dfrac{\sqrt{2}}{3} = 0.47\text{ ms}^{-1}$

Average speed = $\dfrac{4.71}{3} = 1.57\text{ ms}^{-1}$



(iv) For A to A : Distance covered = $2\pi r = 6.28$ m

Displacement = 0

Average velocity = 0

Average speed = $\frac{6.28}{4} = 1.57 \text{ ms}^{-1}$

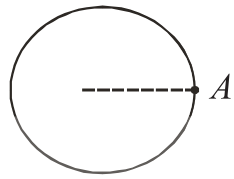


ILLUSTRATION : 3

Displacement x of a particle is given by the equation $x = 3t^2 + 4t + 1$, where x is in metre and t is in second. Find the instantaneous velocity of the particle at (i) $t = 1$s; (ii) $t = 5$s. Also find average velocity of the particle between $t = 1$s and $t = 5$s.

SOLUTION :

We know that, $v_{\text{inst.}} = \frac{dx}{dt}$

So, $v_{\text{inst.}} = (6t + 4)$ ($\because x = 3t^2 + 4t + 1$)

(i) at $t = 1$s, $v_{\text{inst.}} = 6 \times 1 + 4 = 10$ m/s

(ii) at $t = 5$s, $v_{\text{inst.}} = 6 \times 5 + 4 = 34$ m/s

For average velocity we use, $v_{\text{av.}} = \frac{x_2 - x_1}{t_2 - t_1}$

where x_2 = displacement at $t_2 = 5$, $= 3(5)^2 + 4 \times 5 + 1 = 75 + 20 + 1 = 96$ m

x_1 = displacement at $t_1 = 1$, $= 3(1)^2 + 4(1) + 1 = 8$ m

$v_{\text{av.}} = \frac{96 - 8}{5 - 1} = \frac{88}{4} = 22$ m/s

RATE OF CHANGE OF VELOCITY [ACCELERATION]

Average Acceleration

Average acceleration is defined as *the change in velocity divided by the time interval to make the change.*

$\vec{a} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - t_0}$, where $\vec{a}$ is average acceleration, Δv is change in velocity, and Δt is time interval.

Instantaneous Acceleration

Instantaneous acceleration of the particle is the acceleration at particular instant, mathematically, it will be

$$a_{\text{inst.}} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Instantaneous acceleration is also referred to as 'acceleration',

Positive acceleration : If the velocity of an object increases in the same direction, the object has a positive acceleration.

Negative acceleration (Retardation) : If the velocity of a body decreases in the same direction, the body has a negative acceleration or it is said to be retarding e.g, A train slows down.

When velocity of a particle increases with time, it is said to be accelerated motion i.e. both acceleration and velocity will be positive and speed (magnitude of velocity) would increase.

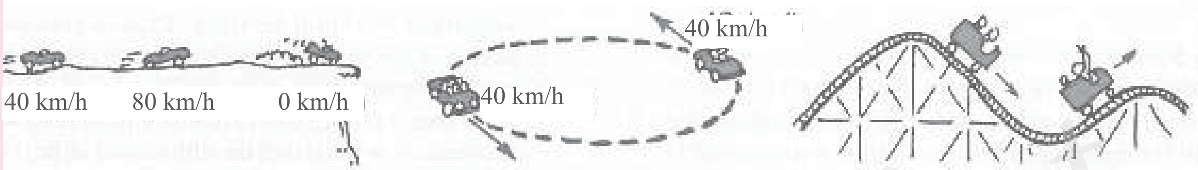
When both acceleration and velocity are negative, that would mean that the direction of motion is in the opposite direction but in this case also speed of particle would increase with time.

If acceleration and velocity are of opposite signs, in that case speed of the particle would decrease. Deceleration is equivalent to negative of acceleration.

idea box

Three ways to accelerate

Acceleration = {Rate of change in velocity} due to {change in speed and/or direction}



Change in speed but *not* direction | Change in direction but *not* speed | Change in speed and dierection

Misconception

A common misconception about velocity and acceleration has to do with their directions. Since velocity has both magnitude and direction, a change in either magnitude (speed) and/or direction will result in a change in velocity, therefore an acceleration. We can accelerate objects either by speeding them up or down (change magnitude) and/or by changing their directions of travel.

For motion in one-dimension, when the velocity and acceleration of an object are in the same direction (they have the same directional signs), the velocity increases and the object speeds up (acceleration). When the velocity and acceleration are in opposite direction, the velocity decreases and the object slows down (deceleration).

CHECK Point

- **Can the velocity of an object ever be in the direction other than the direction of the acceleration of the object?**

Solution

Yes, instantaneous acceleration is independent of instantaneous velocity. So the direction of velocity has no relation to the direction of its acceleration.

idea box

General methods of approaching numerical problems.

1. Draw a 'sketch' diagram wherever possible.
2. Copy down the numerical information given in the question.
3. Write down the relevant formula.
4. Substitute the given values into the formula.
5. Calculate the answer, remembering to show all steps in the working out and giving the correct units for our final answer.

ILLUSTRATION : 4

An object moving to the right has a decrease in velocity from 5.0 m/s to 1.0 m/s in 2.0 s. What is the average acceleration ? What does your result mean ?

SOLUTION :

Given $v_0 = +5.0$ m/s, $v = +1.0$ m/s, $t = 2.0$ s, $\vec{a} = ?$

According to the definition of average acceleration,

$$\vec{a} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t} = \frac{+1.0\text{ m/s} - (+5.0\text{ m/s})}{2.0\text{s}} = \frac{-4.0\text{ m/s}}{2.0\text{ s}} = -2.0\text{ m/s}^2$$

The negative sign means the acceleration is opposite to velocity (deceleration). The result means that the object decreases its velocity by 2.0 m/s every *s* or 2.0 m/s^2.

ILLUSTRATION : 5

A car covers the 1st half of the distance between two places at a speed of 40 km/h and the 2nd half at 60 km/h. What is the average speed of the car ?

SOLUTION :

Suppose the total distance covered is s. Then time taken to cover first half distance with speed 40 km/h, $t1 = \frac{s}{40}$ h.

Time taken to cover second half distance with speed 60 km/h, $t_2 = \frac{s}{60}$ h.

$$\text{Average speed, } v_{av} = \frac{\text{total distance}}{\text{total time}} = \frac{s+s}{\left(\frac{s}{40}+\frac{s}{60}\right)}$$

or $$v_{av} = \frac{2s}{\left(\frac{3s+2s}{120}\right)} = \frac{2s}{5s} \times 120 = 48 km/h$$

ILLUSTRATION : 6

The table below shows the distance (in cm), travelled by the objects *A*, *B* and *C* during each second.

Time	Distance (in cm) covered in each second by *A*, *B* and *C*		
	Object *A*	Object *B*	Object *C*
1st second	20	20	20
2nd second	20	36	60
3rd second	20	24	100
4th second	20	40	140
5th second	20	48	180

(i) Which object is moving with constant speed ? Give a reason for your answer.

(ii) Which object is moving with a constant acceleration ? Give a reason.

(iii) Which object is moving with irregular acceleration ?

SOLUTION :

(i) The object A is moving with constant speed. The reason is that it covers equal distance = 20 cm in each second.

(ii) The object *C* is moving with a constant acceleration. The reason is that for the object *C*, the distance covered increases by the same amount in each second. It can further be verified by drawing graph between *s* (total distance covered) and t^2 (square of time taken). The graph will be a straight line.

t^2	1	4	9	16	25
s	20	80	180	320	500

(iii) The object *b* is moving with irregular acceleration.

EQUATIONS OF MOTION (KINEMATIC EQUATIONS)

Kinematic equations can be used to describe the motion with constant acceleration.

The symbols used in the kinematic are : *v_0 or u initial velocity; v, final velocity; a, acceleration; x, displacement; t, time interval.*

Be aware that the terms initial and final are relative. The end of one event is always the beginning of another. There are three general equations and two algebraic combinations of these equations that provide calculation convenience.

$x = \vec{v}t$, displacement = average velocity × time interval

$\vec{v} = \frac{v+u}{2}$, average velocity = (final velocity + initial velocity) / 2

First equation (Equation for velocity-time relation) :

$v = u + at$, final velocity = initial velocity + acceleration × time interval,

By definition, $\text{acceleration} = \frac{\text{change in velocity}}{\text{time taken}} = \frac{\text{final velocity} - \text{initial velocity}}{\text{time taken}}$

or $a = \frac{v-u}{t} \Rightarrow at = v - u$

or $\boldsymbol{v = u + at}$ .... (1)

Second equation (Equation for position-time relation) : $s = ut + \frac{1}{2}at^2$

Displacement = initial velocity × time interval + $\frac{1}{2}$ × acceleration × time interval^2

Distance travelled = average velocity × time = $\left(\frac{\text{Initial velocity} + \text{final velocity}}{2}\right) \times \text{time}$

or $s = \frac{u+v}{2} \times t$

But from eq. (1), $v = u + at$

$\therefore$ $s = \frac{u + (u + at)}{2} \times t$ or $s = \frac{2u + at}{2} \times t$

or $\boldsymbol{s = ut + \frac{1}{2}at^2}$ .... (2)

Third equation (Equation for position-velocity relation) : $v^2 = u^2 + 2as$

Final velocity^2 = initial velocity^2 + 2 × acceleration × displacement

Distance travelled = average velocity × time

$s = \frac{u+v}{2} \times t$

But from eq. (1), $v = u + at$

$t = \frac{v-u}{a}$ $\therefore$ $s = \frac{u+v}{2} \times \frac{v-u}{a}$

or $s = \frac{v^2 - u^2}{2a}$ $v^2 - u^2 = 2as$

or $\boldsymbol{v^2 = u^2 + 2as}$ .... (3)

The three equations listed can be used to solve the majority of kinematic problems.

Which equation should you select for a particular problem ? The equation you select must have the unknown quantity in it and everything else must be given, because we can only solve for one unknown in one equation.

CONNECTING TOPIC

Distance Covered by a Body in nth Second

$s = ut + \frac{1}{2}at^2$, is the distance covered by a body in t sec

or $s_n = un + \frac{1}{2}an^2$ ....(i) distance covered by a body along straight line in n sec.

$S_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2$ ....(ii) distance covered by a body along straight line in $(n-1)$ sec.

$\therefore$ The distance covered by the body in n^{th} second will be $S_{nth} = S_{n-1}$

$\therefore\ S_{nth} = un + \frac{1}{2}an^2 - \{u(n-1) + \frac{1}{2}a(n-1)^2\}$

$S_{nth} = un + \frac{1}{2}an^2 - \{un - u + \frac{1}{2}a(n+1-2n)\}$

$\Rightarrow\ un + \frac{1}{2}an^2 - \{nu - u + \frac{an^2}{2} + \frac{a}{2} - an\}$

$\Rightarrow\ un + \frac{1}{2}an^2 - nu + u - \frac{an^2}{2} - \frac{a}{2} + an = u + a\left(n - \frac{1}{2}\right) = n + a\left(\frac{2n-1}{2}\right)$

So, the distance covered by body in nth second, $S_{nth} = u + \frac{a}{2}(2n-1)$

ILLUSTRATION : 7

A body covers a distance of 20m in the 7th second and 24m in the 9th second. How much distance shall it cover in 15th second.

SOLUTION :

$s_7\text{th} = u + \frac{a}{2}(2\times 7 - 1)$ but s_7 th = 20 m ....(1)

$\therefore 20 = u + \frac{a}{2}\times 13 \Rightarrow 20 = u + \frac{13a}{2}$

Also $s_9\text{th} = 24\text{m}$

$\therefore\ 24 = u + \frac{17a}{2}$ ....(2)

From equation (1) $u = 20 - \frac{13a}{2}$ ....(3)

Substituting this value in eq (2),

$24 = 20 - \frac{13a}{2} + \frac{17a}{2} \quad \Rightarrow \quad 24 - 20 = \frac{17a}{2} - \frac{13a}{2}$

$4 = \frac{4a}{2} \Rightarrow\ 4 = 2a \Rightarrow a = \frac{4}{2} = 2\ \text{m/s}^2$

Use this value in eq (3),

$u = 20 - \frac{13a}{2}$

$\therefore u = 20 - \frac{13\times 2}{2} \Rightarrow u = 20 - 13\ = 7\ \text{m/s}$

Now, $s_{15}\text{th} = u + \frac{a}{2}(2\times 15 - 1)\ =\ 7 + \frac{2}{2}(29) = 7 + 29 = 36\text{m}$

Tips to solve problems on kinematic equations

1. Make a drawing to represent the situation being studied.
2. Decide which directions are to be called positive (+) and negative (–) relative to a conveniently chosen coordinate origin. Do not change your decision during the course of a calculation.
3. In an organized way, write down the values (with appropriate plus and minus signs) that are given for any of the five kinematic variables (x, a, v, v_0 and t).
4. Before attempting to solve a problem, verify that the given information contains values for at least three of the five kinematics variables.
5. When the motion of an object is divided into segments, remember that the final velocity of one segment is the initial velocity for the next segment.

ILLUSTRATION : 8

An automobile accelerates uniformly from rest to 25 m/s while traveling 100m. What is the acceleration of the automobile ?

SOLUTION :

Given : $v_0 = 0$ (rest), $v = 25$ m/s, $s = 100$m, acceleration a =?

Since $v^2 = v_0^2 + 2as, \Rightarrow \ a = \dfrac{v^2 - v_0^2}{2s} = \dfrac{(25\,\text{m/s})^2 - (0)^2}{2(100\,\text{m})} = 3.1\ \text{m/s}^2$

Since a is positive, it is in the direction of the velocity or motion.

ILLUSTRATION : 9

A car is moving at a speed 50 km/h. After two seconds it is moving at 60 km/h. Calculate the acceleration of the car.

SOLUTION :

Here, $v_0 = 50$ km/h $= 50 \times \dfrac{5}{18}\,\text{m/s} = \dfrac{250}{18}\,\text{m/s}$

$v = 60$ km/h $= 60 \times \dfrac{5}{18} = \dfrac{300}{18}\,\text{m/s}$ and time $t = 2s$

Since $a = \dfrac{v - v_0}{t} = \dfrac{\frac{300}{18} - \frac{250}{18}}{2} = \dfrac{\frac{50}{18}}{2} = \dfrac{50}{36} = 1.39\ m/s^2$

ILLUSTRATION : 10

A body with an initial velocity of 18 km/h accelerates uniformly at the rate of 9 cm s^{-2} over a distance of 200m. Calculate: (i) the acceleration in ms^{-2} (ii) its final velocity in ms^{-1}

SOLUTION :

(i) Acceleration = 9 cm s^{-2} = $\dfrac{9}{100}\,\text{ms}^{-2} = 0.09\ \text{ms}^{-2}$

(ii) Initial velocity u = 18 km h^{-1} = $\dfrac{18000\text{m}}{60 \times 60\text{s}} = 5\ \text{ms}^{-1}$

Acceleration, $a = 0.09$ ms^{-2} and distance $s = 200$m

From equation of motion $v^2 = u^2 + 2as$

$v^2 = (5)^2 + 2 \times 0.09 \times 200$

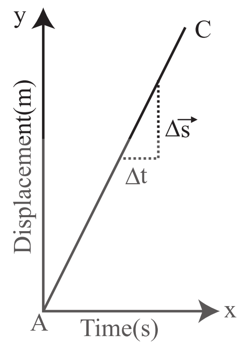
or $v^2 = 25 + 36 = 61 \Rightarrow v = \sqrt{61} = 7.81\ \text{ms}^{-1}$.

Thus, final velocity = 7.81 ms^{-1}.

GRAPHICAL INTERPRETATION OF MOTION IN A STRAIGHT LINE

In physics we often use graphs as important tools for picturing certain concepts. Following are some graphs that help us picture the concepts of displacement, velocity and acceleration.

Displacement-Time Graphs

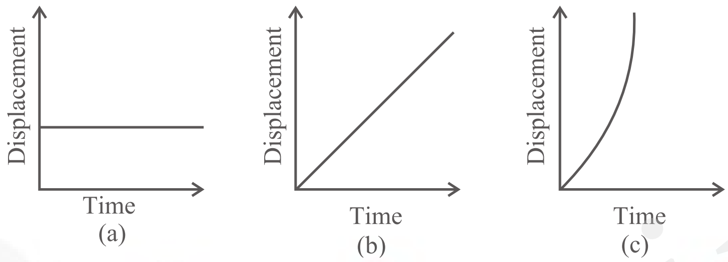


Below is a graph showing the displacement of the cyclist from A to C:
This graph shows us how, in t seconds time, the cyclist has moved from A to C.
We know the gradient (slope) of a graph is defined as the change in y divided by the change in x, i.e $\frac{\Delta y}{\Delta x}$.

In this graph the gradient of the graph is just $\frac{\Delta \vec{s}}{\Delta t}$ and this is just the expression for velocity.

The slope of a displacement-time graph gives the velocity.

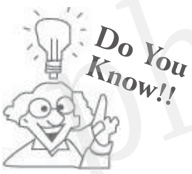
The slope is the same all the way from A to C, so the cyclist's velocity is constant over the entire displacement he travels.
Observe the following displacement-time graphs.



Graph (a) Shows the graph for an object stationary over a period of time. The gradient is zero, so the object has zero velocity.

Graph (b) Shows the graph for an object moving at a constant velocity. You can see that the displacement is increasing as time goes on. The gradient, however, stays constant (remember is the slope of straight line) so the velocity is constant. Here the gradient is positive, so the object is moving in the direction we have defined as positive.

Graph (c) Shows the graph for an object moving at a constant acceleration. You can see that both the displacement and the velocity (gradient of the graph) increases with time. The gradient is increasing with time, thus the velocity is increasing with time and the object is accelerating.

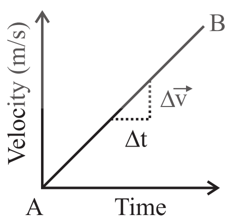


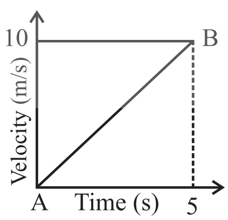
The x-t graph of an object having uniform motion is a straight line inclined to the time-axis. The slope of straight line x-t graph gives velocity of the uniform motion of the object.

Velocity-Time Graphs

This is the velocity-time graph of a cyclist travelling from A to B at a constant acceleration, i.e. with steadily increasing velocity. The gradient of this graph is just $\frac{\Delta \vec{s}}{\Delta t}$ and this is just the expression for acceleration. Because the slope is the same at all points on this graph, the acceleration of the cyclist is constant.

The slope of a velocity-time graph gives the acceleration.





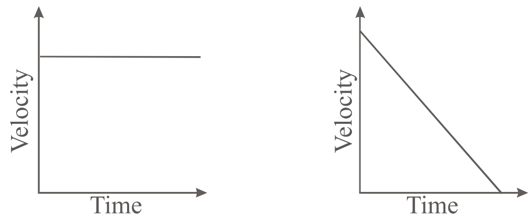
We can also calculate displacement travelled from velocity-time graph.
This graph shows an object moving at a constant velocity of 10m/s for a duration of 5s. The area between the graph and the time

axis of the above plot will give us the displacement of the object during this time. In this case we just need to calculate the area of a rectangle with width 5s and height 10m/s.

Area of rectangle = height × width

$$= \vec{v} \times t = 10 \text{ m/s} \times 5\text{s} = 50 \text{ m} = \text{displacement}$$

Observe the following velocity-time graphs.



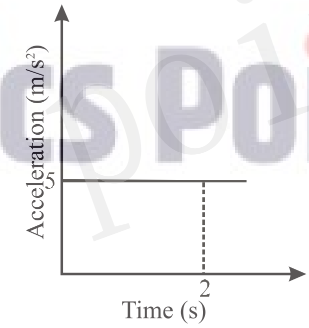
(a) (b)

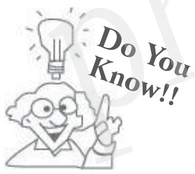
Graph (a) Shows an object moving at a constant velocity over a period of time. The gradient is zero, so the object is not accelerating.

Graph (b) Shows an object which is decelerating. You can see that the velocity is decreasing with time. The gradient, however, stays constant so the acceleration is constant. Here the gradient is negative, so the object is accelerating in the opposite direction to its motion, hence it is decelerating.

We can obtain the velocity of a particle at some given time from an acceleration time graph-it is just given by the area between the acceleration and the time-axis. In the graph below, showing an object at a constant positive acceleration, the increase in velocity of the object after 2 seconds corresponds to the portion.

Area of rectangle $= \vec{a} \times t = 5\dfrac{m}{s^2} \times 2s = 10\dfrac{m}{s} = \vec{v}$

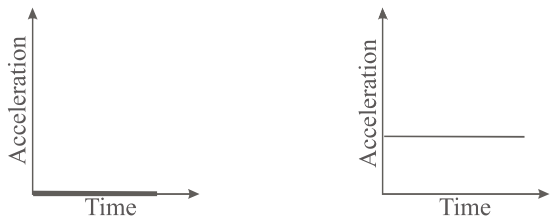




The v-t graph of an object having uniform motion is a straight line parallel to time-axis. The area between v-t graph of an object and time-axis is numerically equal to distance covered by it.

Acceleration-Time Graphs

Observe the following acceleration-time graphs.

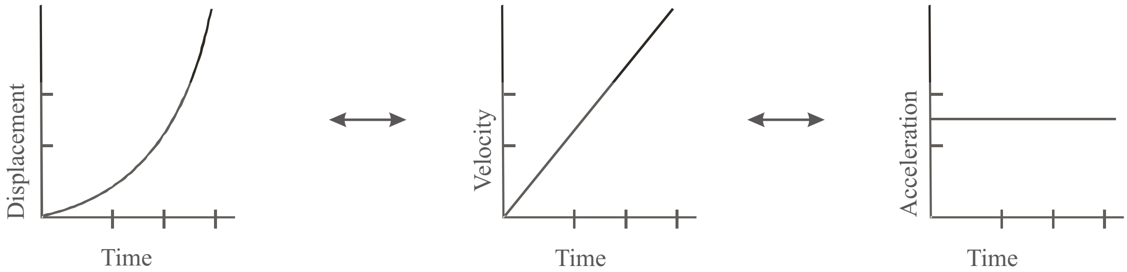


(a) (b)

Graph (a) Shows an object which is either stationary or travelling at a constant velocity. Either way, the acceleration is zero over time.

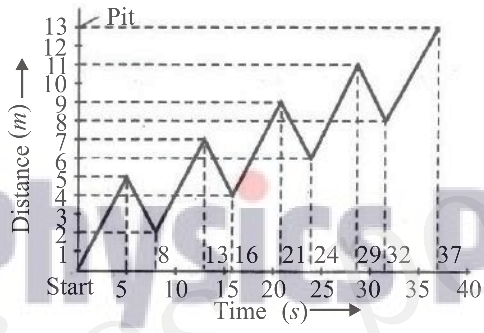
Graph (b) Shows an object moving at a constant acceleration. In this case the acceleration is positive - remember that it can also be negative.

Its useful to remember the set of graphs below when working on problems. Figure shows how displacement, velocity, acceleration and time relate to each other. Given a displacement-time graph like the one on the left, we can plot the corresponding velocity-time graph by remembering that the slope of a displacement-time graph gives the velocity. Similarly, we can plot an acceleration-time graph from the gradient of the velocity-time graph.



CHECK Point

- **A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed again by 5 steps forward and 3 steps backward, and so on. Each step is 1 m long and requires 1 s. Plot the x - t graph of his motion. Determine graphically, how long the drunkard takes to fall in a pit 13m away from the start.**



Solution

Here, length of each step = 1 m
Time required to take one step = 1s
It follows that when the drunkard takes 5 steps forward, he will cover a distance of 5 m in forward direction in a time interval of 5s. On the other hand, when he takes 3 steps backwards, he will cover a distance of 3 m in backward direction in a time interval of 3s. Therefore, x-t graph for the drunkard, till he falls in the pit 13m away will be as shown in fig.
As is evident from the graph, the drunkard will take 37s to fall in the pit.

GRAPHICAL DERIVATION OF EQUATION OF MOTION

(i) First equation (Equation for velocity-time relation)

$$v = u + at$$

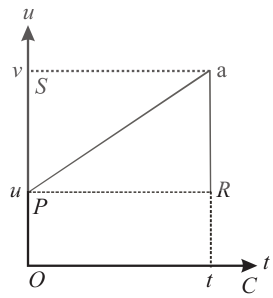
It can be derived from u-t graph, as shown in graph.
From line PQ. The slope of the line = acceleration a

$$a = \frac{QR}{RP} = \frac{SP}{RP}$$

or $\quad SP = a\,RP = at$

As $\quad OS = OP + SP$

Putting values, we get, $v = u + at$



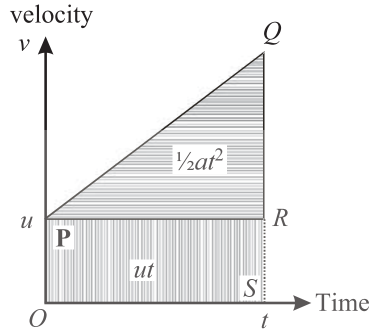
(ii) Second equation (Equation for position-time relation)

$$s = ut + \frac{1}{2}at^2$$

It can also be derived from u-t graph as shown in figure.
Distance covered = area under u-t line
s = area of trapezium $OPQS$

= area of rectangle $OPRS$ + area of triangle PQR

$= OP \times PR + \dfrac{RQ \times PR}{2}$



Putting values, $s = u \times t + \dfrac{1}{2}(v-u) \times t$ $\quad (\because RQ = v - u$ and $PR = OS = t)$

$= u \times t + \dfrac{1}{2} at \times t$ $\quad (\because v - u = at)$

or $\quad \boldsymbol{s = ut + \dfrac{1}{2}at^2}$

(iii) Third equation (Equation for Position-velocity relation)

$v^2 = u^2 + 2as$

From above graph $\quad OP = u,\ SQ = v,\ OP + SQ = u + v$

$a = \dfrac{QR}{PR}$ or $PR = \dfrac{QR}{a} = \dfrac{v-u}{a}$

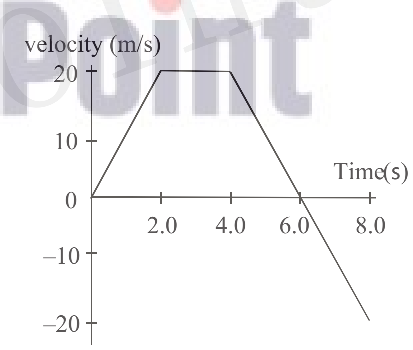
S = area of trapezium OPQS = $\dfrac{OP + SQ}{2} \times PR$

On putting the values $s = \dfrac{v+u}{2} \times \dfrac{v-u}{a} = \dfrac{v^2 - u^2}{2a}$

or $\boldsymbol{v^2 = u^2 + 2as}$

ILLUSTRATION : 11

The graph represents the velocity of a particle as a function of time.



(a) What is the acceleration at 2.0 s ?

(b) What is the acceleration at 3.0 s ?

(c) What is the average acceleration between 0 and 5.0 s ?

(d) What is the average acceleration for the 8.0 s interval ?

(e) What is the displacement for the 8.0 s interval ?

SOLUTION :

(a) Acceleration is the slope of the line

$a = \dfrac{\Delta v}{\Delta t} = \dfrac{20\text{ m/s} - 0\text{ m/s}}{2.0\,s - 0\,s} = 10\text{ m/s}^2$

(b) The slope of the line is zero and hence $a = 0$.

(c) $\vec{a} = \dfrac{\Delta v}{\Delta t} = \dfrac{10\text{ m/s} - 0\text{ m/s}}{5.0\text{ s} - 0\text{ s}} = 2.0\text{ m/s}^2$

(d) $a = \dfrac{\Delta v}{\Delta t} = \dfrac{-20\,m/s - 0\,m/s}{8.0\,s - 0\,s} = -2.5\,m/s^2$

(e) The net area equals the displacement.

The area of a rectangle is length × width and area of a triangle is $\dfrac{1}{2}$ × base × height

$\Delta x_{0-2} = \dfrac{1}{2}(2.0\,s - 0\,s)\,(20\,m/s) = 20\,m$; $\quad \Delta x_{2-4} = (4.0\,s - 2.0\,s)\,(20\,m/s) = 40\,m$

$\Delta x_{4-6} = \dfrac{1}{2}(6.0\,s - 4.0\,s)\,(20\,m/s) = 20\,m$; $\quad \Delta x_{6-8} = \dfrac{1}{2}(8.0\,s - 6.0\,s)\,(-20\,m/s) = -20\,m$

So, $\quad \Delta x = 20\text{m} + 40\text{m} + 20\text{m} + (-20\text{m}) = 60\text{m}$

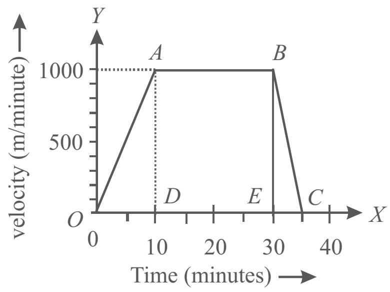
ILLUSTRATION : 12

A train starts from rest and accelerates uniformly at 100m minute^{-2} for 10 minutes. It then maintains a constant velocity for 20 minutes. The brakes are then applied and the train is uniformly retarded. It comes to rest in 5 minutes. Draw a velocity-time graph and use it to find :

(i) the maximum velocity reached
(ii) the retardation in the last 5 minutes
(iii) total distance travelled, and
(iv) the average velocity of the train

SOLUTION :

The velocity-time graph is shown in figure.



$$\text{Acceleration} = \frac{\text{final velocity} - \text{initial velocity}}{\text{time interval}} = \frac{\text{final velocity} - 0}{\text{time interval}}$$

or Final velocity = acceleration × time interval

$$= \frac{100\text{m}}{\text{minute}^2} \times 10 \text{ minute} = 1000\text{m minute}^{-1}$$

(i) The maximum velocity reached = 1000m minute^{-1}.

(ii) The retardation in the last 5 minutes = slope of the line BC

$$= \frac{BE}{EC} = \frac{(0-1000)\text{ m minute}^{-1}}{(35-30)\text{ minute}} = \frac{-1000\text{ m minute}^{-1}}{5\text{ minute}} = -200\text{m minute}^{-2}$$

(iii) Total distance travelled = area of trapezium $OABC$

$$= \frac{1}{2}(OC + AB) \times AD = \frac{1}{2}(35+20)\times 1000 = 55 \times 500 = 27500\text{ m (or 27.5 km)}$$

(iv) Average velocity $= \dfrac{\text{total distance travelled}}{\text{total time of travel}} = \dfrac{27500\text{m}}{35\text{ minute}} = 785.7\text{ m minute}^{-1}$

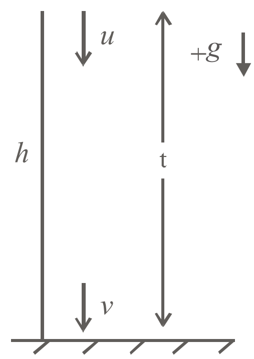
CONNECTING TOPIC

MOTION UNDER GRAVITY

It is a common experience that when a body is dropped form a certain height it experiences acceleration due to gravity and its motion is in a straight path. Similarly, when a body is thrown vertically up, it goes to a certain height and then starts falling again, experiencing acceleration due to gravity throughout the motion. The value of acceleration due to gravity (g) is taken as 9.8 m/s^2, 980 cm/s^2 or 32 ft/s^2. Let us consider the three cases discussed below.

Case-I : Body thrown downward :

In this case, initial motion of the body is downward so according to the sign convention, downward direction will be taken as positive and upward direction as negative. So, the kinematic equations will be :



$$v = u + gt$$

$$h = ut + \frac{1}{2}gt^2$$

$$v^2 = u^2 + 2gh$$

$$h^{\text{nth}} = h + \frac{1}{2}g(2n-1)$$

In a special case when the body is dropped/let fall/allowed to fall we will take the initial velocity(u) as zero, then equation becomes

$$v = gt\ ;\ h = \frac{1}{2}gt^2\ ;\ v^2 = 2gh\ ;\ h^{\text{nth}} = \frac{1}{2}g(2n-1)$$

Case-II : Body thrown upward :

If a body is thrown vertically up with an initial velocity (u).

Hence $a = -g$. Kinematic equations will be:

(i) $v = u - gt$ (ii) $h = ut - \frac{1}{2}gt^2$

(iii) $v^2 - u^2 = -2gh$ (iv) $h_n = u - g\left(n - \frac{1}{2}\right)$

Maximum height reached by the body

From equation $v^2 = u^2 + 2gh$

$$H = \frac{u^2}{2g} \quad [\because v = 0]$$

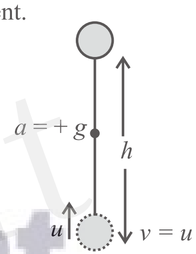
Therefore, the maximum height reached by the body is directly proportional to the square of the initial velocity.

Time of ascent (t_a): The time taken by a body thrown up to reach maximum height 'h' is called its time of ascent.

$$t_a = \frac{u}{g}$$

Hence time of ascent t_a is directly proportional to the initial velocity u.

Time of descent (t_d): The time taken by a freely falling body to reach the ground is called the time of descent.



$$t_d = \sqrt{\frac{2h}{g}}$$

and $$h = \frac{v^2}{2g}, t_d = \sqrt{\frac{2 \times v^2}{2g \times g}} = \frac{v}{g}$$

But, we know that $u = v$ i.e., projected velocity of a body is equal to the velocity of the body on reaching the ground.

$t_d = \frac{u}{g}$ = time of ascent (t_a) $\therefore$ Time of ascent = time of descent

Time of flight : Time of flight is the time for which a body remains in the air and is given by sum of time of ascent and time of descent.

Therefore, $t = t_a + t_d = \frac{u}{g} + \frac{u}{g}$

$\therefore$ Time of flight, $t = \frac{2u}{g}$

Velocity on reaching ground : When a body is dropped from a height h, its initial velocity is zero. Let the final velocity on reaching the ground be v. For a freely falling body.

$v^2 - u^2 = 2gh$

but $u = 0$

$\therefore$ $v^2 - 0 = 2gh$ or, $v = \sqrt{2gh}$

Case-III : Body projected vertically up from the top of a tower :

If a body is projected vertically up from the top of a tower of height 'h' with velocity 'u'. Then

Displacement after time t is $s = ut - \frac{1}{2}gt^2$

Velocity after time t is $v = u - gt$.

Its velocity on reaching the ground is $\sqrt{u^2 + 2gh}$

Its maximum height above the ground is $\{h + (u^2/2g)\}$

ILLUSTRATION : 13

A body is allowed to fall from a height of 98 m. Find the time taken by the body to hit the ground, its velocity before hitting the ground and the distance travelled by it in the last second of motion. (acc. due to gravity, g = 9.8 m/s²)

SOLUTION :

Given $u = 0$ (allowed to fall) ; $h = 98\ m$; $g = 9.8$ m/s^2

Since, the body is falling downward we will use the following equations.

$$h = ut + \frac{1}{2}gt^2$$

$$98 = 0 + \frac{1}{2} \times 9.8 \times t^2$$

$t^2 = 20 \Rightarrow t = 4.47$s (time taken by the body to hit the ground)

For velocity, $v = u + gt = 0 + 9.8 \times 4.47$

$\therefore v = 43.83$ m/s

For distance travelled in the last second of motion

$$h_{nth} = u + \frac{1}{2}g(2n - 1) = 0 + \frac{1}{2} \times 9.8\ (2 \times 4.47 - 1) = 38.91 \text{ m}$$

ILLUSTRATION : 14

A stone is thrown vertically upwards with a speed of 80 m/s, simultaneously another stone is thrown vertically downward from a tower of height 400 m with a speed of 20 m/s. Find when and where the two stones meet. (take $g = 10$ m/s^2)

SOLUTION :

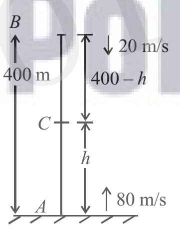
Suppose the two stones meet after time 't' at height h above the ground.

For the stone thrown up : upward direction +ve, downward direction –ve.

so, $h = 80t - \frac{1}{2}gt^2$ ...(i)

For the stone thrown downward : downward direction +ve as it is the initial direction of motion upward direction –ve,

so, $400 - h = 20\,t + \frac{1}{2}gt^2$ ...(ii)



Adding eq. (i) and (ii)

$400 = 100\,t \Rightarrow t = 4$ sec

Substituting $t = 4$s in eq. (i) $h = 80 \times 4 - \frac{1}{2} \times 10 \times 16$

$h = 320 - 80 = 240$ m

MOTION IN A PLANE (OR TWO DIMENSIONAL) AS A COMBINATION OF TWO MOTIONS IN A STRAIGHT LINE

When position of a moving particle can be completely described by two dimensions it is said to be a two dimensional motion. If a particle is at A at $t = 0$ and reaches at B after time 't' experiencing accelerations a_x and a_y along x and y axes respectively then we can use the following equations.

For motion along x-axis

(i) $v_x = u_x + a_x t$

(ii) $x = u_x t + \frac{1}{2}a_x t^2$

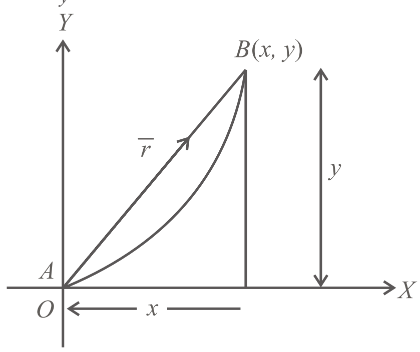
(iii) $v_x^2 = u_x^2 + 2a_x \cdot x$

For motion along y-axis

(i) $v_y = u_y + a_y t$

(ii) $y = u_y t + \frac{1}{2}a_y t^2$

(iii) $v_y^2 = u_y^2 + 2a_y \cdot y$



In vector form :

$\vec{r} = x\hat{i} + y\hat{j}$ $\quad |\vec{r}| = \sqrt{x^2 + y^2}$

$\vec{v} = v_x\hat{i} + v_y\hat{j}$ $\quad |\vec{v}| = \sqrt{v_x^2 + v_y^2}$

Examples of motion in two dimensions are (i) *projectile motion* (ii) *circular motion* and (iii) *relative motion*.

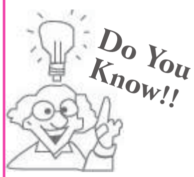
PROJECTILE MOTION

Projectile is the name given to a body thrown with some initial velocity in any arbitrary direction and then allowed to move under the influence of a constant acceleration. The motion of a projectile is called projectile motion.

Example : A football kicked by the player, a stone thrown from the top of building, a bomb released from a plane.

Terms Related to Projectile Motion

1. **Trajectory :** The path followed by a projectile is called its trajectory. Generally, the trajectory of a projectile is parabolic.
2. **Maximum height (H) :** When a projectile moves, it covers a maximum distance in vertical direction. This maximum distance is called the maximum height attained by the projectile.
3. **Horizontal range (R) :** The horizontal distance between the point of projection and the point of landing of a projectile is called its horizontal range.
4. **Time of flight (T) :** The time taken by the projectile to reach the point of landing from the point of projection is called the time of flight.



A projectile returns to ground at the same angle and with the same velocity with which it is projected.

Study of Motion of a Projectile

Let us consider a projectile projected from a point O on a level ground with a velocity v at an angle α to the horizontal, reaching ground level again at a point A.

The different kinematic parameters along horizontal and vertical directions are given below-

Horizontal direction

Initial velocity, $u_x = v\cos\alpha$

Acceleration, $a_x = 0$

Vertical direction

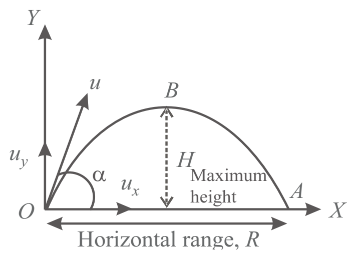
Initial velocity, $u_y = u\sin\alpha$

Acceleration, $a_y = -g$

The time of flight (T): This is the time taken for the particle to travel along its path from O to A.

At any time t, $y = (u\sin\alpha)\,t - \frac{1}{2}gt^2$

When the particle is at A, $y = 0$



Therefore, $(u\sin\alpha)\,t - \frac{1}{2}gt^2 = 0 \Rightarrow t = 0$ or $t = \frac{2u\sin\alpha}{g}$

i.e., *the time of flight* $T = \frac{2u\sin\alpha}{g}$

The maximum height (H) : It is the height at the midpoint of the path.

At any time t, $y = u\sin\alpha - gt$

When the particle is at B, it is moving horizontally, i.e. $y = 0$

So $u\sin\alpha - gt = 0 \Rightarrow t = \frac{u\sin\alpha}{g}$ (Note that this is half the total time of flight)

Then, $y = (u\sin\alpha)t - \frac{1}{2}gt^2$ gives $h = \frac{u^2\sin^2\alpha}{g} - \frac{u^2\sin^2\alpha}{2g}$

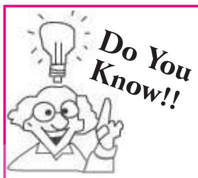
i.e., *the maximum height* $H = \frac{u^2\sin^2\alpha}{2g}$

The horizontal range (R) : This is the distance from the initial position to the final position on a horizontal plane through the point of projection, i.e. OA.

At any time t, $x = ut\cos\alpha$ but at A, $t = \frac{2u\sin\alpha}{g}$

So, for OA, $x = \frac{2u^2\sin\alpha\cos\alpha}{g} = \frac{u^2\sin 2\alpha}{g}$

i.e., the range $R = \frac{u^2\sin 2\alpha}{g}$



When a projectile is at the highest point of its trajectory,

(i) it possesses velocity only along horizontal and

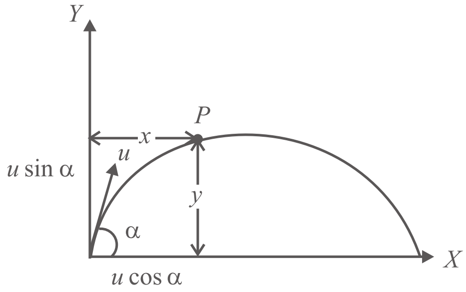
(ii) the velocity and acceleration of the projectile are perpendicular to each other.

Equation of trajectory : Let us say particle is at P at any time t.

$x = u\cos\alpha\, t,$

$$y = u\sin\alpha\, t - \frac{1}{2}gt^2 = u\sin\alpha \frac{x}{u\cos\alpha} - \frac{1}{2}g\frac{x^2}{u^2\cos^2\alpha}$$

$$y = x\tan\alpha - \frac{gx^2}{2u\cos^2\alpha}$$



Equation is of type $y = ax - bx^2$ where $a = \tan\alpha$; $b = \dfrac{g}{2u^2\cos^2\alpha}$

This represents a parabola.

Another form of the equation of trajectory : Multiply by sin α in numerator and denominator in second term

$$y = x\tan\alpha - \frac{gx^2\sin\alpha}{2u^2\cos\alpha\sin\alpha\cos\alpha} = x\tan\alpha - \frac{x^2\tan\alpha}{\left(\frac{2u^2\cos\alpha\sin\alpha}{g}\right)}$$

$$y = x\tan\alpha - \frac{x^2\tan\alpha}{R} = x\tan\alpha\left(1 - \frac{x}{R}\right)$$

Various Common Results

1. **Velocity at any time t :** Let us say particle projected with a velocity u is at P at any time t. To calculate velocity at P, calculate its x and y component.

In projectile motion x-component of velocity remains constant

$v_x = u\cos\theta$, $v_y = u\sin\theta - gt$

$$\vec{v} = v_x\hat{i} + v_y\hat{j} = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\,\hat{j}$$

Magnitude : $|\vec{v}| = \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2} = \sqrt{u^2\cos^2\theta + u^2\sin^2\theta + g^2t^2 - 2ug\,t\sin\theta} = \sqrt{u^2 + g^2t^2 - 2ugt\sin\theta}$

Direction : $\tan\alpha = \dfrac{v_y}{v_x} = \dfrac{u\sin\theta - gt}{u\cos\theta}$ $[\alpha \neq \theta]$

2. **Velocity at any position $P(x, y)$:** $v_x^2 = u^2\cos^2\theta$, $v_y^2 = u^2\sin^2\theta - 2gy$ $[\because v^2 = u^2 + 2gh]$

$$v^2 = v_x^2 + v_y^2 = u^2\cos^2\theta + u^2\sin^2\theta - 2gy = u^2 - 2gy$$

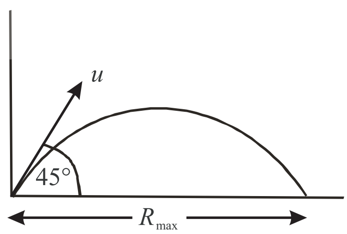
$\Rightarrow v = \sqrt{u^2 - 2gy}$ independent of x co-ordinate.

3. **Maximum range :** For given speed, range varies with angle of projection. It is maximum at θ = 45º. $R = \dfrac{u^2\sin 2\theta}{g}$

$u \to$ constant, $R \to$ maximum , $\sin 2\theta \to$ maximum

$2\theta = 90^\circ \Rightarrow \theta = 45^\circ$

In many practical applications (like javelin throw, shot put etc.), the initial and final elevations may not be equal, and other consideration are important. For example, in the shot put, the ball ends its flight when it hits the ground, but it is projected from an initial height of about 2m above the ground. This causes the range to be maximum at an angle somewhat lower than 45º (app. 42º in practice). When calculating the range of artillery shells, air resistance must be taken into account to predict the range accurately. As expected, air resistance reduces the range for a given angle of projection. It also decreases the optimum angle of projection slightly. If the initial and final elevations were the same, the 45º trajectory would have the greater range.



idea box

When a projectile is thrown at 45° with horizontal,
(i) its range is maximum and is equal to u^2/g.
(ii) the maximum height attained by the projectile is equal to one fourth of its maximum range i.e. equal to $u^2/4g$.

4. **Two angles of projection for same range :** If speed is constant then for angle of projection $\theta_1, R_1 = \frac{u^2 \sin 2\theta_1}{g}$; and for angle of projection $\theta_2, R_2 = \frac{u^2 \sin 2\theta_2}{g}$; $R_1 = R_2$

$\sin 2\theta_1 = \sin 2\theta_2$

$\sin 2\theta_1 = \sin(180° - 2\theta_2)$

$2\theta_1 = 180° - 2\theta_2 \Rightarrow \theta_1 = 90° - \theta_2 = \theta_1 + \theta_2 = 90°$

Thus for given u there are two angle of projection whose sum is 90º for same range,

like $\theta, 90º - \theta \Rightarrow$ 30º, 60º ; $45º - \theta, 45º + \theta$

For these two angles of projection range is same but maximum height and time of flight will be different.

CHECK Point

1. **As a projectile moves in its parabolic path, is there any point along its path where the velocity and acceleration vectors are (a) perpendicular to each other? (b) parallel to each other?**
2. **In the absence of air resistance, why does the horizontal component of a projectile's motion not change, while the vertical component does?**

Solution

1. (a) At the top of its flight, v is horizontal and a is vertical. This is the only point at which the velocity and acceleration vectors are perpendicular.
   (b) If the object is thrown straight up or down, then v and a will be parallel throughout the downward motion. Otherwise, the velocity and acceleration vectors are never parallel.
2. In the absence of air resistance, the force acting on the projectile is only the force of gravity acting vertically downward. No component of the force due to gravity acts in the horizontal direction therefore the horizontal component of the projectile motion remains unchanged.
   Whereas the vertical component of motion undergoes changes. The vertical component of motion decreases while rising against gravity and increases during downward motion.

ILLUSTRATION : 15

A ball is thrown at an angle of 30º with horizontal with a speed of 30 ms^{-1}. Calculate maximum height, time of flight and horizontal range. (Use $g = 10$ m/s^2).

SOLUTION :

Given $u = 30$ ms^{-1} : $\theta_0 = 30º$

Using $h = \frac{u^2 \sin^2 \theta_0}{2g}$; maximum height, $h = \frac{(30)^2 \times \sin^2 30º}{2 \times 10} =$ 11.25 m

Time of flight, $T = \frac{2u \sin \theta_0}{g} = \frac{2 \times 30 \times \sin 30º}{10} = 3$ sec

Horizontal range, $R = \frac{u^2 \sin 2\theta_0}{g} = \frac{(30)^2 \times \sin 60º}{10} = 77.85$ m

ILLUSTRATION : 16

A body is projected at an angle of 45º if its horizontal range is 400 m, find corresponding maximum height.

SOLUTION :

At 45º a projectile has maximum range (R) and the corresponding height is, $h = \frac{R}{4}$; $h = \frac{400}{4} = 100\,\text{m}$

ILLUSTRATION : 17

A particle projected with some velocity at an angle 30º with horizontal and another particle with the same speed but an angle of 60º. Find the ratio of the range and maximum height.

SOLUTION :

As 30º and 60º are complimentary angles their horizontal range will be the same so, $\frac{R_1}{R_2} = 1:1$

For vertical heights, we can use $\frac{H_1}{H_2} = \tan^2\theta$, where $\theta = 30°$

So, $$\frac{H_1}{H_2} = \tan^2 30° = \frac{1}{3} = 1:3$$

ILLUSTRATION : 18

A body, when projected at angles of 30º and 60º the corresponding time of flights are 10s and 20s, respectively find horizontal range of projectile. (Use g = 10 m/s^2)

SOLUTION :

Remember, $R = \frac{gT_1T_2}{2}$ where T_1 and T_2 are time of flights

$$R = \frac{10\times10\times20}{2} = 1000\text{ m}$$

ILLUSTRATION : 19

A staircase contains 3 steps each 10 cm high and 20 cm wide. What should be the minimum horizontal velocity of a ball rolling off the uppermost plane so as to hit the lowest plane?

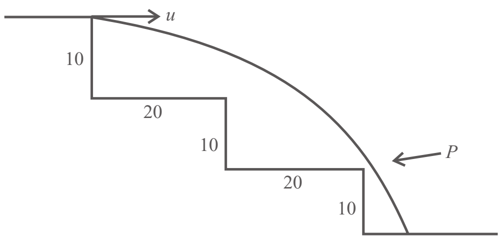
SOLUTION :

In order to hit the lower most plane and 'u' be the minimum, let us assume that the ball almost touches the 2nd staircase and then hits the 3rd step.

So, $h = 2 \times 10 = 20$ cm $= 0.2$ m
and $x = 2 \times 20 = 40$ cm $= 0.4$ m

Now using $h = \frac{1}{2}\frac{gx^2}{u^2}$

$$0.2 = \frac{1}{2}\frac{10\times0.4\times0.4}{u^2} \Rightarrow u = 2\,\text{m/s}$$



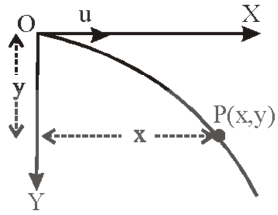
Note : *If y is height of each step and x is width then the minimum speed for a ball to hit the n*th *step is given by* $u = \sqrt{\frac{g(n-1)x^2}{2y}}$.

HORIZONTAL PROJECTILE MOTION

When a body is projected horizontally from a certain height 'y' vertically above the ground with initial velocity u. If friction is considered to be absent, then there is no other horizontal force which can affect the horizontal motion. The horizontal velocity therefore remains constant and so the object covers equal distance in horizontal direction in equal intervals of time.

Trajectory of horizontal projectile motion

The horizontal displacement x is governed by the equation



$x = ut$ ... (1)

$\Rightarrow t = \frac{x}{u}$

The vertical displacement y is governed by

$y = -\frac{1}{2}gt^2$ ... (2) (since initial vertical velocity is zero)

By substituting the value of t in equation (2) $y = -\frac{1}{2}\frac{gx^2}{u^2}$ (– (ve) sign, as curve is in 4^{th} quadrant)

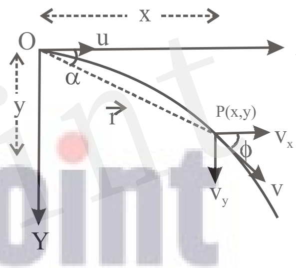
Displacement of Projectile ($\vec{r}$)

After time t, horizontal displacement x = ut and vertical displacement $y = \frac{1}{2}gt^2$

So, the position vector, $\vec{r} = ut\hat{i} + \frac{1}{2}gt^2\hat{j}$

$$\mathbf{r = ut\sqrt{1+\left(\frac{gt}{2u}\right)^2}} \quad \text{and} \quad \mathbf{á = tan^{-1}\left(\frac{gt}{2u}\right)}$$

Therefore, $\alpha = \tan^{-1}\left(\frac{\sqrt{2gy}}{u}\right)\left(\text{as } t = \sqrt{\frac{2y}{g}}\right)$



Instantaneous velocity

Throughout the motion, the horizontal component of the velocity is $v_x = u$.

The vertical component of velocity increases with time and is given by

$v_y = 0 + gt = gt$ (from v = u + gt)

So, $\vec{v} = v_x\hat{i} + v_y\hat{j} = u\hat{i} + gt\hat{j}$, i.e., $v = \sqrt{u^2 + (gt)^2} = u\sqrt{1+\left(\frac{gt}{u}\right)^2}$

Again, $\vec{v} = u\hat{i} + \sqrt{2gy}\hat{j}$ i.e., $\mathbf{v = \sqrt{u^2 + 2gy}}$

Direction of instantaneous velocity

$\tan\phi = \frac{v_y}{v_x} \Rightarrow \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{\sqrt{2gy}}{u}\right)$ or $\mathbf{\phi = tan^{-1}\left(\frac{gt}{u}\right)}$

Where φ is the angle of instantaneous velocity from the horizontal.

Time of flight : If a body is projected horizontally from a height *h* with velocity *u* and time taken by the body to reach the ground is T, then

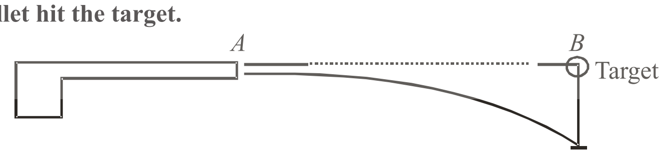
$$h = 0 + \frac{1}{2}gT^2 \Rightarrow \mathbf{T = \sqrt{\frac{2h}{g}}} \quad \text{(for vertical motion)}$$

Horizontal range : Let R is the horizontal distance travelled by the body

$$R = uT + 0T^2 \Rightarrow \mathbf{R = u\sqrt{\frac{2h}{g}}} \quad \text{(for horizontal motion)}$$

CHECK Point

- **A bullet is fired horizontally towards a target directly in front of the gun. The target starts falling freely the moment bullet is fired, will the bullet hit the target.**



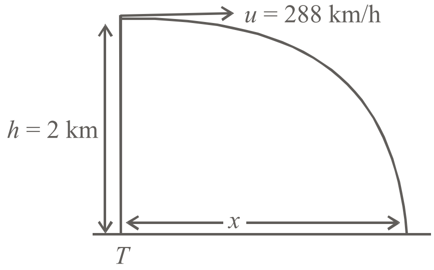
Solution

Suppose the bullet takes '*t*' time to reach to the point *B* after being fired from the gun at *A*. Both the bullet and the target have their initial vertical velocity as zero. The vertical distance covered by both the bullet and the falling target will be $\frac{1}{2}gt^2$. Hence the bullet will surely hit the target.

ILLUSTRATION : 20

An aeroplane flying horizontally at an altitude of 2 km with a speed of 288 km/h. When it is just above a target it drops a bomb find by what distance, the bomb will miss the target and the time taken by it to hit the ground. (Take g = 10 m/s)

SOLUTION :



Here, $u = 288$ km/h $= 288 \times \frac{5}{18} = 80$ m/s

$h = 2$ km $= 2000$ m

Using, $h = \frac{1}{2} g \frac{x^2}{u^2}$ $\quad x^2 = \frac{2hu^2}{g} = \frac{2 \times 2000 \times 80 \times 80}{10}$

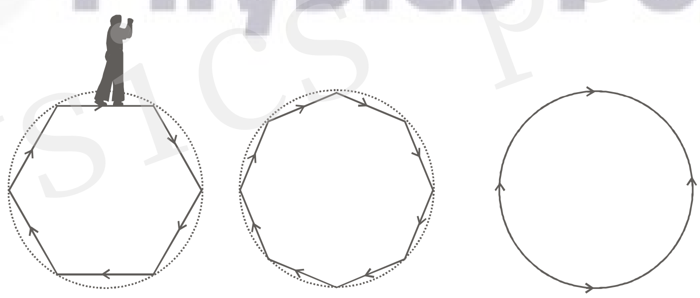
i.e. the bomb will miss the target by a distance $x = 1600$ m

For time $t = \frac{x}{u}$; $\quad t = \frac{1600}{80} = 20$ sec.

CIRCULAR MOTION

Motion of a particle (small body) along a circle (circular path), is called a circular motion. *If the body covers equal distances along the circumference of the circle, in equal intervals of time, the motion is said to be a **uniform circular motion***. A uniform circular motion is a motion in which speed remains constant but direction of velocity changes.

Explanation : Consider a boy running along a regular hexagonal track (path) as shown in fig. As the boy runs along the side of the hexagon at a uniform speed, he has to take a turn at each corner changing direction but keeping the speed same. In one round he has to take six turns at regular intervals. If the same boy runs along the side of a regular octagonal track with same uniform speed, he will have to take eight turns in one round at regular intervals but the interval, will become smaller.

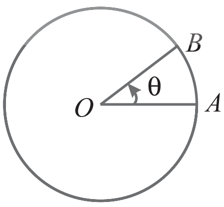


By increasing the number of sides of the regular polygon, we find the number of turns per round becomes more and the interval between two turns become still shorter. A circle is a limiting case of a polygon with an infinite number of sides on the circular track, the turning becomes a continuous process without any gap in between the boy running along the sides of such a track will be performing a circular motion. Hence, circular motion is the motion of a body along the sides of a polygon of infinite number of sides with uniform speed, the direction changing continuously.

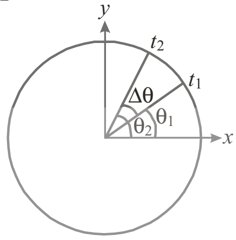
Examples of uniform circular motion are (i) motion of moon around the earth. (ii) motion of satellite round its planet.

Angular Position, Displacement and Velocity

Angular position : Angle made by radius to the particle with a reference radius is known as angular position. In figure θ is known as angular position. *OA* is known as reference radius, with respect to which the angle is measured.



Angular displacement : Angular displacement is defined as the angle traced out by the radius vector at the axis of the circle in a given time. $\theta_1 \rightarrow$ angular position at time t_1, $\theta_2 \rightarrow$ angular position at time t_2. Time taken $\Delta t = t_2 - t_1$.



Here $\Delta\theta = \theta_2 - \theta_1$ is known as angular displacement. It is a vector quantity, provided $\Delta\theta$ is small because commutative law of vector addition is not valid for large $\Delta\theta$.
Its units are degree, radian revolution, etc.

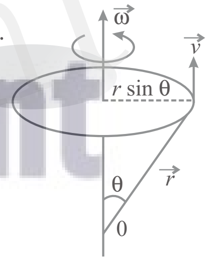
Angular velocity: It is defined as the rate of change of angular displacement. If $\Delta\theta$ be the change in angular displacement during time Δt then angualr velocity (ω) is given by $\omega = \frac{\Delta\theta}{\Delta t}$

Unit of angular velocity : radian/second (rad/s), rotations per minute (rpm), rotation per second (rps), etc.

Some Terms Related to Circular Motion

Time period : It is defined as the time taken by the particle to complete one revolution on its circular path. It is denoted by T.

Frequency : It is defined as number of revolutions completed by particle in its circular path in unit time. It is denoted by f. Its unit is s^{-1} or Hz.



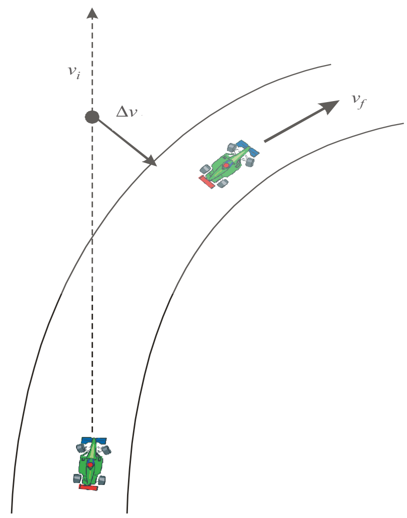
Relation between time period and frequency : $T = \frac{1}{f}$

Relation between ω, f and T : $\omega = \frac{2\pi}{T} = 2\pi f$

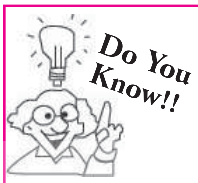
idea box

Nothing will follow a circular path unless it is forced to.

Acceleration and Force in Uniform Circular Motion



Any change of velocity-speeding up, slowing down, or turning a corner is an acceleration. We distinguish these different kinds of accelerated motion by assigning a direction to the acceleration. In other words, acceleration is a vector. If you are traveling in a straight line and speed up, your change in velocity $(\Delta\vec{v})$ is in the direction you are going. Then your acceleration is also in that direction. This is the first rule for the direction of an acceleration : when the acceleration is in the same direction as the velocity, the result is an increase in speed.
Now suppose you do not change speed but are making a turn. This is a change in only the direction of velocity, since velocity is a vector, this is surely a change in velocity-an acceleration. Figure shows how to find the direction of this acceleration. The dotted line shows what the path of the car would be if its motion were not accelerated that is, if it traveled at constant speed in a straight line.
The arrow is a vector representing the change in the velocity of the car, that is, the vector that must be added to the old velocity to get the new velocity.



1. *If circular motion of the object is uniform, the object will possess only centripetal acceleration.*
2. *If circular motion of the object is non-uniform, the object will possess both centripetal and transverse accelerations.*

The acceleration is in the same direction as the change in velocity. When the car turns to the right, its acceleration is to the right, perpendicular to its velocity, this acceleration is called ***centripetal acceleration*** and its value is $\frac{v^2}{r}$ where v is the magnitude of the velocity and r is the radius of the path.

If m be the mass of the object then it experiences a force which directs towards the centre of the path and has a magnitude given by

$$F_c = \frac{mv^2}{r}$$

This force is known as the **centripetal force.**

ILLUSTRATION : 21

A grinding wheel (radius 7.6 cm) is rotating at 1750 rpm.

(a) What is the speed of a point on the outer edge of the wheel ?

(b) What is the centripetal acceleration of the point ?

SOLUTION :

(a) Speed, $v = \frac{2\pi r}{T}$

The period of motion is $T = (1/1750)(60\text{s}) = 3.43 \times 10^{-2}$ s

So, $v = \frac{2\pi\,(7.6\times10^{-2}\,\text{m})}{3.43\times10^{-2}\,\text{s}} = 14\ \text{m/s}$

(b) Centripetal acceleration $a_C = \frac{v^2}{r} = \frac{14\ \text{m/s}^2}{7.6\times10^{-2}\,\text{m}} = 2.6\times10^{3}\,\text{m/s}^2$

ILLUSTRATION : 22

Rishabh, a 20 kg child on his bicycle moving with a speed of 10 ms^{-1} takes a turn on a circular turning of radius 20 m. Calculate (i) the centripetal acceleration and (ii) the centripetal force acting on Rishabh.

SOLUTION : Here, $m = 20$ kg ; $v = 10$ ms^{-1} ; $r = 20$ m

(i) Centripetal acceleration, $a_c = \frac{v^2}{r} = \frac{(10)^2}{20} = \frac{100}{20} = 5\,\text{ms}^{-2}$

(ii) Centripetal force, $F_c = \frac{mv^2}{r} = ma = (20 \times 5) = 100$ N

RELATIVE VELOCITY

The relative velocity of an object B w.r.t. object A when both are in motion is the rate of change of position of object B w.r.t. object A.

Relative velocity of object B w.r.t. object A,

$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$

Relative velocity of object A w.r.t. object B,

$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$

i.e., $\vec{V}_{AB} = -\vec{V}_{BA}$

When both the object A and B move in the same direction

$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$

When the object B moves in the opposite direction of A

$\vec{V}_{AB} = \vec{V}_A + \vec{V}_B$

Same for relative acceleration.

When $\vec{V}_A$ and $\vec{V}_B$ are inclined to each other at angle θ.

$$\vec{V}_{AB} = \sqrt{V_A^2 + V_B^2 + 2V_A V_B \cos(180^o - \theta)}$$

$$= \sqrt{V_A^2 + V_B^2 - 2V_A V_B \cos\theta}$$

If V_{AB} makes angle α with V_A, then

$$\tan\theta = \frac{V_B \sin\theta}{V_A - V_B \cos\theta}$$

Relative velocity concept is very useful to solve :

(1) River-boat problems
(2) Aircraft-wind problems
(3) Rain-man problems

River-Boat Problems

In river-boat problems we come across the following three terms :

$\vec{v}_r$ = absolute velocity of river.

$\vec{v}_{br}$ = velocity of boatman with respect to river or velocity of boatman in still water

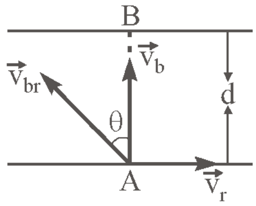
and $\vec{v}_b$ = absolute velocity of boatman.

Hence, it is important to note that $\vec{v}_{br}$ is the velocity of boatman with which he steers and $\vec{v}_b$ is the actual velocity of boatman relative to ground.Further,

$\vec{v}_b = \vec{v}_{br} + \vec{v}_r$

Now, let us derive some standard results and their special cases.

A boatman starts from point A on one bank of a river with velocity $\vec{v}_{br}$ in the direction shown in figure. River is flowing along positive x-direction with velocity $\vec{v}_r$. Width of the river is d. Then



$$\vec{v}_b = \vec{v}_r + \vec{v}_{br}$$

Therefore, $v_{bx} = v_{rx} + v_{brx} = v_r - v_{br} \sin\theta$

and $v_{by} = v_{by} + v_{bry} = 0 + v_{br}\cos\theta = v_{br}\cos\theta$

Now, time taken by the boatman to cross the river is :

$$t = \frac{d}{v_{by}} = \frac{d}{v_{br}\cos\theta} \quad \text{or} \quad t = \frac{d}{v_{br}\cos\theta} \quad \text{........... (i)}$$

Further, displacement along x-axis when he reaches on the other bank (also called drift) is

$$x = v_{bx}\ t = (v_r - v_{br}\sin\theta)\frac{d}{v_{br}\cos\theta}$$

or $$x = \left(v_r - v_{br}\sin\theta\right)\frac{d}{v_{br}\cos\theta} \quad \text{........... (ii)}$$

Special Cases:

(i) When the boatman crosses the river in shortest interval of time.

From eq. (i) we can see that time (t) will be minimum when θ = 0º, i.e., the boatman should steer his boat perpendicular to the river current.

$$t_{min} = \frac{d}{V_{br}} \quad \text{as } \cos\theta = 1$$

(ii) When the boatman wants to reach at a point just opposite (B) from where he started (shortest distance)

In this case, the drift (x) should be zero.

$$x = 0$$

or $(v_r - v_{br} \sin) \dfrac{d}{v_{br} \cos\theta} = 0 \Rightarrow v_r = v_{br} \sin\theta$ or $\sin\theta = \dfrac{v_r}{v_{br}}$ or $\theta = \sin^{-1}\left(\dfrac{v_r}{v_{br}}\right)$

Hence, to reach point just opposite from where he started should row at an angle

$\theta = \sin^{-1}\left(\dfrac{v_r}{v_{br}}\right)$ upstream from AB.

Since sin 1 so, if v_r v_{br}, the boatman can never reach at point B. Because if $v_r = v_{br}$, $\sin\theta = 1$ or $\theta = 90^o$ and it is just impossible to reach at B if $= 90^o$. Similarly, if $v_r > v_{br}$, $\sin\theta > 1$, i.e., no such angle exists. Practically it can be realized in this manner that it is not possible to reach at B if river velocity (v_r) is too high.

(iii) Shortest path

(a) When $V_r < V_{br}$:

S_{min} = width of the river (d) at angle $\theta = \sin^{-1}\left(\dfrac{V_r}{V_{br}}\right)$

(b) When $V_r > V_{br}$:

$S_{min} = d\left(\dfrac{V_r}{V_{br}}\right)$ at angle $\theta = \sin^{-1}\left(\dfrac{V_{br}}{V_r}\right)$

Aircraft wind Problems

This is similar to river-boat problems replacing river by aircraft and boat by wind.

Rain-Man Problems

If $\vec{V}_r$ = velocity of rain w.r.t. ground

$\vec{V}_m$ = velocity of man w.r.t. ground

then, relative velocity of rain w.r.t. man

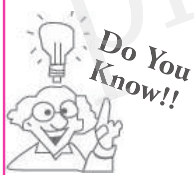
$\vec{V}_{rm} = \vec{V}_r - \vec{V}_m$

CHECK Point

- **When will the relative velocity of two moving objects be zero ?**

Solution

When the two objects are moving with equal velocities (*i.e.* same speed and in the same direction.)



The relative velocity of an object A w.r.t. another object B is equal to resultant of the velocity of A and negative of the velocity of B. Mathematically, $\vec{v}_{AB} = \vec{v}_A + (-\vec{v}_B)$

ILLUSTRATION : 23

A man standing on a road has to hold his umbrella at 30º with the vertical to keep the rain away. He throws the umbrella and starts running at 10 km/h. He finds that raindrops are hitting his head vertically. Find the speed of raindrops with respect to (a) road (b) the moving man.

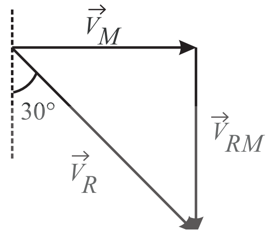
SOLUTION :

Given that the velocity of rain drops with respect to road is making an angle 30º with the vertical, and the velocity of the man is 10 km/h, also the velocity of rain drops with respect to main is vertical. We have

$$\vec{v}_{RM} = \vec{v}_R - \vec{v}_M$$

hence $\overrightarrow{v_R} = \overrightarrow{v_{RM}} - \vec{v}_M$

The situation is shown in velocity triangle in figure.



It shows clearly that, (a) $v_R = V_M \operatorname{cosec}\theta = 10 \times 2 = 20$ km/h

and (b) $V_{RM} = V_M \cos\theta = 10 \times \sqrt{3} = 10\sqrt{3}\ \text{km/h}$

ILLUSTRATION : 24

A man swims at an angle $\theta = 120^\circ$ to the direction of water flow with a speed $v_{mw} = 5$km/h relative to water. If the speed of water $v_w = 3$km/h, find the speed of the man.

SOLUTION :

$$\vec{v}_{mw} = \vec{v}_m - \vec{v}_w \quad \Rightarrow \vec{v}_m = \vec{v}_{mw} + \vec{v}_w$$

$$\Rightarrow \quad \vec{v}_m = |\vec{v}_{mw} + \vec{v}_w| = \sqrt{v^2{}_{mw} + v^2{}_w + 2v_{mw}.v_w \cos\theta}$$

$$\Rightarrow \quad \vec{v}_m = \sqrt{5^2 + 3^2 + 2(5)(3)\cos 120^\circ}$$

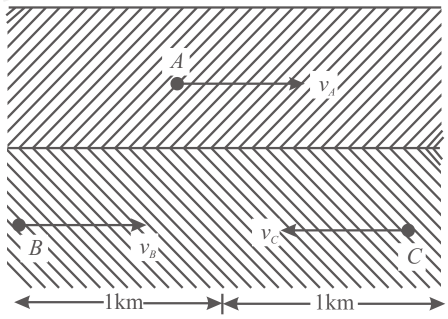
$$\Rightarrow \quad \vec{v}_m = \sqrt{25 + 9 - 15} = \sqrt{19}\ \text{m/sec}$$

ILLUSTRATION : 25

On a two-lane road, car *A* is travelling with a speed of 36 km h^{-1}. Two cars *B* and *C* approach car *A* in opposite directions with a speed of 54 km h^{-1} each. At a certain instant, when the distance *AB* is equal to *AC*, both being 1 km, *B* decides to overtake *A* before *C* does. What minimum acceleration of car B is required to avoid an accident ?

SOLUTION :

Here $v_A = 36\ \text{km h}^{-1} = 36 \times \frac{5}{18}\text{ms}^{-1} = 10\text{ms}^{-1}$



$V_B = V_C = 54\ \text{KM H}^{-1} = 54 \times \frac{5}{18}\text{ms}^{-1} = 15\text{ms}^{-1}$

Relative velocity of *B* w.r.t. *A*, $v_{BA} = 5\ \text{ms}^{-1}$

Relative velocity of *C* w.r.t. *A*, $v_{CA} = 5\ \text{ms}^{-1}$

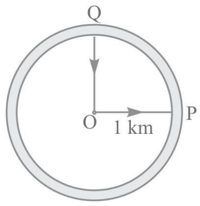
Time taken by *C* to cover distance $AC = \frac{1000\text{m}}{25\text{ms}^{-1}} = 40\text{s}$

Now, for *B*, $1000 = 5 \times 40 + \frac{1}{2} a \times 40 \times 40$

On simplification, $a = 1\ \text{ms}^{-2}$ the minimum acceleration to avoid an accident.

MISCELLANEOUS SOLVED EXAMPLES

1. A cyclist travels from centre *O* of a circular park of radius 1 km and reaches point *P*. After cycling 1/4 th of the circumference along *PQ*, he returns to the centre of the park *QO*. If the total time taken is 10 minute, calculate



(i) net displacement
(ii) average velocity and
(iii) average speed of the cyclist.

Sol. (i) The net displacement becomes zero.

(ii) As net displacement is zero, so average velocity $v_{av} = \frac{\text{Net displacement}}{\text{time taken}} = 0.$

(iii) Total distance covered = $OP + \text{Arc } PQ + OQ = r + \frac{2\pi r}{4} + r = 1 + \frac{2 \times 22 \times 1}{7 \times 4} + 1 = \frac{25}{7}\, km$

Time taken = 10 min = 1/6 h.

$$\text{Average speed} = \frac{\text{total distance covered}}{\text{time taken}} = \frac{\left(\frac{25}{7}\right)}{\left(\frac{1}{6}\right)} = 21.43 \text{ km/h}$$

2. A bus moving with a velocity of 60 km/h is brought to rest in 20 seconds by applying brakes. Find its acceleration.

Sol. Here v_0 = 60 km/h = $60 \times \frac{5}{18} = \frac{50}{3}$ m/s ; time t = 20s; acceleration a = ?

$v = 0$ (as bus comes to rest)

as $a = \frac{v - v_0}{t}$ $\therefore$ $a = \frac{0 - \frac{50}{3}}{20} = \frac{-50}{3 \times 20} = \frac{-5}{6} = -0.83 \text{ m/s}^2$ retardation

3. A bullet moving with 10 m/s hits a wooden plank. The bullet is stopped when it penetrates the plank 20 cm deep. Calculate the retardation of the bullet.

Sol. Given: v_0 = 10 m/s, $v = 0$, s = 20 cm = $\frac{2}{100} = 0.02$m, retardation $(-a)$ = ?

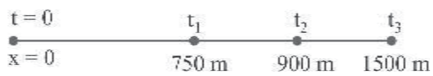
Using, $v^2 - v_0^2 = 2as$

$0 - (10)^2 = 2a\,(0.2) \Rightarrow \frac{-100}{2 \times 0.02} = a$ or $a = -2500 \text{ m/s}^2$

Retardation = 2500 m/s^2

4. An athlete runs a distance of 1500 m in the following manner. (i) Starting from rest, he accelerates himself uniformly at 2m/s^2 till he covers a distance of 900 m. (ii) He, then runs the remaining distance of 600 m at the uniform speed developed. Calculate the time taken by the athlete to cover the two parts of the distance covered.

Sol. The situation is shown in figure.



For the motion between $t = 0$ to t_1; s = 750 m. We know that

$$s = ut + \frac{1}{2}at^2 \qquad 750 = 0 + \frac{1}{2} \times 2 \times t_1^2$$

$\therefore$ t_1 = 27.4 s

(i) For the motion from $t = 0$ to $t = t_2$; $s = 900$ m

$$900 = 0 + \frac{1}{2} \times 2 \times t_2^2 \qquad \therefore \qquad t_2 = 30 \text{ s.}$$

(ii) Let v is the velocity of the athlete at $t = t_2$, then

$v = 0 + 2 \times 30 = 60$ m/s.

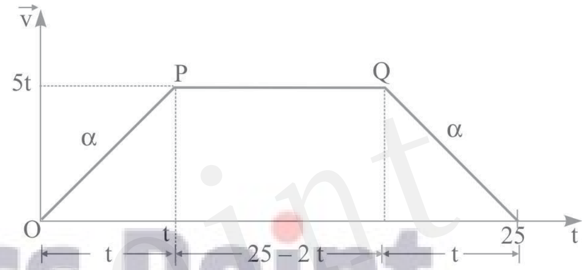
For the motion between t_2 and t_3; $s = 600$ m.

If t is the time of motion, then

$$t = \frac{\text{distance}}{\text{speed}} = \frac{600}{60} = 10\ s.$$

5. A car starts moving rectilinearly, first with acceleration a = 5 m/s^2 (the initial velocity is equal to zero), uniformly, and finally, decelerating at the same rate α comes to a stop. The total time of motion equals τ = 25 s. The average velocity during that time is equal to <v> = 72 km/h. How long does the car move uniformly?

Sol. (i) Let t be the time upto which car accelerates or decelerates. The maximum velocity attained in this duration is $5\,t$. The time upto which car move uniformly $= 25 - 2t$. The velocity – time graph of the motion of car is drawn as in figure.



Given the average velocity in whole time of motion

$$v_{av} = \frac{72 \times 5}{18} = 20 \text{ m/s.}$$

The average velocity from the graph can be obtained as

$$v_{av} = \frac{\text{total displacement}}{\text{total time}} = \frac{\text{area of } \vec{v} - t \text{ graph}}{\text{total time}}$$

$$\therefore \qquad 20 = \frac{\frac{1}{2} \times [25 + (25 - 2t)] \times 5t}{25} = \frac{\frac{1}{2} \times [50 - 2t)] \times 5t}{25}$$

or $\quad 200 = 50t - 2t^2$

or $\quad t^2 - 25t + 100 = 0$

$(t - 20)(t - 5) = 0$

$t = 5$ s or 20 s

But $t = 20$ is not possible

$\therefore \quad t = 5\ s.$

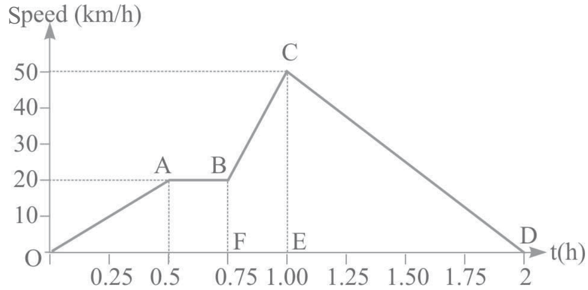
The time upto which car moves uniformly

$= 25 - 2t = 25 - 2 \times 5 = 15\ s$

6. A train moves from one station to another in two hours time. Its speed time-graph during the motion is shown in figure.

(i) Determine the maximum acceleration during the journey

(ii) Also calculate the distance covered during the time interval from 0.75 hour to 1 hour.



Sol. (i) As the part BC of the graph has maximum slope, so acceleration is maximum in this duration.

$\therefore$ Maximum acceleration a = slope of line BC

$$= \frac{v_2 - v_1}{t_2 - t_1} = \frac{(50 - 20)}{1.00 - 0.75} = 120 \text{ km/h}^2.$$

(ii) Distance travelled in the duration 0.75 to 1 hour.

$$= \text{Area of trapezium } BCEF = \frac{1}{2}[20 + 50] \times (1.00 - 0.75) = 8.75 \text{ km.}$$

SOLVED EXAMPLES BASED ON CONNECTING TOPICS

7. A body covers 12 m in 2nd and 20 m in 4th second. How much distance will it cover in 4 second after the 5th second ?

Sol. Given : $s_{2nd} = 12$ m, $s_{4th} = 20$ m.

We known that $s_n = u + \frac{a}{2}(2n - 1)$

$\therefore \quad 12 = u + \frac{a}{2}(2\times2-1) = u + \frac{3a}{2}$ ...(i)

and $\quad 20 = u + \frac{a}{2}(2\times4-1) = u + \frac{7a}{2}$ ...(ii)

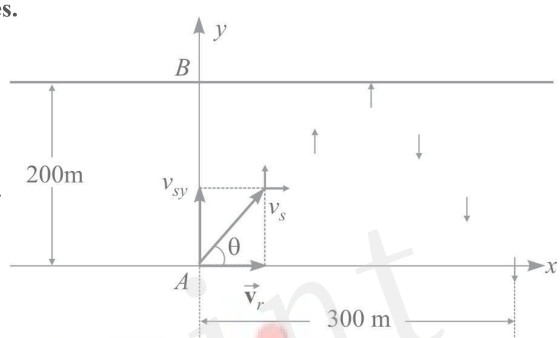
Solving equations (i) and (ii), we get

$a = 4\text{m/s}^2$ and $u = 6$ m/s.

Now distance covered in 4 second after 5th second = $s_9 - s_5$

$$= \left(6\times9+\frac{1}{2}\times4\times9^2\right)-\left(6\times5+\frac{1}{2}\times4\times5^2\right) = 136 \text{ m.}$$

8. A swimmer crosses a 200 m wide channel with straight bank and return in 10 minute at a point 300 m below the starting point (downstream). Find the magnitude and the direction of the velocity of the swimmer relative to the bank if he heads towards the bank to the channel all the time at right angles.



Sol. Time to cross the river = 5 min.

The displacement perpendicular to flow = 200 m.

$\therefore$ Velocity of swimmer

$$v_{sy} = \frac{200}{5\times60} = \frac{2}{3}\text{m/s}$$

The displacement covered in the direction of flow = 300 m in 10 min.

$\therefore$ The velocity of river flow

$$v_r = \frac{300}{10\times60} = \frac{1}{2}\text{m/s}$$

The velocity of swimmer in the direction of flow

$\vec{v}_{sx} = \vec{v}_r + \vec{v}_s$ or $v_{sx} = \frac{1}{2}+0 = \frac{1}{2}$ m/s

His velocity with respect to bank

$$v_s = \sqrt{v_{sx}^2 + v_{sy}^2} = \sqrt{\left(\frac{2}{3}\right)^2+\left(\frac{1}{2}\right)^2} = \frac{5}{6}\text{m/s} = 3 \text{ km/h}$$

The velocity $\vec{v}_s$ makes an angle θ with the bank, then $\tan\theta = \frac{v_{sy}}{v_{sx}} = \frac{2/3}{1/2} = \frac{4}{3}$ or $\theta = \tan^{-1}(4/3)$

9. An aeroplane flying horizontally with a speed of 49 m/s releases a bomb at a height of 490 m. Find the time taken by the bomb to reach the ground and also the magnitude and the direction of velocity with which it strikes the ground.

Sol. Time taken by the bomb to reach the ground, $T = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2\times100}{9.8}} = 10$ sec

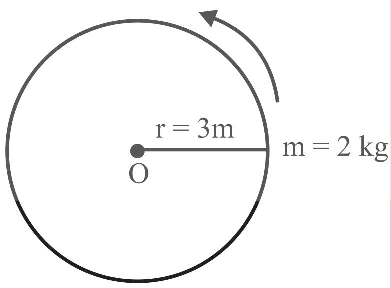
$v_x = u_x = 49$ m/s

$v_y = u_y + gt = 0 + 9.8 \times 10 = 98$ m/s

$\therefore \quad v = \sqrt{v_x^2+v_y^2} = \sqrt{(49)^2+(98)^2} = 49\sqrt{5} = 109.6$ m

$\Rightarrow \tan\theta = \frac{v_y}{v_x} = \frac{98}{49} = 2 \quad \therefore \theta = \tan^{-1}(2)$ below horizontal.

10. A body of mass 2kg lying on a smooth surface is attached to a string 3m long and then whirled round in a horizontal circle making 60 revolution per minute. Find (i) the angular velocity, (ii) the linear velocity, (iii) the centripetal acceleration and (iv) the tension in the string.



Sol. Given that the mass of the particle m = 2kg, radius of the circle r = 3m.

(i) Angular velocity = 60 revolutions / minute

$$= \frac{60\times2\pi}{60} \text{ rad/sec} = 2\pi \text{ rad / sec}$$

because angle described during 1 revolution is 2π radian.

(ii) The linear velocity v = ω r = 2π × 3m /s = 6π m /s

(iii) The centripetal acceleration

$$= \frac{v^2}{r} = \frac{(6\pi)^2}{3}\text{ m/s}^2 = 12\pi^2 \text{ m/s}^2 = 118.4 \text{ m/s}^2$$

(iv) The tension in the string = the centrifugal force

$$= m\frac{v^2}{r} = (2\text{kg})\,(118.4 \text{ m/s}^2) = 236.8 \text{ N}$$

1 EXERCISE

Fill in the Blanks :

DIRECTIONS : *Complete the following statements with an appropriate word / term to be filled in the blank space(s).*

1. Distance travelled divided by elapsed time gives ....
2. If a car starts at rest and accelerates uniformly, the distance it travels is proportional to the .... of the time it travels.
3. If a car is going northward and the driver jams on its brakes, the direction of its acceleration is ....
4. A ball thrown vertically upwards return to its starting point in 4s. If $g = 10\ m/s^2$, its initial speed was ....
5. A body, dropped from a tower with zero velocity, reaches the ground in 4 sec. The height of the tower is about ....*m*
6. The magnitude of average velocity .... equal to the average speed.
7. When negative accelaration acts on a moving body its velocity.......... .
8. A body moving with a uniform speed along a circle has .... velocity.
9. The speed-time graph of a moving object is a straight line parallel to the time-axis. It means the speed is.............. .
10. A stone is let to fall from a buillding of height 30m. The ratio of heights fallen by it after 2s and 3s is ....

True/False :

DIRECTIONS : *Read the following statements and write your answer as true or false.*

1. Area under velocity-time graph shows displacement.
2. Magnitude of displacement can be equal to or lesser than distance.
3. If particle speed is constant, acceleration of the particle must be zero
4. A particle moving with a uniform velocity must be along a straight line.
5. A particle is known to be at rest at time $t = 0$. If its acceleration at $t = 0$ must be zero.
6. The equation $s = ut + \frac{1}{2}at^2$ with the usual notation is vectorial in nature.
7. In a journey, numerical value of displacement ≤ distance.
8. A particle in one dimensional motion with positive value of acceleration must be speeding up.
9. Magnitude of acceleration is constant in the rotating motion along a circular path.

Match the Columns :

DIRECTIONS : *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column I have to be matched with statements (p, q, r, s, t) in Column II.*

1. Match the Column :

Column I	Column II
(A) Uniform speed	(p) Unequal distance in equal time
(B) Constant speed	(q) Zero acceleration
(C) Uniform acceleration	(r) Unequal velocity in equal time change
(D) Non-uniform acceleration	(s) Equal distance in equal time
(E) Non-uniform speed	(t) Equal velocity change in equal time

2.

Column I	Column II
(A) Average velocity	(p) $\frac{v-u}{T}$
(B) Acceleration	(q) D/T
(C) Final velocity	(r) $ut + \frac{1}{2}at^2$
(D) Distance	(s) $\frac{v+u}{t}$
(E) Speed	(t) $u + at$

3.

Column I	Column II
(A) Slope of displacement-time graph	(p) Acceleration
(B) Slope of velocity-time graph	(q) Velocity
(C) Area under velocity-time graph intercepted with time-axis.	(r) Change in velocity
(D) Area under acceleration-time graph intercepted with time-axis.	(s) Displacement
	(t) Speed

Very Short Answer Questions :

DIRECTIONS : *Give answer in one word or one sentence.*

1. What does the speedometer record – the average speed or instantaneous speed ?

2. Can a body moving with a uniform velocity be in equilibrium?
3. Two particles *A* and *B* are moving along the same straight line with *B* ahead of *A*. Velocity remaining unchanged, what would be the effect on the magnitude of relative velocity, if *A* is ahead of *B*?
4. Under what condition the average velocity of a body is equal to its instantaneous velocity?
5. When the magnitude of average velocity is same as that of average speed?
6. Can a particle has varying speed but a constant velocity?
7. What is the acceleration of a particle moving with uniform velocity?
8. Under what condition will the distance and displacement of a moving object have the same magnitude?
9. What does the slope of position and time graph represent for uniform motion?
10. If the velocity of a particle is non-zero, can its acceleration ever be zero ? Explain.
11. If the velocity of a particle is zero, can its acceleration ever by non-zero ? Explain.
12. If a car is traveling eastward, can its acceleration be westward ? Explain.
13. What does slope of *v-t* graph represent ?
14. A ball is thrown straight up. What is its velocity and acceleration at the top?

Short Answer Questions :

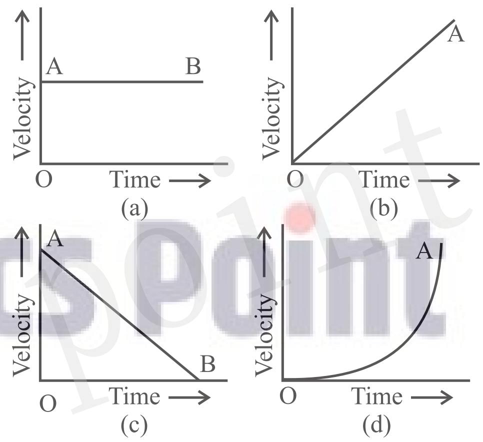
DIRECTIONS : *Give answer in 2-3 sentences.*

1. A stone released with zero velocity from the top of the tower reaches the ground in 4 second. What is the approximate height of the tower?
2. A stone is thrown upwards with a velocity *v* from the top of a tower. It reaches the ground with a velocity 3*v*. What is the height of the tower?
3. A body is projected vertically upwards with a velocity of 96 ft/s. What will the total time for which the body remains in air? Assume, acceleration due to gravity, $g = 32$ fts^{-2}.
4. The velocity of a body moving with a uniform acceleration of 2ms^{-2} is 10 ms^{-1}. What is its velocity after an interval of 4 second?.
5. A motor car moving with a uniform velocity of 20 ms^{-1} comes to a stop, on the application of brakes, after travelling a distance of 10 m. What is its acceleration?
6. A wooden block of mass 10 g is dropped from the top of a cliff 100 m high. Simultaneously, a bullet of mass 10 g is fired from the foot of the cliff upwards with a velocity 100 ms^{-1}. After what time, the bullet and the block meet ?
7. Draw the position time graph for particle moving with positive and negative velocities.
8. A car travels half the distance with constant velocity 30 km h^{-1} and another half with a constant velocity of 40 km h^{-1} . What is the average velocity of the car ?
9. For a particle in one dimensional motion, the instantaneous speed is always equal to the magnitude of instantaneous velocity. Why ?

Long Answer Questions :

DIRECTIONS : *Give answer in four to five sentences.*

1. What types of motions are represented by the following velocity-time graphs?



2. A circular track has a circumference of 3140 m with *AB* as one of its diameter. A scooterist moves from *A* to *B* along the circular path with a uniform speed of 10 m/s. Find
   (a) distance covered by the scooterist,
   (b) displacement of the scooterist, and
   (c) time taken by the scooterist in reaching from *A* to *B*.
3. Establish the three equations of uniformly accelerated motion graphically.
4. (a) Draw velocity-time graph of a uniformly accelerated motion.
   (b) Draw position-time graph of an uniformly accelerated motion.
5. Derive the distance travelled by body performing a motion with constant acceleration at the nth second.
6. Define (a) instantaneous velocity (b) instantaneous acceleration (c) average velocity and (d) average acceleration.
7. What do you mean by the distance and displacement covered by a particle? Explain with examples.

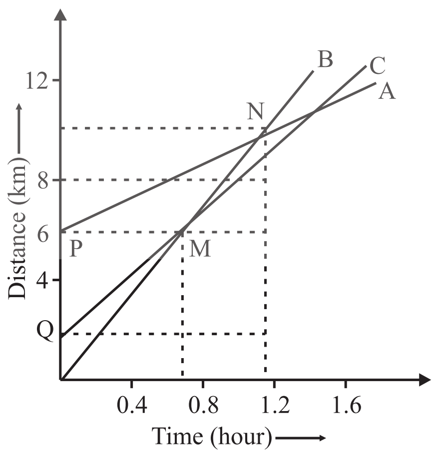
2 EXERCISE

Text-Book Questions :

1. An object has moved through a distance. Can it have zero displacement ? If yes, support your answer with an example.
2. A farmer moves along the boundary of a square field of side 10 m in 40 s. What will be the magnitude of displacement of the farmer at the end of 2 minutes 20 seconds from his initial position ?
3. Which of the following is true for displacement?
   (a) It cannot be zero.
   (b) Its magnitude is greater than the distance travelled by the object.
4. Distinguish between speed and velocity.
5. Under what condition (s) is the magnitude of average velocity of an object equal to its average speed ?
6. What does the odometer of an autombile measure?
7. What does the path of an object look like when it is in uniform motion?
8. During an experiment, a signal from a spaceship reached the ground station in five minutes. What was the distance of the spaceship from the ground station ? The signal travels at the speed of light, that is, 3×10^8 ms^{-1}.
9. When will you say a body is in (i) uniform acceleration, (ii) non-uniform acceleration?
10. A bus decreases its speed from 80 kmh^{-1} to 60 kmh^{-1} in 5s. Find the acceleration of the bus.
11. A train starting from the railway station and moving with a uniform acceleration attains a speed of 40 kmh^{-1} in 10 minutes. Find its acceleration.
12. What is the nature of the distance-time graphs for uniform and non-uniform motion of an object?
13. What can you say about the motion of an object whose distance-time graph is a straight line parallel to the time axis?
14. What can you say about the motion of an object if its speed-time graph is a straight line parallel to the time axis?
15. What is the quantity which is measured by the area occupied below velocity-time graph?
16. A bus starting from rest moves with a uniform acceleration of 0.1 ms^{-2} for 2 minutes. Find (a) the speed acquired, (b) the distance travelled.
17. A train is travelling at a speed of 90 kmh^{-1}. Brakes are applied so as to produce a uniform acceleration of –0.5 ms^{-2}. Find how far the train will go before it is brought to rest ?
18. A trolley while going down an inclined plane has an acceleration of 2 cms^{-2}. What will be its velocity 3 s after the start ?
19. A racing car has a uniform acceleration of 4ms^{-2}. What distance will it cover in 10 s after start?
20. A stone is thrown in a vertically upward direction with a velocity of 5ms^{-1}. If the acceleration of the stone during its motion is 10ms^{-2} in the downward direction, what will be the height attained by the stone and how much time will it take to reach there?

Text-Book Exercise :

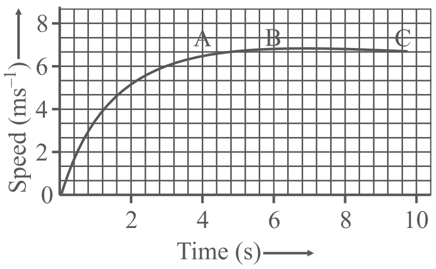
1. An athlete completes one round of a circular track of diameter 200 m in 40 s. What will be the distance covered and the displacement at the end of 2 minutes 20 s ?
2. Joseph jogs from one end A to the other end B of a straight 300 m road in 2 minutes 30 seconds and then turns around and jogs 100 m back to point C in another 1 minute. What are Joseph's average speeds and velocities in jogging (a) from A to B and (b) from A to C?
3. Abdul, while driving to school, computes the average speed for his trip to be 20 km h^{-1}. On his return trip along the same route, there is less traffic and the average speed is 30 km h^{-1}. What is the average speed for Abdul's trip?
4. A motorboat starting from rest on a lake accelerates in a straight line at a constant rate of 3.0 m s^{-2} for 8.0 s. How far does the boat travel during this time?
5. A driver of a car travelling at 52 km h^{-1} applies the brakes and accelerates uniformly in the opposite direction. The car stops in 5 s. Another driver going at 34 km h^{-1} in another car applies his brakes slowly and stops in 10 s. On the same graph paper, plot the speed versus time graphs for the two cars. Which of the two cars travelled farther after the brakes were applied?
6. Fig. shows the distance-time graph of three objects A, B and C. Study the graph and answer the following questions:



(a) Which of the three is travelling the fastest?
(b) Are all three ever at the same point on the road?
(c) How far has C travelled when B passes A?
(d) How far has B travelled by the time it passes C?

7. A ball is gently dropped from a height of 20 m. If its velocity increases uniformly at the rate of 10 m s^{-2}, with what velocity will it strike the ground? After what time will it strike the ground?

8. The speed-time graph for a car is shown in Fig.



(a) Find how far does the car travel in the first 4 seconds. Shade the area on the graph that represents the distance travelled by the car during the period.
(b) Which part of the graph represents uniform motion of the car?

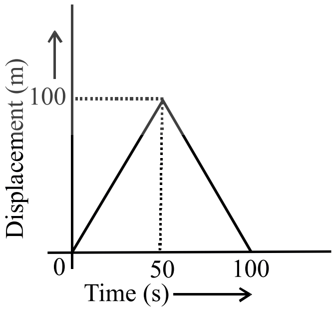
9. State which of the following situations are possible and give an example for each of these:
(a) an object with a constant acceleration but with zero velocity.
(b) an object moving in a certain direction with an acceleration in the perpendicular direction.

10. An artificial satellite is moving in a circular orbit of radius 42250 km. Calculate its speed if it takes 24 hours to revolve around the earth.

Exemplar Questions :

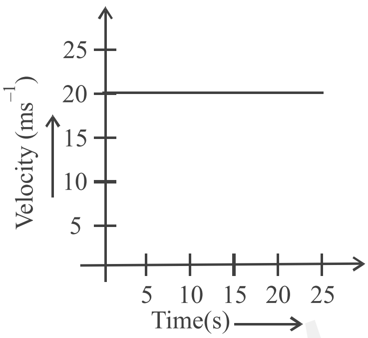
1. The displacement of a moving object in a given interval of time is zero. Would the distance travelled by the object also be zero? Justify your answer.

2. A girl walks along a straight path to drop a letter in the letterbox and comes back to her initial position. Her displacement-time graph is shown in Fig. Plot a velocity-time graph for the same.



3. A car starts from rest and moves along the x-axis with constant acceleration 5 m s–2 for 8 seconds. If it then continues with constant velocity, what distance will the car cover in 12 seconds since it started from the rest?

4. The velocity-time graph (Fig) shows the motion of a cyclist. Find (i) its acceleration (ii) its velocity and (iii) the distance covered by the cyclist in 15 seconds.



5. An object is dropped from rest at a height of 150 m and simultaneously another object is dropped from rest at a height of 100 m. What is the difference in their heights after 2s if both the objects drop with same accelerations? How does the difference in heights vary with time?

6. An object starting from rest travels 20 m in first 2s and 160 m in next 4s. What will be the velocity after 7s from the start?

Hots Questions :

1. Sailing fun, especially on a windy day. Consider the top views of the two boats below, one sailing with the wind and the other across the wind. Which can sail faster than wind speeds?

2. What will be the acceleration of a rock thrown straight upward at the moment it reaches the tippity-top of its trajectory?

3. Two balls of different masses are thrown vertically upwards with the same speed. They pass through the point of projection in their downward motion with the same speed (Neglect air resistance). Is it possible?

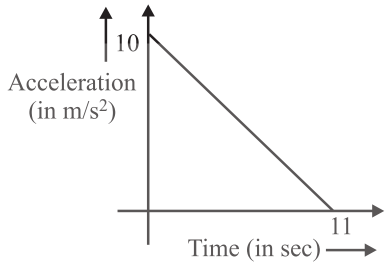
4. Two bodies, *A* (of mass 1 kg) and *B* (of mass 3 kg), are dropped from heights of 16m and 25m, respectively. What is the ratio of the time taken by them to reach the ground ?

5. A body starts from rest and moves with a uniform acceleration. Find the ratio of the distance covered in the n^{th} sec to the distance covered in n sec.

6. A rocket is fired upward from the earth's surface such that it creates an acceleration of 19.6 ms^{-2}. If after 5 s, its engine is switched off, what would be the maximum height of the rocket from earth's surface?

7. When a particle is thrown vertically upward with some initial velocity, draw velocity-time and acceleration-time graph.

8. A particle starts from rest. Its acceleration (a) versus time (t) graph is as shown in the figure. What will be the maximum speed of the particle ?

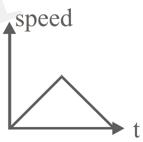


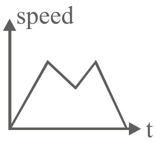
3 EXERCISE

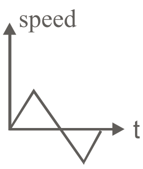
Single Option Correct :

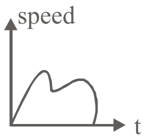
DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. If a car at rest accelerates uniformly to a speed of 144 km/h in 20 sec., it covers a distance of
   (a) 20 cm (b) 400 m
   (c) 1440 cm (d) 2980 cm
2. The displacement of a particle is given by
   $y = a + bt + ct^2 - d\,t^4$
   The initial velocity and acceleration are respectively
   (a) $b, -4d$ (b) $-b, 2c$
   (c) $b, 2c$ (d) $2c, -4d$
3. Which of the following speed time graphs is not possible?

(a)


(b)


(c)


(d)


4. The displacement x of a particle moving along a straight line at time t is given by
   $x = a_0 + a_1 t + a_2 t^2$
   What is the acceleration of the particle?
   (a) a_1 (b) a_2
   (c) $2a_2$ (d) $3a_2$
5. The displacement-time graphs of two particles A and B are straight lines making angles of respectively 30º and 60º with the time axis.
   If the velocity of A is v_A and that of B is v_B, the value of v_A/v_B is
   (a) 1/2 (b) $1/\sqrt{3}$
   (c) $\sqrt{3}$ (d) 1/3
6. A person travels along a straight road for the first half time with a velocity v_1 and the second half time with a velocity v_2. Then the mean velocity $\bar{v}$ is given by
   (a) $\bar{v} = \dfrac{v_1 + v_2}{2}$ (b) $\dfrac{2}{\bar{v}} = \dfrac{1}{v_1} + \dfrac{1}{v_2}$
   (c) $\bar{v} = \sqrt{\dfrac{v_2}{v_1}}$ (d) $\bar{v} = \sqrt{\dfrac{v_2}{v_1}}$
7. A passenger travels along the straight road for half the distance with velocity v_1 and the remaining half distance with velocity v_2. Then average velocity is given by
   (a) $v_1 v_2$ (b) v_2^2/v_1^2
   (c) $(v_1 + v_2)/2$ (d) $2v_1v_2/(v_1 + v_2)$
8. A point moves with uniform acceleration and v_1, v_2 and v_3 denote the average velocities in t_1, t_2 and t_3 sec. Which of the following relation is correct?
   (a) $(v_1 - v_2) : (v_2 - v_3) = (t_1 - t_2) : (t_2 + t_3)$
   (b) $(v_1 - v_2) : (v_2 - v_3) = (t_1 + t_2) : (t_2 + t_3)$
   (c) $(v_1 - v_2) : (v_2 - v_3) = (t_1 - t_2) : (t_1 - t_3)$
   (d) $(v_1 - v_2) : (v_2 - v_3) = (t_1 - t_2) : (t_2 - t_3)$
9. A bus starts moving with acceleration 2 m/s^2. A cyclist 96 m behind the bus starts simultaneously towards the bus at 20 m/s. After what time will he be able to overtake the bus?
   (a) 4 sec (b) 8 sec
   (c) 12 sec (d) 16 sec
10. When the speed of a car is v, the minimum distance over which it can be stopped is s. If the speed becomes nv, what will be the minimum distance over which it can be stopped during same retardation
    (a) s/n (b) ns
    (c) s/n^2 (d) n^2s

11. The velocity of a particle at an instant is 10 m/s. After 5 sec, the velocity of the particle is 20 m/s. The velocity at 3 seconds before from the instant when velocity of a particle is 10 m/s.
(a) 8 m/s (b) 4 m/s
(c) 6 m/s (d) 7 m/s

12. A rubber ball is dropped from a height of 5 metre on a plane where the acceleration due to gravity is not known. On bouncing, it rises to a height of 1.8 m. On bouncing, the ball loses its velocity by a factor of
(a) $\frac{3}{5}$ (b) $\frac{9}{25}$
(c) $\frac{2}{5}$ (d) $\frac{16}{25}$

13. A particle covers half of the circle of radius r. Then the displacement and distance of the particle are respectively
(a) $2\pi r, 0$ (b) $2r, \pi r$
(c) $\frac{\pi r}{2}, 2r$ (d) $\pi r, r$

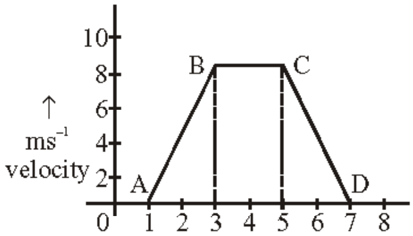
14. A body covers 26, 28, 30, 32 meters in 10^{th}, 11^{th}, 12^{th} and 13^{th} seconds respectively. The body starts
(a) from rest and moves with uniform velocity
(b) from rest and moves with uniform acceleration
(c) with an initial velocity and moves with uniform acceleration
(d) with an initial velocity and moves with uniform velocity

15. The speed of the car is v, the minimum distance over which it can be stopped is x. If the speed becomes nv, then the minimum distance in which it can be stopped in same time is
(a) x/n (b) nx
(c) x/n^2 (d) n^2x

16. A car moving with a speed of 40 km/hour can be stopped by applying brakes after at least 2m. If the same car is moving with a speed of 80km/hour, what is the minimum stopping distance.
(a) 8m (b) 6m
(c) 4m (d) 2m

17. An automobile travelling with a speed of 60 km/h, can brake to stop within a distance of 20m. If the car is going twice as fast i.e., 120 km/h, the stopping distance will be
(a) 60 m (b) 40 m
(c) 20 m (d) 80 m

18. For the velocity time graph shown in the figure below the distance covered by the body in the last two seconds of its motion is what fraction of the total distance travelled by it in all the seven seconds?



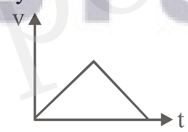
(a) $\frac{1}{2}$ (b) $\frac{1}{4}$
(c) $\frac{2}{3}$ (d) $\frac{1}{3}$

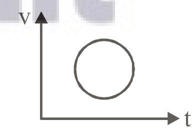
19. If a body is moving at constant speed in a circular path, its
(a) velocity is constant and its acceleration is zero
(b) velocity and acceleration are both changing direction only
(c) velocity and acceleration are both increasing
(d) velocity is constant and acceleration is changing direction

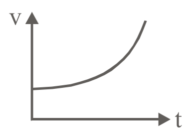
20. A graph is plotted showing the velocity of a car as a function of time. If the graph is a straight line, it means that
(a) the car started at rest
(b) acceleration was constant
(c) acceleration was increasing
(d) velocity was constant

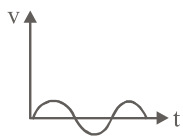
21. If a car is traveling north on a straight road and its brakes are applied, it will
(a) have no acceleration
(b) accelerate to the south
(c) accelerate to the north
(d) accelerate either east or west

22. Which of the following curves do not represent motion of a body?

(a)


(b)


(c)


(d)


23. The acceleration of a car that speeds up from 12 meters per second to 30 meters per second in 15 seconds–
(a) 2.4 m/s² (b) 1.2 m/s²
(c) 2 m/s² (d) 5.2 m/s²

24. A car going at 24 meters per second passes a motorcycle at rest. As it passes, the motorcycle starts up, accelerating at 3.2 meters per second squared. If the motorcycle can keep up that acceleration, how long will it take for it to catch the car
(a) 12 s (b) 14s
(c) 20s (d) 18s

25. Mohan takes 20 minutes to cover a distance of 3.2 kilometers due north on a bicycle, his velocity in kilometer/hour–
(b) 8.1 (b) 9.6
(c) 1.2 (d) 7.2

26. A body moving along a straight line at 20 m/s undergoes an acceleration of -4 m/s^2. After two seconds its speed will be
(a) –8 m/s (b) 12 m/s
(c) 16 m/s (d) 28 m/s

27. A particle experiences constant acceleration for 20 seconds after starting from rest. If it travels a distance s1 in the first 10 seconds and distance s2 in the next 10 seconds, then
 (a) s2 = s1 (b) s2 = 2s1
 (c) s2 = 3s1 (d) s2 = 4s1

28. In which of the following cases, the object does not possess an acceleration or retardation when it moves in
 (a) upward direction with decreasing speed
 (b) downward direction with increasing speed
 (c) with constant speed along circular path
 (d) with constant speed along horizontal direction.

More than One Option Correct :

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

1. A vector quantity is a physical quantity which needs
 (a) magnitude (b) direction
 (c) both (a) and (b) (d) time

2. Which of the following are examples of uniform velocity?
 (a) Motion of moon around earth
 (b) Motion of planet around sun
 (c) Motion of car on crowded road
 (d) Motion of a moving fan

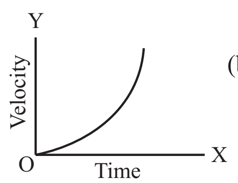
3. Average velocity can be calculated by
 (a) $\frac{\text{Distance travelled along given direction}}{\text{Time taken}}$
 (b) $\frac{\text{Initial velocity}}{2}$
 (c) $\frac{\text{Initial velocity + Final velocity}}{2}$
 (d) $\frac{\text{Final velocity}}{2}$

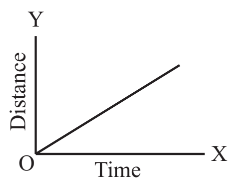
4. Which of the following are vector quantities ?
 (a) Speed (b) Distance
 (c) Velocity (d) Acceleration

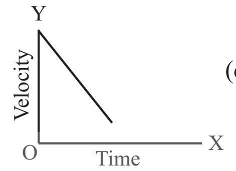
5. If a body starts from rest, its
 (a) $u = 0$ (b) $a = 0$
 (c) velocity increases (d) velocity decreases

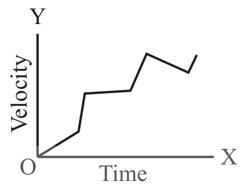
6. If a body moves with uniform velocity, its
 (a) $u = v$ (b) $v = 0$
 (c) $u = 0$ (d) $a = 0$

7. Velocity time graph of a body moving with variable acceleration is

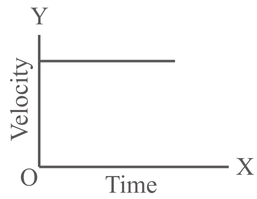
(a) 

(b) 

(c) 

(d) 

8. Which of the following is correct about the given below graph?



 (a) Velocity is zero
 (b) Velocity is constant
 (c) Acceleration is zero
 (d) Acceleration is constant

9. Which of the following statements are true for displacements?
 (a) It can be zero
 (b) It cannot be zero
 (c) Its magnitude is greater than distance travelled
 (d) Its magnitude is lesser than distance travelled

10. The speed of an object is
 (a) distance per unit time
 (b) a scalar quantity
 (c) displacement per unit time
 (d) a vector quantity

Multiple Matching Questions :

DIRECTIONS : *Following question has four statements (A, B, C and D) given in Column I and five or six statements (p, q, r, s...) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in Column I with entries in Column II.*

1.

Column I	Column II
(A) v	(p) $ut + ½ at^2$
(B) u	(q) $v - u/t$
(C) a	(r) Final velocity
(D) s	(s) $u + at$
	(t) Initial velocity
	(u) $\sqrt{u^2 + 2as}$

	A	B	C	D
(a)	r, s, u	t	q	p
(b)	p, q, r, s	q	p, q, r, s	p, q, r, s
(c)	p, s	q	r, s, t	r
(d)	p,	q, r	r	s

2. **Column I** | **Column II**

Column I	Column II
(A) Acceleration	(p) m/s^2
(B) Speed	(q) centripetal force
(C) Circular motion	(r) m
(D) Displacement	(s) m/s
	(t) scalar quantity
	(u) vector quantity

	A	B	C	D
(a)	p,q	q	r, s	q, r
(b)	p, u	s, t	q	r, u
(c)	p, s	q	r, s, t	r
(d)	r, t	t, u	s	r

3. A body is in motion for some time, then at a certain instant of time

Column I	Column II
(A) Distance	(p) may be positive
(B) Displacement	(q) may be negative
(C) Speed	(r) may be zero
(D) Velocity	(s) may be increasing
	(t) may be decreasing

	A	B	C	D
(a)	s, p	r, t, s	p, t	r
(b)	p, q, r, s	q	p, q, r, s	p, q, r, s
(c)	p, s	q	r, s, t	r
(d)	p, s	p, q, r, s, t	p, r, s, t	p, q, r, s, t

Passage Based Questions :

DIRECTIONS : *Study the given paragraph(s) and answer the following questions.*

PASSAGE - I

Consider the motion of a batsman in a cricket game. The length of the pitch is 18 m. Suppose the batsman complete one run and he is now 18 m away from his batting crease or his starting point. Then he turns back and gets run out, when he is exactly mid way through his second run.

1. The total distance travelled by batsman is
(a) 18 m (b) 36 m
(c) 9 m (d) 27 m

2. How far is the batsman from his starting point ?
(a) 9 m (b) 0 m
(c) 18 m (d) 27 m

3. What is the net displacement of the batsman ?
(a) 9 m (b) 0 m
(c) 18 m (d) 27 m

PASSAGE - II

The velocity of any moving body, in ordinary circumstances, does not always remain constant. For example, a bus gains velocity while leaving a station and loses velocity while approaching a station. Similarly, when a store is dropped from a certain height, its velocity goes on increasing as it comes to the ground. But if a stone is thrown upwards its velocity goes on decreasing, till it becomes zero and then its velocity starts increasing as it approaches the ground.

4. The rate of change of velocity with time is known as
(a) Acceleration (b) Speed
(c) Intial velocity (d) Final velocity

5. What can be concluded about acceleration if velocity increases continously ?
(a) Acceleration is positive (b) Acceleration is constant
(c) Acceleration is negative(d) Acceleration is zero

6. Which of the following case represents a negative acceleration?
(a) Car starting from rest
(b) A stone falling from height
(c) Train coming to halt
(d) Bus moving with uniform velocity

PASSAGE-III

A thief is running on a motorcycle at a constant speed of 25 ms^{-1}. A police jeep starts chasing from a point 1.25 km behind him with a uniform acceleration of $2ms^{-2}$.

7. After how much time will the police catch the thief ?
(a) 1 minute (b) 1.5 minute
(c) 2 miniutes (d) 50 seconds

8. How much distance will the jeep cover to reach the thief ?
(a) 2.5 km (b) 3.75 km
(c) 5 km (d) 1.25 km

9. What should be the minimum acceleration of the thief to escape from police.
(a) 1 ms^{-2} (b) 1.25 ms^{-2}
(c) 1.5 ms^{-2} (d) 1.75 ms^{-2}

Assertion & Reason :

DIRECTIONS : *Each of these questions contains an Assertion followed by Reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.
(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.
(c) If **Assertion** is **correct** but **Reason** is **incorrect**.
(d) If **Assertion** is **incorrect** but **Reason** is **correct**.

1. **Assertion :** The distance-time graph of uniform motion is a straight line.
Reason : Independent variable is taken along x-axis and dependent variable along y-axis.

2. **Assertion :** The velocity of a body is a vector quantity.
Reason : A vector quantity has only magnitude and no direction.

3. **Assertion :** Motion of moon around earth is a non-uniform motion.
Reason : The size of moon is smaller than that of earth.

4. **Assertion:** If a body moves with uniform velocity, its acceleration is zero.
Reason : Rate of change of velocity is zero in case of body moving with uniform velocity.

5. **Assertion :** Instantaneous speed is the speed of a body over a long period of time.
Reason : The graph representing non-uniform speed will be a curve with increasing or decreasing slope.

6. **Assertion:** Displacement of a body may be zero when distance travelled by it is not zero.
Reason: The displacement is the longest distance between initial and final positions.

7. **Assertion:** The displacement time graph of a body moving with uniform acceleration is a straight line.
Reason: The displacement is proportional to square of time for uniformly accelerated motion.

Integer/Numeric type Questions :

RECTIONS : *Following are integer based/Numeric based questions. Each question, when worked out will result in one integer or numeric value.*

1. A train is travelling with a velocity of 40 km h^{-1}
(i) What should be the acceleration on it so that it may reach a point 10 km ahead in 8 minutes?
(ii) What will be its velocity on reaching that point?

2. The change in the velocity of a motor bike is 54 km h^{-1} in one minute. Calculate its acceleration in (a) ms^{-2} (b) km h^{-2}

3. Find the initial velocity of a car which is stopped in 10 seconds by applying brakes. The retardation due to brakes is 2.5 m/s^2.

4. A cheetah the fastest animal can achieve a peak value of velocity 100 km/h upto a distance less than 500 m. If a cheetah spots his prey at a distance of 100 m, what is the minimum time it will take to get its prey?

5. Usha swims in a 90 m pool. She covers 180 m in one minute by swimming from one end to the other and back along a straight path. Find the average velocity of Usha.

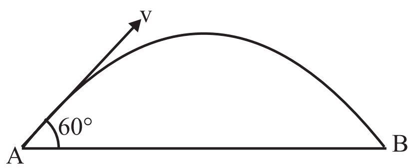
4 ADVANCED EXERCISE
BASED ON CONNECTING TOPICS

DIRECTIONS (Qs. 1–25): *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. A body dropped from a height '*h*' with an initial speed zero, strikes the ground with a velocity 3 km/hour. Another body of same mass dropped from the same height '*h*' with an initial speed *u*' = 4 km/hour. Find the final velocity of second mass, with which it strikes the ground
(a) 3 km/h (b) 4 km/h
(c) 5 km/h (d) 6 km/h

2. A ball is dropped downwards, after 1 sec another ball is dropped downwards from the same point. What is the distance between them after 3 sec?
(a) 25 m (b) 20 m
(c) 50 m (d) 9.8 m

3. A projectile of mass m is thrown with a velocity v making an angle 60° with the horizontal. Neglecting air resistance, the change in momentum from the departure A to its arrival at B, along the vertical direction is



(a) 2 mv (b) $\sqrt{3}$mv
(c) mv (d) $\frac{mv}{\sqrt{3}}$

4. Two trains are each 50 m long moving parallel towards each other at speeds 10 m/s and 15 m/s respectively. After what time will they pass each other?
(a) $5\sqrt{\frac{2}{3}}$ sec (b) 4 sec
(c) 2 sec (d) 6 sec

5. A stone is thrown vertically upwards. When the particle is at a height half of its maximum height, its speed is 10 m/sec; then maximum height attained by particle is (g = 10 m/sec^2)
(a) 8 m (b) 10 m
(c) 15 m (d) 20 m

6. A parachutist after bailing out falls 50 m without friction. When parachute opens, it decelerates at 2 m/s^2 . He reaches the ground with a speed of 3 m/s. At what height, did he bail out ?
(a) 182 m (b) 91 m
(c) 111 m (d) 293 m

7. The range of a particle when launched at an angle of 15º with the horizontal is 1.5 km. What is the range of the projectile when launched at an angle of 45º to the horizontal?

(a) 1.5 km (b) 3.0 km
(c) 6.3 km (d) 0.75 km

8. A gun fires two bullets at 60º and 30º with horizontal. The bullets strike at some horizontal distance. The ratio of maximum height for the two bullets is in the ratio
(a) 2 : 1 (b) 3 : 1
(c) 4 : 1 (d) 1 : 1

9. Two balls are projected at an angle θ and $(90^\circ - \theta)$ to the horizontal with the same speed. The ratio of their maximum vertical heights is
(a) 1 : 1 (b) $\tan\theta : 1$
(c) $1 : \tan\theta$ (d) $\tan^2\theta : 1$

10. The velocity of projection of a body is increased by 2%. Other factors remaining unchanged, what will be the percentage change in the maximum height attained?
(a) 1% (b) 2%
(c) 4% (d) 8%

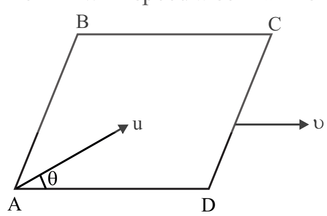
11. A body is thrown with a velocity of 9.8 ms^{-1} making an angle of 30º with the horizontal. It will hit the ground after a time
(a) 3.0 s (b) 2.0 s
(c) 1.5 s (d) 1 s

12. If the horizontal range of a projectile is equal to the maximum height reached, then the corresponding angle of projection is
(a) $\tan^{-1}(1)$ (b) $\tan^{-1}(\sqrt{3})$
(c) $\tan^{-1}(4)$ (d) $\tan^{-1}(12)$

13. Which of the following statements is FALSE for a particle moving in a circle with a constant angular speed ?
(a) The acceleration vector points to the centre of the circle
(b) The acceleration vector is tangent to the circle
(c) The velocity vector is tangent to the circle
(d) The velocity and acceleration vectors are perpendicular to each other.

14. In uniform circular motion, the velocity vector and acceleration vector are
(a) perpendicular to each other
(b) same direction
(c) opposite direction
(d) not related to each other

15. A smooth square platform *ABCD* is moving towards right with a uniform speed *v*. At what angle θ must a particle be projected from *A* with speed *u* so that it strikes the point *B*



(a) $\sin^{-1}\left(\frac{u}{v}\right)$ (b) $\cos^{-1}\left(\frac{v}{u}\right)$
(c) $\cos^{-1}\left(\frac{u}{v}\right)$ (d) $\sin^{-1}\left(\frac{v}{u}\right)$

16. The circular motion of a particle with constant speed is
(a) periodic but not simple harmonic
(b) simple harmonic but not periodic
(c) periodic and simple harmonic
(d) neither periodic nor simple harmonic

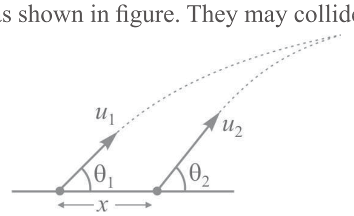
17. An aeroplane flying at a constant speed releases a bomb. As the bomb moves away from the aeroplane, it will
(a) always be vertically below the aeroplane only if the aeroplane was flying horizontally
(b) always be vertically below the aeroplane only if the aeroplane was flying at an angle of 45° to the horizontal
(c) always be vertically below the aeroplane
(d) gradually fall behind the aeroplane if the aeroplane was flying horizontally.

18. The time of flight of a projectile on an upward inclined plane depends upon
(a) angle of inclination of the plane
(b) angle of projection
(c) the value of acceleration due to gravity
(d) all of these.

19. At the highest point on the trajectory of a projectile, its
(a) potential energy is minimum
(b) kinetic energy is maximum
(c) total energy is maximum
(d) kinetic energy is minimum.

20. Two bullets are fired horizontally, simultaneously and with different velocities from the same place. Which bullet will hit the ground earlier ?
(a) It would depend upon the weights of the bullets.
(b) The slower one.
(c) The faster one.
(d) Both will reach simultaneously.

21. Two particles are projected simultaneously from the level ground as shown in figure. They may collide after a time :



(a) $\frac{x\sin\theta_2}{u_1}$ (b) $\frac{x\cos\theta_2}{u_2}$
(c) $\frac{x\sin\theta_2}{u_1\sin(\theta_2-\theta_1)}$ (d) $\frac{x\sin\theta_1}{u_2\sin(\theta_2-\theta_1)}$

22. If a particle is projected at 45°, then
(a) $R = 4H$ (b) $4R = H$
(c) $2H = R$ (d) none of these

23. A cyclist moving at a speed of 20 m/s takes a turn, if he doubles his speed then chance of overturn
(a) is doubled (b) is halved
(c) becomes four times (d) becomes 1/4 times

24. A projectile can have the same range '*R*' for two angles of projection. If 't_1' and 't_2' be the times of flights in the two cases, then the product of the two time of flights is proportional to

(a) $\frac{1}{R^2}$ (b) R^2

(c) R (d) $\frac{1}{R}$

25. The relative velocity v_{AB} or v_{BA} of two bodies A & B may be

(1) greater than velocity of body A
(2) greater than velocity of body B
(3) less than the velocity of body A
(4) less than the velocity of body B

(a) (1) and (2) only (b) (3) and (4) only
(c) (1), (2) and (3) only (d) (1), (2), (3) and (4).

DIRECTIONS (Qs. 26–33) : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

26. A balloon starts rising from the ground with an acceleration of 2.5ms^{-2}. After 4 seconds, a stone is released from the balloon. If g = 10 m/s^2, the stone will
(a) have a displacement of 25 m
(b) cover a total distance of 30 m
(c) reach the ground in 3.2 s
(d) begin to move down after being released.

27. A man throws a stone, vertically up with a speed of 20 ms^{-1} from top of a high rise building. Two seconds later, an indentical stone is thrown vertical downward with the same speed 20 ms^{-1}. Then
(use g = 10m/s^2)
(a) the relative acceleration between the two is equal to zero
(b) the time interval between their hitting the ground is 2 seconds
(c) both will have the same K.E., when they hit the ground
(d) the relative velocity between the two stones remains constant till one hits the ground.

28. The following quantities may remain constant during uniform circular motion :
(a) magnitude of acceleration
(b) acceleration
(c) speed
(d) velocity

29. A cart moves with a constant speed along a horizontal circular path. From the cart, a particle is thrown up vertically with respect to the cart. Then the particle will :
(a) land outside the circular path
(b) land somewhere on circular path
(c) follow a parabolic path
(d) follow an elliptical path

30. Which of the following represent the projectiles ?
(a) A stone thrown horizontally from the top of the tower
(b) A bullet fired from the gun
(c) A rocket fired into space
(d) A ball thrown upwards

31. A body is moving in a circle of radius r with a uniform speed v, angular frequency ω, time period T and frequency ν. The centripetal acceleration is given by :

(a) $\frac{4\pi^2 r}{T}$ (b) ωv

(c) $\frac{v^2}{r}$ (d) $4\pi^2 r \nu^2$

32. Two particles are projected from the same point with the same speed, at different angles θ_1 and θ_2 to the horizontal. They have the same horizontal range. Their time of flight are t_1 and t_2 respectively. Then

(a) $\theta_1 + \theta_2 = 90°$ (b) $\frac{t_1}{\sin\theta_1} = \frac{t_2}{\sin\theta_2}$

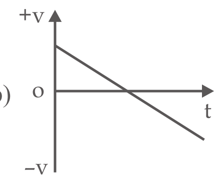
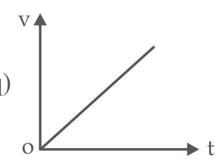
(c) $\frac{t_1}{t_2} = \tan\theta_2$ (d) $\frac{t_1}{t_2} = \tan\theta_1$

33. Let a_r and a_t represent radial and tangential acceleration. The motion of a particle may be circular if

(a) $a_r \neq 0$ and $a_t \neq 0$ (b) $a_r \neq 0$ and $a_t = 0$
(c) $a_r = a_t = 0$ (d) $a_r = 0$ and $a_t \neq 0$

DIRECTIONS (Qs. 34–35) : *Following question has four statements (A, B, C and D) given in Column I and five statements (p, q, r, s and t) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in Column I with entries in Column II.*

34. Match the following :

	Column I		Column II
(A)	When a body is dropped from some height.	(p)	
(B)	When a body is projected vertically upward.	(q)	
(C)	In case of motion under gravity time taken to go up.	(r)	$= \frac{u}{g}$
(D)	In case of motion under gravity time of descent	(s)	$> \frac{u}{g}$
		(t)	$< \frac{u}{g}$

	A	B	C	D
(a)	q	p	r	r
(b)	p, q, r, s	q	p, q, r, s	p, q, r, s
(c)	p, s	q	r, s, t	r
(d)	p,	q, r	r	s

35. A particle is projected from ground at a certain angle. Match the following for the motion of the particle.

Column I	Column II
(A) Speed of the particle	(p) Constant
(B) Acceleration of the particle	(q) Variable
(C) Horizontal component of velocity	(r) First increases then decreases
(D) Vertical displacement	(s) First decreases then increases
	(t) Always downwards

	A	B	C	D
(a)	p,q	q	r, s	q, r
(b)	p, q, r, s	q	p, q, r, s	p, q, r, s
(c)	r, t	s	p, q	t
(d)	q, s	t	p	q, r

DIRECTIONS (Qs. 36–41) : *Study the given paragraph(s) and answer the following questions.*

PASSAGE-I

A thief is running on a motorcycle at a constant speed of 25 ms^{-1}. A police jeep starts chasing from a point 1.25 km behind him with a uniform acceleration of $2ms^{-2}$.

36. After how much time will the police catch the thief ?
(a) 1 minute (b) 1.5 minute
(c) 2 miniutes (d) 50 seconds

37. How much distance will the jeep cover to reach the thief ?
(a) 2.5 km (b) 3.75 km
(c) 5 km (d) 1.25 km

38. What should be the minimum acceleration of the thief to escape from police.
(a) 1 ms^{-2} (b) 1.25 ms^{-2}
(c) 1.5 ms^{-2} (d) 1.75 ms^{-2}

PASSAGE-II

A particle is moving along a circular track of radius 3.5 m with a constant speed 4 ms^{-1}. Answer the following for the motion of the particle.

39. How much time will it take to complete one complete round ?
(a) 5.5 s (b) 11 s (c) 22 s (d) 3.5 s

40. What is the average velocity of the particle in one complete round ?
(a) 4 ms^{-1} (b) 2.6 ms^{-1} (c) 2 ms^{-1} (d) Zero

41. The acceleration of the particle is
(a) $\frac{42}{7}ms^{-2}$ (b) $\frac{32}{7}ms^{-2}$ (c) $\frac{18}{7}ms^{-2}$ (d) 0

DIRECTIONS (Qs. 42–44) : *Each of these questions contains an Assertion followed by Reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.
(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.
(c) If **Assertion** is **correct** but **Reason** is **incorrect**.
(d) If **Assertion** is **incorrect** but **Reason** is **correct**.

42. **Assertion** : In a uniform circular motion, acceleration of the body is constant.
Reason : In uniform circular motion, the body has only centripetal acceleration.

43. **Assertion** : Horizontal component of velocity of projectile is constant.
Reason : Acceleration of the projectile is along the vertical.

44. **Assertion** : In projectile motion, acceleration is always perpendicular to the velocity.
Reason : Path of projectile is a parabola.

DIRECTIONS (Qs. 45-50) : *Following are integer based/ Numeric based questions. Each question, when worked out will result in one integer or numeric value.*

45. A ball is thrown horizontally from the top of a tower with a speed of 50 m/s. Find the velocity and position at the end of 3 second. [$g = 9.8 m/s^2$]

46. A particle is projected with a velocity u so that its horizontal range is twice the greatest height attained. Find the horizontal range of it.

47. The angular velocity of a particle moving in a circle of radius 50 m is increased in 5 minutes from 100 revolutions per minute to 400 revolution per minute. Find (a) angular acceleration (b) linear aceeleration.

48. A stone tied to the end of string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 seconds, what is the magnitude and direction of the acceleration of the stone ?

49. A cyclist is riding with a speed of 27 km h^{-1}. As he approaches a circular turn on the road of radius 80 m, he applies brakes and reduces his speed at the constant rate of 0.5 m s^{-1}. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn ?

50. What is the centripetal acceleration in (m/s^2) of the earth as it moves in its orbit around the sun?

SOLUTIONS

Brief Explanations of Selected Questions

1 EXERCISE

FILL IN THE BLANKS :

1. average speed
2. Square
3. south
4. 20 m/s
5. 80
6. may or may not be
7. decreases
8. variable
9. uniform
10. 4:9

TRUE/FALSE :

1.	True	2.	True	3.	False
4.	True	5.	False	6.	True
7.	True	8.	False	9.	True

MATCH THE COLUMNS :

1. (A) → (s); (B) → (q); (C) → (t); (D) → (r); (E) → (p)
2. (A) → (s); (B) → (p); (C) → (t); (D) → (r); (E) → (q)
3. (A) → (q); (B) → (p); (C) → (s); (D) → (r)

VERY SHORT ANSWER QUESTIONS :

1. Instantaneous speed.
2. Yes, since the net force acting on it is zero.
3. No effect.
4. When the body is moving with constant velocity, the average velocity of a body is equal to its instantaneous velocity.
5. When the body is moving with a constant velocity, the magnitude of average velocity is same as that of average speed.
6. No since both speed and velocity remain constant in uniform motion.
7. Zero.
8. If the object is moving along a straight line.
9. Velocity.
13. Acceleration.
14. Velocity at the top = 0 and acceleration at the top = 9.8 m s^{-2} (downwards).

SHORT ANSWER QUESTIONS :

1. $u = 0$, $a = 10\text{ms}^{-2}$, $t = 4\text{s}$,

   Therefore, $h = \frac{1}{2} \times 10 \times (4)^2 = 80$ m

2. We know, $v^2 - u^2 = 2as$.

   From this relation, we get, $9v^2 - v^2 = 2gh$.

   Therefore, $h = 8v^2/2g = 4v^2/g$

3. $0 = -96 + 32t$ or, $t = 3s$. Total time = $(3 + 3)\ s = 6\ s$.
4. $v = 10 + 2 \times 4 = 18\ \text{ms}^{-1}$.
5. $u = 20\ \text{ms}^{-1}$, $s = 10\text{m}$

   We know, $v^2 - u^2 = 2as$.

   Therefore, $a = -400/(2 \times 10)\ \text{ms}^{-2} = -20\ \text{ms}^{-2}$

6. Let the block and the bullet meet at a height x from the ground. Considering direction as +ve and the upward direction as negative, we get

   for block, $u = 0$, $s = 100 - x$, $a = +g$

   For bullet, $u = -100\ \text{ms}^{-1}$, $a = +g$, $s = -x$

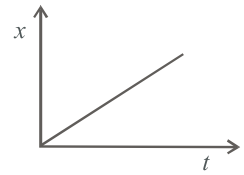
   We get, $-x = -100t + \frac{1}{2}gt^2$

   $100 - x = \frac{1}{2}gt^2$

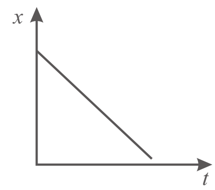
   Therefore, $-x = -100t + 100 - x$

   Therefore, $t = 1s$.

7. The x-t graph for an object moving with a positive velocity.



   The x-t graph for an object moving with a negative velocity.



8. Let s be the total distance covered by the car; and t_1 and t_2 be the time taken by the car to cover first half and second half respectively. Then, total time taken, $t = t_1 + t_2$

   $= \frac{S/2}{30} + \frac{S/2}{40} = \frac{S}{60} + \frac{S}{80} = \frac{7S}{240}$h

   Therefore, average velocity,

   $v_{av} = \frac{S}{t} = \frac{S}{7S/240} = \frac{240}{7} = 34.3\ \text{km h}^{-1}$

9. In an accelerated motion, the velocity of an object always keeps on changing. Therefore, one has to measure the instantaneous velocity. However, when accelerated motion takes place along a straight line, the velocity of the body changes only due to change in magnitude of velocity.

Therefore, the instantaneous speed is always equalto the magnitude of instantaneous velocity of the particle in one dimensional motion.

LONG ANSWER QUESTIONS :

1. (a) The body has uniform velocity.

(b) The body has uniform acceleration and its initial velocity is zero.

(c) The body has some initial velocity and is under uniform retardation.

(d) The body has zero initial velocity and it has variable acceleration.

2. (a) Distance covered $= \frac{1}{2} \times$ circumference

$$= \frac{1}{2} \times 3140 = 1570 \text{ m}$$

(b) Displacement = Diameter AB

$$= \frac{\text{Circumference}}{\pi} = \frac{3140 \text{ m}}{3.14} = 1000 \text{ m}$$

(c) Time taken $= \frac{\text{Distance}}{\text{Speed}}$

$$= \frac{1570 \text{ m}}{10 \text{ ms}^{-1}} = 157 \text{ s}$$

2 EXERCISE

TEXT BOOK QUESTIONS

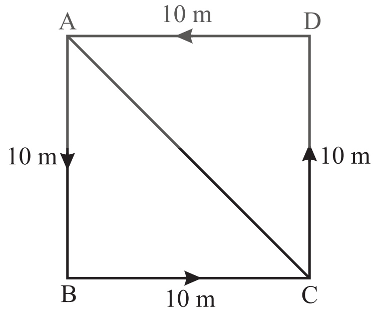
1. Yes, it can have zero displacement.

If we take a round trip and reach back at the starting point, then we have travelled some distance, but our displacement will be zero.

2. If the farmer starts from point A, then at the end of 2 minutes and 20 seconds (= 140 seconds), he will reach the diagonally opposite corner C. The magnitude of displacement of the farmer is :

$$AC = \sqrt{AB^2 + BC^2} = \sqrt{10^2 + 10^2}$$

$$= 10\sqrt{2} = 10 \times 1.414 = 14.14 \text{ m}$$



3. Neither (a) nor (b) is true.

4.

	Speed	Velocity
1.	The distance travelled by a moving body per unit time is called its speed.	The distance travelled by a moving body in a particular direction per unit time is called its velocity.
2.	Speed is a scalar quantity.	Velocity is a vector quantity.

5. Average speed $= \frac{\text{Total path length}}{\text{Time interval}}$

Average velocity $= \frac{\text{Displacement}}{\text{Time interval}}$

When an object moves along a straight line in the same direction, its total path length is equal to the magnitude of displacement. Hence, its average speed is equal to the magnitude of average velocity.

6. The odometer of an automobile measures the distance moved by it.

7. Straight line path.

8. Here, t =5 minutes = 300 s, $v = 3 \times 10^8 \text{ ms}^{-1}$

Distance of spaceship, $s = vt = 3 \times 10^8 \times 300$

$$= 9 \times 10^{10} \text{ m.}$$

9. (i) If a body travels in a straight line and its velocity changes by equal amounts in equal intervals of time, however small these time intervals may be, then the body is said to be in uniform acceleration.

(ii) If the velocity of a body changes by unequal amounts in equal intervals of time, then the body is said to be in non-uniform acceleration.

10. Initial speed, $u = 80 \frac{\text{km}}{\text{h}} = \frac{80 \times 1000 \text{ m}}{3600 \text{ s}}$

$$= \frac{800}{36} \text{ms}^{-1} = 22.22 \text{ ms}^{-1}$$

Final speed, $v = 60 \frac{\text{km}}{\text{h}} = \frac{60 \times 1000 \text{ m}}{3600 \text{ s}}$

$$= \frac{600}{36} \text{ms}^{-1} = 16.66 \text{ ms}^{-1}$$

Acceleration, $a = \frac{v-u}{t} = \frac{(16.66 - 22.22)}{5} = -1.11 \text{ ms}^{-2}$

11. Initial speed, $u = 0$

Final speed, $v = 40 \frac{\text{km}}{\text{h}}$

$$= \frac{40 \times 1000 \text{ m}}{3600 \text{ s}} = \frac{100}{9} \text{ms}^{-1}$$

Time, t = 10 min = 600 s

Acceleration, $a = \frac{v-u}{t} = \frac{\frac{100}{9} - 0}{600}$

$$= \frac{100}{9 \times 600} \text{ms}^{-2}$$

$$= \frac{1}{54} \text{ms}^{-2} = 0.018 \text{ ms}^{-2}$$

12. (i) For uniform motion, the distance-time graph is a straight line inclined with the time-axis.
(ii) For non-uniform motion, the distance-time graph is a curve.

13. The object is at rest.

14. The object is moving with a uniform speed.

15. Displacement covered by the body in the given time interval.

16. Here, u = 0, a = 0.1 ms^{-2}, t = 2 min = 120s

(a) $v = u + at = 0 + 0.1 \times 120 = 12\text{ms}^{-1}$

(b) $s = ut + \frac{1}{2}at^2 = 0 \times 120 + \frac{1}{2} \times 0.1 \times (120)^2$

$= 720m$

17. Here, $u = 90$ km/h = $90 \times \frac{5}{18}$ ms^{-1} = 25 ms^{-1}

$a = -0.5\ \text{ms}^{-2}$, $v = 0$

As $v^2 - u^2 = 2as$

$\therefore\ 0^2 - 25^2 = 2 \times (-0.5) \times s$

or $s = \frac{25 \times 25}{2 \times 0.5} = 625\text{m}$

18. Here, $u = 0, a = 2$ cm s^{-2}, t = 3s

v = u + at = = 0 + 2 × 3 = 6 cm s^{-1}

19. Here, $u = 0, a = 4\text{ms}^{-2}$, $t = 10s$

$$s = ut + \frac{1}{2}at^2 = 0 \times 10 + \frac{1}{2} \times 4 \times (10)^2 = 200\,m.$$

20. Here , $u = 5\text{ms}^{-1}$

As the acceleration acts in the opposite direction of initial velocity, so it is negative.

$a = -10\ \text{ms}^{-2}$

At the highest point, $v = 0$

Using, $v^2 - u^2 = 2as$, we get

$0^2 - 5^2 = 2 \times (-10) \times s$

$s = \frac{25}{20} = 1.25$ m

∴ Height attained by the stone = 1.25 m.

Again, $v = u + at$

⇒ $0 = 5 - 10 \times t$

or $t = \frac{5}{10} = 0.5\text{s}$

∴ Time taken by the stone to reach the highest point = 0.5 s.

Text-Book Exercise :

1. Diameter of circular path (d) = 200 m

∴ Radius of circular path, r = 100 m

Time period (T) = 40 s

Time (t) = 2 min, 20 s

= (120 + 20)s = 140 s

Therefore, number of revolutions

$$= \frac{\text{Time taken}}{\text{Time period}} = \frac{140\text{s}}{40\text{s / revolution}} = 3.5 \text{ revolution}$$

Distance travelled

$= 2\pi r \times$ No. of revolutions

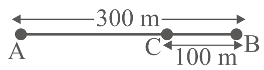
$= 2 \times \frac{22}{7} \times 100 \text{ m} \times 3.5 = 2200$ m

Now during 3 revolutions, the displacement is zero, since the athlete has reached the starting point.

Therefore, **actual displacement** is due to half revolution

= Diameter of circular track = 200 m

2. (a) For motion from A to B :



Distance covered = 300 m

Displacement = 300 m

Time taken = 2 minutes 30 seconds

= 2 × 60 + 30 = 150 s

$$\text{Average speed} = \frac{\text{Distance covered}}{\text{Time taken}}$$

$$= \frac{300 \text{ m}}{150 \text{ s}} = 2 \text{ ms}^{-1}.$$

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time taken}} = \frac{300 \text{ m}}{150 \text{ s}}$$

$= 2\ \text{ms}^{-1}$.

(b) For motion from A to B to C :

Distance covered = 300 + 100 = 400 m

Displacement = $AB - CB$

= 300 – 100 = 200 m

Time taken = 150 + 60 = 210 s

$$\text{Average speed} = \frac{300 \text{ m}}{150 \text{ s}} = \frac{400 \text{ m}}{210 \text{ s}} = 1.90 \text{ ms}^{-1}.$$

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time taken}}$$

$$= \frac{200 \text{ m}}{210 \text{ s}} = 0.952 \text{ ms}^{-1}.$$

3. Let one way distance = x km

Time taken in forward trip at a speed of 20 km/h

$$= \frac{\text{Distance}}{\text{Speed}}$$

$$t_1 = \frac{x}{20}\text{h}$$

Time taken in return trip at a speed of 30 km/h

$$t_2 = \frac{x}{30}\text{h}$$

Total time for the whole trip

$$= \frac{x}{20} + \frac{x}{30} = \frac{3x + 2x}{60} = \frac{5x}{60} h$$

Total distance covered = x + x = 2x km

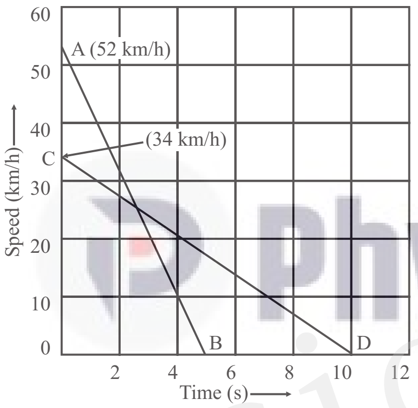
$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{2x}{5x/60}$$

$$= \frac{2x \times 60}{5x} = 24 \text{ km h}^{-1}.$$

4. Here, $u = 0$, $a = 3 \text{ ms}^{-2}$, $t = 8$ s

$$s = ut + \frac{1}{2}at^2 = 0 \times 8 + \frac{1}{2} \times 3 \times (8)^2 = 96 \text{ m}.$$

5. In the given Fig. AB and CD represent the speed-time graphs for the two cars whose initial speeds are 52 km/h and 34 km/h, respectively.



Distance covered by first car before coming to rest = Area of triangle AOB

$$= \frac{1}{2} \times AO \times BO = \frac{1}{2} \times 52 \text{ km/h} \times 5 \text{ s}$$

$$= \frac{1}{2} \times \frac{52 \times 5}{18} \text{ m/s} \times 5 \text{ s} = 36.1 \text{ m}$$

Distance covered by the second car before coming to rest

$$= \text{Area of triangle COD} = \frac{1}{2} \times CO \times DO$$

$$= \frac{1}{2} \times 34 \text{ km/h} \times 10 \text{ s}$$

$$= \frac{1}{2} \times \frac{34 \times 5}{18} \text{ m/s} \times 10 \text{ s} = 47.2 \text{ m}.$$

Thus, the second car travelled farther than the first car after the brakes were applied.

6. (a) Speed of A = Slope of $PN = \frac{10 - 6}{1.1 - 0}$

$$= \frac{40}{11} \text{ km/h} = 3.63 \text{ km/h}$$

Speed of B = Slope of $OM = \frac{6 - 0}{0.7 - 0}$

$$= \frac{60}{7} \text{ km/h} = 8.57 \text{ km/h}$$

Speed of C = Slope of $QM = \frac{6 - 2}{0.7 - 0}$

$$= \frac{40}{7} \text{ km/h} = 5.71 \text{ km/h}$$

The slope of line B is more than that of line A and C hence B is travelling the fastest.

(b) No, all three do not meet at any point on the road.

(c) When B passes A at point N (at 1.2 hours), C is at a distance of approximately 8 km from the origin O.

(d) B passes C at 0.7 hours. During this time B covers distance of 6 km.

7. Here, $u = 0$, $s = 20$ m, $a = 10 \text{ ms}^{-2}$, $v = ?$, $t = ?$

As $v^2 - u^2 = 2as$

$\therefore$ $v^2 - 0^2 = 2 \times 10 \times 20 = 400$

or $v = 20 \text{ ms}^{-1}$.

and $v = u + at$, $t = \frac{v - u}{a} = \frac{20 - 0}{10} = 2$ s.

8. (a) On horizontal axis, 5 small divisions = 2 s

On vertical axis, 3 small divisions = 2 ms^{-1}

$\therefore$ Area of 15 small squares

$$= 2 \text{ s} \times 2 \text{ ms}^{-1} = 4 \text{ m}$$

$\therefore$ Area of 1 small square $= \frac{4}{15}$ m

Total area under the speed-time graph from time 0 to 4 s

$$= 57 \text{ small squares} + \frac{1}{2} \times 6 \text{ small squares}$$

= 60 small squares

Distance travelled by the car in first 4 seconds

= Area under the speed-time graph from 0 to 4 s

= 60 small squares

$$= 60 \times \frac{4}{15} \text{ m}$$

= 16 m.

(b) After 6 s, the car has a uniform motion for straight part of graph BC.

9. (a) Yes, a body can have acceleration even if its velocity is zero. When a body is thrown up, at highest point its velocity is zero but it has acceleration equal to acceleration due to gravity.

(b) Yes, when an object moves on a circular path, it has centripetal acceleration which is perpendicular to displacement.

10. Here, r = 42,250 km = 42,250 × 1000 m

T = 24 h = 24 × 60 × 60 s.

$$\text{Speed} = \frac{\text{distance}}{\text{time}},$$

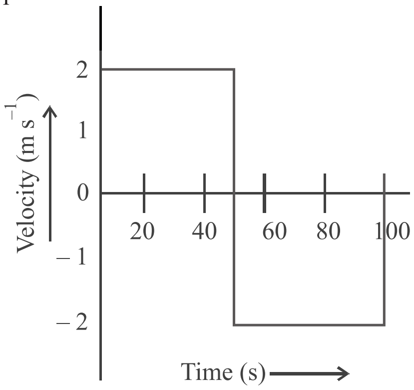
$$v = \frac{2\pi r}{T} = \frac{2 \times 3.14 \times 42,250 \times 1000}{24 \times 60 \times 60} \text{ m/s}$$

$= 3070.9$ m/s $\simeq 3.07$ km/s.

Exemplar Questions :

1. No. Though the moving object comes back to its initial position the distance travelled is not zero.

2.



3. The distance travelled in first 8 s,

$x_1 = 0 + \frac{1}{2}(5)(8)^2 = 160$ m.

At this point the velocity v = u + at = 0 + (5 × 8) = 40 ms^{-1}.

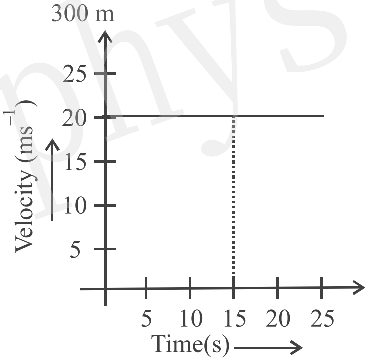
Therefore, the distance covered in last four seconds, x_2 = (40 × 4)m = 160 m

Thus, the total distance x = $x_1 + x_2$ = (160 + 160)m = 320 m

4. (i) Since velocity is not changing, acceleration is equal to zero.

(ii) Reading the graph, velocity = 20 ms^{-1}

(iii) Distance covered in 15 seconds, s = u × t = 20 × 15 = 300 m



5. Initial difference in height = (150 – 100)m = 50 m

Distance travelled by first body in 2s = h_1

$= 0 + \frac{1}{2}g(2)^2 = 2g$

Distance travelled by another body in 2s = h_2

$= 0 + \frac{1}{2}g(2)^2 = 2g$

After 2s, height at which the first body will be = h_1' = 150 – 2 g

After 2s, height at which the second body will be = h_2' = 100 – 2 g

Thus, after 2s, difference in height = 150 – 2 g – (100 – 2 g) = 50 m = initial difference in height

Thus, difference in height does not vary with time.

6. $s_1 = ut + \frac{1}{2}at^2$ or $20 = 0 + \frac{1}{2}a(2)^2$ or a = 10 ms^{-2},

v = u + at = 0 + (10 × 2) = 20 ms^{-1}

$s_2 = 160 = vt' + \frac{1}{2}a'(t)^2 = (20 \times 4) + \left(\frac{1}{2}a' \times 16\right) \Rightarrow a' = 10$ ms^{-2}

Since acceleration is the same, we have v′ = u + at = 0 + (10 × 7) = 70 ms^{-1}

Hots Questions :

1. The boat that sails directly with the wind can sail no faster than wind speed. Why? Even sailing as fast as the wind, there would be no wind impact against the sail. It would sag. But when sailing crosswind, there would still be wind impact against the sail, and speeds greater than wind speed can be achieved.

2. Although its speed and velocity at the top will both instantaneously be zero, its acceleration will be g or 9.8 m/s^2. Remember, acceleration is not speed or velocity — it is the rate at which velocity changes. A moment before or after the rock reaches the top, it is moving, which is evidence that its velocity is changes at every instant. The rock undergoes a change as it passes through the zero value of velocity just as it undergoes the same rate of change passing through any other value of velocity or look at it via Newton's 2nd law. At the top or anywhere in its path, the rock has both weight and mass, and

$$a = \frac{F}{m} = \frac{mg}{m} = g$$

3. Yes.

When the two balls are thrown vertically upwards with the same speed *u* then their final sp eed *v* at the point of projection is

$v^2 - u^2 = 2 \times g \times s$

Here, $s = 0$ ∴ $v = u$ for both the cases

Thus, we find that final velocity is independent of mass.

4. Let t_1 & t_2 be time taken to reach the ground then from the formula,

$$h = \frac{1}{2}gt^2,$$

For first body, $16 = \frac{1}{2}gt_1^2$

For second body, $25 = \frac{1}{2}gt_2^2$

$$\therefore \frac{16}{25} = \frac{t_1^2}{t_2^2} \Rightarrow \frac{t_1}{t_2} = \frac{4}{5}.$$

5. $s_n = u + \frac{a}{2}(2n-1)$ or $s_n = 0 + \frac{a}{2}(2n-1)$ ...(1)

Further distance covered in *n* seconds is

$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}an^2$ ...(2)

$\therefore \frac{s_n}{s} = \frac{\frac{a}{2}(2n-1)}{(an^2/2)} = \frac{2}{n} - \frac{1}{n^2}$

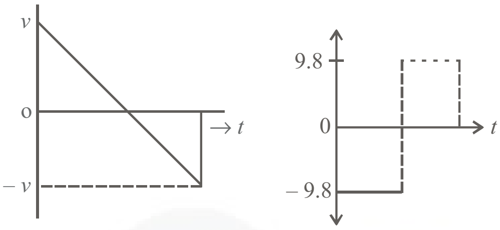
6. Velocity when the engine is switched off
$v = 19.6 \times 5 = 98\, m\, s$

$h_{max} = h_1 + h_2$ where $h_1 = \frac{1}{2}at^2$ & $h_2 = \frac{v^2}{2a}$

$h_{max} = \frac{1}{2} \times 19.6 \times 5 \times 5 + \frac{98 \times 98}{2 \times 9.8}$

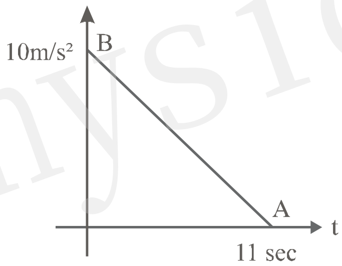
$= 245 + 490 = 735$ m

7.



8. The area under acceleration time graph gives change in velocity. As acceleration is zero at the end of 11 sec.
i.e. vmax = Area of D OAB

$\frac{1}{2} \times 11 \times 10 = 55$ m/s



3 EXERCISE

SINGLE OPTION CORRECT :

1. (b) $v = [144 \times 1000/(60 \times 60)]$ m/sec.
$v = u + at$
or $(144 \times 1000)/(60 \times 60) = 0 + a \times 20$
$\therefore a = \frac{144 \times 1000}{60 \times 60 \times 20} = 2\,\text{m/sec}^2$
Now $s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 2 \times (20)^2 = 400$ m

2. (c) $v = \frac{dy}{dt} = b + 2ct - 4dt^3$
$v_0 = b + 2c(0) - 4d(0)^3 = b$
($\because$ for initial velocity, $t = 0$)

Now $a = \frac{dv}{dt} = 2c - 12dt^2$
$\therefore a_0 = 2c - 12d(0)^2 = 2c$, (at $t = 0$)

3. (c) This is because speed can never be negative.

4. (c) $v = \frac{dx}{dt} = a_1 + 2a_2t$ $\therefore a = \frac{dv}{dt} = 2a_2$

5. (d) $v_A = \tan 30^\circ$ and $v_B = \tan 60^\circ$
$\therefore \frac{v_A}{v_B} = \frac{\tan 30^\circ}{\tan 60^\circ} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3}$

6. (a) Let for the first half time t, the person travels a distance s_1. Hence $v_1 = \frac{s_1}{t}$ or $s_1 = v_1 t$
For second half time, $v_2 = \frac{s_2}{t}$ or $s_2 = v_2 t$
Now, $\bar{v} = \frac{\text{Total displacement}}{\text{Total time}} = \frac{s_1 + s_2}{2t}$
$= \frac{v_1 t + v_2 t}{2t} = \frac{v_1 + v_2}{2}$

7. (d) $\frac{\frac{x}{2} + \frac{x}{2}}{\frac{x}{2v_1} + \frac{x}{2v_2}} = \frac{1}{\left(\frac{v_2 + v_1}{2v_1 v_2}\right)} = \frac{2v_1 v_2}{v_1 + v_2}$

8. (b) Let u be the initial velocity
$\therefore v_1' = u + at_1,\ v_2' = u + a(t_1 + t_2)$
and $v_3' = u + a(t_1 + t_2 + t_3)$
Now $v_1 = \frac{u + v_1'}{2} = \frac{u + (u + at_1)}{2} = u + \frac{1}{2}at_1$
$v_2 = \frac{v_1' + v_2'}{2} = u + at_1 + \frac{1}{2}at_2$
$v_3 = \frac{v_2' + v_3'}{2} = u + at_1 + at_2 + \frac{1}{2}at_3$
So, $v_1 - v_2 = -\frac{1}{2}a(t_1 + t_2)$
and $v_2 - v_3 = -\frac{1}{2}a(t_2 + t_3)$
$\therefore (v_1 - v_2) : (v_2 - v_3) = (t_1 + t_2) : (t_2 + t_3)$

9. (b) Let after a time t, the cyclist overtake the bus. Then
$96 + \frac{1}{2} \times 2 \times t^2 = 20 \times t$ or $t^2 - 20t + 96 = 0$
$\therefore t = \frac{20 \pm \sqrt{400 - 4 \times 96}}{2 \times 1}$
$= \frac{20 \pm 4}{2} = 8$ sec and 12 sec

10. (d) $v^2 = u^2 + 2\,a\,s$ or $v^2 - u^2 = 2\,a\,s$

Maximum retardation, $a = v^2/2\,s$

When the initial velocity is nv, then the distance over which it can be stopped is given by

$$s_n = \frac{u_0^2}{2a} = \frac{(nv)^2}{2(v^2/2s)} = n^2 s$$

11. (b) $u = 10$ m/s, $t = 5$ sec, $v = 20$ m/s, $a = ?$

$$a = \frac{20-10}{5} = 2\ ms^{-2}$$

From the formula $v_1 = u_1 + a\,t$, we have

$10 = u_1 + 2 \times 3$ or $u_1 = 4$ m/sec.

12. (c) Downward motion

$$v^2 - 0^2 = 2 \times 9.8 \times 5 \quad \Rightarrow \quad v = \sqrt{98} = 9.9$$

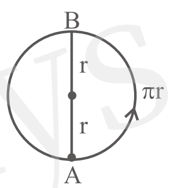
Also for upward motion

$$0^2 - u^2 = 2 \times (-9.8) \times 1.8 \quad \Rightarrow \quad u = \sqrt{3528} = 5.94$$

$$\text{Fractional loss} = \frac{9.9 - 5.94}{9.9} = 0.4 = \frac{2}{5}$$

13. (b) When a particle cover half of circle of radius r, then displacement is AB = $2r$

& distance = half of circumference of circle = πr



14. (c) The distance covered in n^{th} second is

$$S_n = u + \frac{1}{2}(2n-1)a$$

where u is initial velocity & a is acceleration then

$$26 = u + \frac{19a}{2} \quad \text{....(1)}$$

$$28 = u + \frac{21a}{2} \quad \text{....(2)}$$

$$30 = u + \frac{23a}{2} \quad \text{....(3)}$$

$$32 = u + \frac{25a}{2} \quad \text{....(4)}$$

From equations (1) & (2) we get $u = 7$m/sec, $a = 2$m/sec^2

$\therefore$ The body starts with initial velocity $u = 7$m/sec and moves with uniform acceleration $a = 2$m/sec^2

15. (b) In first case retardations,

$$a = \frac{0-v}{t} = -\frac{v}{t} \quad \therefore \quad x = vt + \frac{1}{2}at^2 = vt - \frac{vt}{2} = \frac{vt}{2}$$

In second case retardation is,

$$a' = \frac{0-nv}{t} = -\frac{nv}{t} \quad \therefore \quad s = nvt - \frac{1}{2}\frac{nv}{t}.t^2 = \frac{nvt}{2}$$

but $\frac{vt}{2} = x \quad \therefore s = nx$

16. (a) From third equation of motion : $v^2 = u^2 + 2as$

for first case $u = \frac{40 \times 10}{36}$ m/sec,

$v = 0$, $a = ?$, $s = 2$ m

So, $a = \left(\frac{40 \times 10}{36}\right)^2 \frac{1}{4}$ m/sec^2

for second case $u = \frac{80 \times 10}{36}$ m/sec, $v = 0$,

So $s_2 = \left(\frac{80 \times 10}{36}\right)^2 / 2 \times \frac{1}{4} \times \left(\frac{40 \times 10}{36}\right)^2 = 8$ meter

17. (d) Speed $v_1 = 60 \times \frac{5}{18}$ m/s $= \frac{50}{3}$ m/s

$d_1 = 20$ m, $v_1' = 120 \times \frac{5}{18} = \frac{100}{3}$ m/s

Let deceleration be a

$\therefore 0 = v_1^2 - 2ad_1$ ....(1)

or $v_1^2 = 2ad_1$

$(2v_1)^2 = 2ad_2$ ...(2)

(2)divided by (1) gives,

$$4 = \frac{d_2}{d_1} \Rightarrow d_2 = 4 \times 20 = 80\,\text{m}$$

18. (b) Distance in last two second

$= \frac{1}{2} \times 10 \times 2 = 10$ m.

Total distance $= \frac{1}{2} \times 10 \times (6 + 2) = 40$ m.

19. (b) **20.** (b) **21.** (b)

22. (b) At a particular time, two values of velocity are not possible.

23. (b) **24.** (b) **25.** (b) **26.** (b)

27. (c) **28.** (d)

More Than One Option Correct :

1. (a, b) **2.** (a, b) **3.** (a, c) **4.** (c, d)

5. (a, c) **6.** (a, d) **7.** (a, d) **8.** (b, c)

9. (a, d) **10.** (a, b)

Multiple Matching Questions :

1. (a) (A) → (r, s, u); (B) → (t); (C) → (q); (D) → (p)

2. (b) (A) → (p, u); (B) → (s, t); (C) → (q); (D) → (r, u)

3. (d) (A) → (p, s); (B) → (p, q, r, s, t); (C) → (p, r, s, t); (D) → (p, q, r, s, t)

Passage Based Questions :

1. (d) The total distance travelled is (18 + 9) m = 27 m

2. (a) 9 m **3.** (a) Net displacement = 9 m

4. (a) **5.** (a) **6.** (c)

7. (d) Initial velocity of police jeep w.r.t thief's motorcycle,

$u_{pt} = u_p - u_t = -25\ ms^{-1}$

Acceleration of police jeep w.r.t thief's motorcycle,

$a_{pt} = a_p - a_t = 2\ ms^{-2}$

Using, $s = ut + \frac{1}{2}at^2$, we get

$1250 = -25t + t^2$

or $t^2 - 1250 = 0$

or $t = 50s, -25s$

but time cannot be –ve, $\therefore\ t = 50s$

8. (a) Distance covered by jeep in 50*s*, $s = ut + \frac{1}{2}at^2$ = 2500 m = 2.5 km.

9. (b) Let *a* be the acceleration of thief, then

$a_{pt} = a_p - a_t = 2 - a$

We have $u_{pt}{}^2 - u_{pt}{}^2 = 2a_{pt}s_{pt}$

$u_{pt}{}^2 = u_{pt}{}^2 + 2a_{pt}s_{pt}$

Thief will escape from police if, $v_{pt}{}^2 \le 0$

$u_{pt}{}^2 + 2a_{pt}s_{pt} \le 0$

or $625 + 2(2 - a) \times 1250 \le 0$

or $a \ge \frac{5}{4}$ or $a \ge 1.25\ ms^{-2}$

Assertion & Reason :

1. (b) **2.** (c) **3.** (d) **4.** (a) **5.** (d)

6. (c) Displacement is shortest possible distance between initial and final position.

7. (d) $s \times t^2$, hence $s - t$ graph for uniformly accelerated motion is parabola.

Integer/Numeric type Qustions :

1. (i) $0.04\ ms^{-2}$ (ii) $30.31\ ms^{-1}$

2. (a) $0.25\ ms^{-2}$ (b) $3240\ km\ hr^{-2}$

3. $25 ms^{-1}$

4. 4 s

5. $0\ ms^{-1}$

4 ADVANCED EXERCISE
BASED ON CONNECTING TOPICS

1. (c) From third equation of motion, $v^2 = u^2 + 2as$

where v & u are final & initial velocity, a is acceleration, *s* is distance.

For first case v_1= 3km/hour

$u_1 = 0,\ a_1 = g\ \&\ s_1 = ?$

$s_1 = \frac{9 \times 100}{36 \times 36 \times 20}$ metre

For second case $v_2 = ?,\ u_2 = 4$km/hour ,

a_2= g = 10m/sec

$\&\ s_1 = s_2 = \frac{9 \times 100}{36 \times 36 \times 20}$

so $v_2^2 = \frac{16 \times 1000 \times 1000}{3600 \times 3600} + \frac{2 \times 10 \times 9 \times 100}{20 \times 36 \times 36}$

or $v_2 = 5$ km/hour

2. (a) $s = ut + \frac{1}{2}at^2$ here $a = g$

For first body $u_1 = 0 \Rightarrow s_1 = \frac{1}{2}g \times 9$

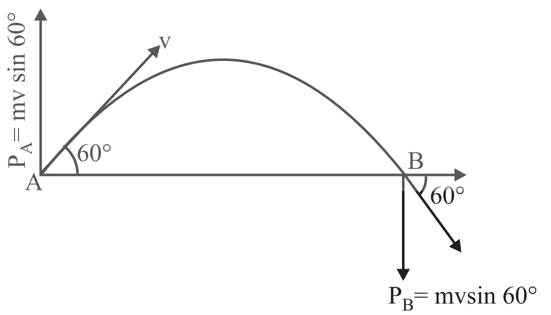
For second body $u_2 = 0 \Rightarrow s_2 = \frac{1}{2}g \times 4$

So difference between them after 3 sec.

$= s_1 - s_2 = \frac{1}{2}g \times 5$

If $g = 10$m/sec^2 then $s_1 - s_2 = 25$m.

3. (b)



As the figure drawn above shows that at points A and B the vertical component of velocity is v sin 60° but their directions are opposite.

Hence, change in momentum is given by :

$$\Delta p = mv\sin 60° - (-mv\sin 60°) = 2mv\sin 60°$$

$$= 2mv\frac{\sqrt{3}}{2} = \sqrt{3}mv$$

4. (b) Relative speed of each train with respect to each other be, v = 10 + 15 = 25 m/s

Here distance covered by each train = sum of their lengths = 50 + 50 = 100 m

$\therefore$ Required time $= \frac{100}{25} = 4$ sec.

5. (b) From third equation of motion

$v^2 = u^2 - 2gh \quad (\because a = -g)$

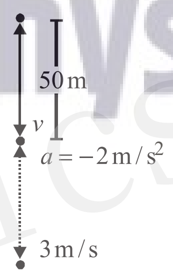
Given, v = 10 m/sec at $\frac{h}{2}$. But v = 0, when particle attained maximum height h.

Therefore $(10)^2 = u - \frac{2gh}{2}$

or 100 = 2gh – 2gh/2 $\qquad (\because 0 = u^2 - 2gh)$

$\Rightarrow h = 10$ m

6. (d) Initial velocity of parachute after bailing out,



$u = \sqrt{2gh}$

$u = \sqrt{2 \times 9.8 \times 50} = 14\sqrt{5}$

The velocity at ground,

v = 3m/s

$$S = \frac{v^2 - u^2}{2 \times 2} = \frac{3^2 - 980}{4} \approx 243\,\text{m}$$

Initially he has fallen 50 m.

$\therefore$ Total height from where he bailed out

= 243 + 50 = 293 m

7. (b) $R_1 = \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin 30°}{g}$

or $\quad 1.5 = \frac{u^2}{2g} \quad or \quad \frac{u^2}{g} = 3$

$$R_2 = \frac{u^2 \sin 90°}{g} = \frac{u^2}{g} = 3\,\text{km}$$

8. (b) The bullets are fired at the same initial speed

$$\frac{H}{H'} = \frac{u^2 \sin^2 60°}{2g} \times \frac{2g}{u^2 \sin^2 30°} = \frac{\sin^2 60°}{\sin^2 30°}$$

$$= \frac{(\sqrt{3}/2)^2}{(1/2)^2} = \frac{3}{1}$$

9. (d) $\frac{H_1}{H_2} = \frac{u^2 \sin^2 \theta / 2g}{u^2 \sin^2 (90° - \theta)/2g} = \tan^2 \theta$

10. (c) We know that, $y_m = H = \frac{(u \sin\theta)^2}{2g} = \frac{u^2 \sin^2\theta}{2g}$

$\therefore \quad \frac{\Delta H}{H} = \frac{2\Delta u}{u}$. Given $\frac{\Delta u}{u} = 2\%$

$\therefore \quad \frac{\Delta H}{H} = 2 \times 2 = 4\%$

11. (d) Time of flight $= \frac{2u\sin\theta}{g}$

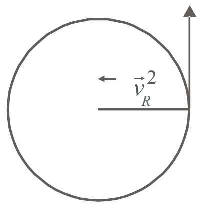
$$= \frac{2 \times 9.8 \times \sin 30°}{9.8} = 2 \times \frac{1}{2} = 1\,\text{sec}.$$

12. (c) $\frac{u^2\, 2\sin\theta\cos\theta}{g} = \frac{u^2 \sin^2\theta}{2g}$ or tan θ = 4.

13. (b) Acceleration vector is always radial (i.e . towards the centre) for uniform circular motion.

14. (a) In uniform circular motion speed is constant. So, no tangential acceleration.

It has only radial acceleration $a_R = \frac{v^2}{R}$ [directed towards center]

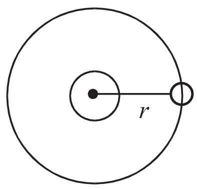


and its velocity is always in tangential direction. So these two are perpendicular to each other.

15. (b) Particle will strike the point B if velocity of particle with respect to platform is along AB or component of its relative velocity along AD is zero, i.e. $u\cos\theta = v$

or $\theta = \cos^{-1}\left(\frac{v}{u}\right)$

16. (a) In circular motion of a particle with constant speed, particle repeats its motion after a regular interval of time but does not oscillate about a fixed point. So, motion of particle is periodic but not simple harmonic.



17. (c) Since horizontal component of the velocity of the bomb will be the same as the velocity of the aeroplane, therefore horizontal displacements remain the same at any instant of time.

18. (d) $T = \frac{2u\sin(\theta - \alpha)}{g\cos\alpha}$

19. (d) Velocity and hence kinetic energy is minimum at the highest point.

$K.E = \frac{1}{2}mv^2\cos^2\theta$

20. (d) The initial velocity in the vertically downward direction is zero and same height has to be covered.

21. (c) $x + u_2\cos\theta_2 t = u_1\cos\theta_1 t$

$\therefore \quad t = \frac{x}{u_1\cos\theta_1 - u_2\cos\theta_2}$ ...(i)

Also $u_1\sin\theta_1 = u_2\sin\theta_2$ ...(ii)

After solving above equations, we get

$t = \frac{x\sin\theta_2}{u_1\sin(\theta_2 - \theta_1)}$

22. (a) We have,

$R = \frac{u^2\sin 2\theta}{g} = \frac{u^2}{g} \quad (\because \theta = 45°)$

$H = \frac{u^2\sin^2\theta}{2g} = \frac{u^2}{4g} (for\ \theta = 45°)$

$\Rightarrow R = 4H$

23. (d) When a cyclist moves on a circular path, it experiences a centrifugal force which is equal to mv^2/r. It tries to overturn the cyclist in outward direction. If speed increases twice, the value of centrifugal force too increases to 4 times its earlier value. Therefore the chance of overturning is 1/4 times.

24. (c) The time of flight of a projectile is given as,

$t = \frac{2u\sin\theta}{g}$

A projectile can have same range if angle of projection are complementary i.e., θ and (90 – θ)

$\therefore t_1 = \frac{2u\sin\theta}{g}$

and $t_2 = \frac{2u\sin(90-\theta)}{g} = \frac{2u\cos\theta}{g}$

$\therefore t_1 t_2 = \frac{4u^2\sin\theta\cos\theta}{g^2}$

$= \frac{4u^2}{g^2} \times \frac{sin 2\theta}{2} = \frac{2u^2}{g^2}\sin 2\theta = \frac{2R}{g}$

where range $(R) = \frac{u^2\sin 2\theta}{g}$

25. (d) All options are correct :

(i) When two bodies A & B move in opposite directions then relative velocity between A & B either v_{AB} or v_{BA} both are greater than v_A & v_B.

(ii) When two bodies A & B move in parallel direction then $v_{AB} = v_A - v_B \Rightarrow v_{AB} < v_A$

$v_{BA} = v_B - v_A \Rightarrow v_{BA} < v_B$

26. (b, c) Taking upward motion of the balloon for 4 seconds, we have, u = 0; a = 2.50 ms^{-2};

t = 4s; v = ?; s = ?

v = u + at = 0 + 2.5 × 4 = 10m^{s-1}

$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 2.5 \times 4^2 = 20m.$

When stone is released from the balloon at the height of 20m, it retains the velocity of balloon i.e. 10m/s upwards but not its acceleration.

Taking downward motion of stone, when released from balloon to ground, we have

u = – 10ms^{-1}; a = 10ms^{-2}, s = 20m, t = ?

As $s = ut + \frac{1}{2}at^2$; so $20 = -10t + \frac{1}{2} \times 10t^2$

or $5t^2 - 10t - 20 = 0$

On solving t = 3.2 s

Distance covered by stone after being released from balloon up to highest point of its path is

$v^2 = u^2 + 2as$

$0 = 10^2 + 2(-10)s$ or s = 5m

Total distance travelled = 5 + 5 × 20 = 30m.

27. (a, c) A stone thrown vertically up with velocity 20 ms^{-1} from top of building will go up and return to the point of projection with downward velocity 20 ms^{-1}.

So both will strike the ground with same speed and hence same K.E. Acceleration of each stone is acceleration due to gravity acting downwards. So the relative acceleration of two stones is zero.

28. (a, c)

In uniform circular motion, magnitude of acceleration and velocity is constant but not their directions.

29. (a, c)

The particle has initially horizontal as well as vertical velocity hence its motion is projectile.

30. (a, b)

31. (b, c, d)

Centripetal acceleration,

$a_c = \frac{v^2}{r} = \omega^2 r = 4\pi^2 v^2 r = \omega v = \frac{4\pi^2 r}{T^2}$

32. (a, b, d)

For same horizontal range

$\theta_1 + \theta_2 = 90°$

$\therefore \frac{f_1}{f_2} = \frac{\sin\theta_1}{\sin\theta_2} = \frac{\sin\theta_1}{\cos\theta_1} = \tan\theta_1$ or $\frac{f_1}{\sin\theta_1} = \frac{f_2}{\sin\theta_2}$

33. (a, b)

For circular motion $a_r \neq 0$

34. (a) (A)–(q); (B)–(p); (C)–(r); (D)–(r)

When a body is dropped from some height then it falls with a constant acceleration g and when a body is projected vertically upward its velocity at the highest point is zero but its acceleration = g towards the earth.

In case of motion under gravity time of ascent = time of descent = $\frac{u}{g}$

35. (d) (A)–(q, s); (B)–(t); (C)–(p); (D)–(q, r)

36. (d) Initial velocity of police jeep w.r.t thief's motorcycle,

$u_{pt} = u_p - u_t = -25 \text{ ms}^{-1}$

Acceleration of police jeep w.r.t thief's motorcycle,

$a_{pt} = a_p - a_t = 2 \text{ ms}^{-2}$

Using, $s = ut + \frac{1}{2}at^2$, we get

$1250 = -25t + t^2$

or $t^2 - 1250 = 0$

or $t = 50s, -25s$

but time cannot be –ve, $\therefore t = 50s$

37. (a) Distance covered by jeep in 50s, $s = ut + \frac{1}{2}at^2$ = 2500 m = 2.5 km.

38. (b) Let a be the acceleration of thief, then

$a_{pt} = a_p - a_t = 2 - a$

We have $u_{pt}{}^2 - u_{pt}{}^2 = 2a_{pt}s_{pt}$

$u_{pt}{}^2 = u_{pt}{}^2 + 2a_{pt}s_{pt}$

Thief will escape from police if, $v_{pt}{}^2 \leq 0$

$u_{pt}{}^2 + 2a_{pt}s_{pt} \leq 0$

or $625 + 2(2 - a) \times 1250 \leq 0$

or $a \geq \frac{5}{4}$ or $a \geq 1.25 \text{ ms}^{-2}$

39. (a) $t = \frac{2\pi r}{v} = \frac{2 \times \frac{22}{7} \times 3.5}{4} = 5.5\text{s}$

40. (d) Displacement of the particle in one complete round is zero, hence average velocity of the particle in one complete round is zero.

41. (b) Acceleration

$a = \frac{v^2}{r} = \frac{4^2}{3.5} = \frac{32}{7}\text{ms}^{-2}$

42. (c) Centripetal acceleration is not constant as it varies in direction.

43. (a) Acceleration due to gravity acts vertically downwards hence horizontal velocity does not change.

44. (d) In projectile motion, acceleration is perpendicular to velocity only at the heighest point of its trajectory.

45. $u = 50$ m/s, $t = 3s$, $x = ut = 150$ m;

$y = \frac{1}{2}gt^2 = 44.1\text{m}$

$v_x = u_x + a_x t = 50$ m/s; $v_y = u_y + a_y t = 29.4$ m/s ;

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{50^2 + (29.4)^2} = 58$ m/s

$\tan\theta = \frac{v_y}{v_x} = \frac{29.4}{50} = 0.588 = \tan 30°27'$

46. $R = 2H \Rightarrow \frac{u^2 \sin 2\theta}{g} = 2 \times \frac{u^2 \sin^2\theta}{2g}$

$\Rightarrow 2\sin\theta\cos\theta = \sin^2\theta \Rightarrow \tan\theta = 2$

$\therefore \sin\theta = \frac{2}{\sqrt{5}}$ and $\cos\theta = \frac{1}{\sqrt{5}}$

$\therefore R^2 = \frac{2u^2}{g}.\frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{5}} = \frac{4u^2}{5g}$

47. $\alpha = \frac{\omega - \omega_0}{t} = \frac{(400-100)\times 2\pi}{60 \times 5 \times 60} = \frac{\pi}{30}$ rad/s^2

48. Here, $r = 80$ cm ; $T = \frac{25}{14}s$

Now, $\omega = \frac{2\pi}{T} = \frac{2\pi}{25/14} = \frac{28\pi}{25}$ rad s^{-1}

The acceleration of uniform circular motion is given by

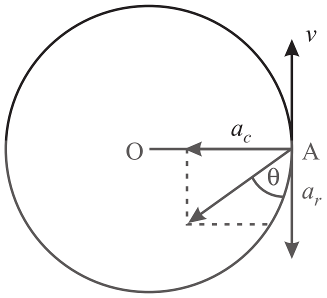
$a = \omega^2 r = \left(\frac{28\pi}{25}\right)^2 \times 80 = 990.4$ cm s^{-2}

The acceleration is directed along the radius of the circular path and *towards the centre* of the circle.

49. Here, $v = 27$ km h^{-1} = 7.5 m s^{-1}; $r = 80$ m

Centripetal acceleration, $a_c = \frac{v^2}{r} = \frac{(7.5)^2}{80} = 0.7$ m s^{-2}

Suppose that the cyclist applies brakes at the point A of the circular turn. Then, retardation produced due to the brakes, say a_T will act opposite to the velocity v [Fig]



Thus, $a_T = -0.5 \text{ m s}^{-2}$

Therefore, total acceleration is given by

$$a = \sqrt{a_c{}^2 + a_T{}^2} = \sqrt{(0.7)^2 + (-0.5)^2}$$

$$= \sqrt{0.49 + 0.25} = \sqrt{0.74} \quad = 0.86 \text{ m s}^{-2}$$

If θ is the angle between the total acceleration and the velocity of the cyclist, then,

$$\tan\theta = \frac{a_c}{a_T} = \frac{0.7}{0.5} = 1.4$$

or, $\theta = 54° - 28'$

50. $$a_c = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2}$$

$$= \frac{4\pi^2 (1.5\times10^{11}\text{m})}{(1yr)^2}\left(\frac{1yr}{3.15\times10^7 s}\right)^2$$

$$= 6.0\times10^{-3} \text{ m/s}^2$$

Chapter

2

FORCE AND LAWS OF MOTION

INTRODUCTION

In last chapter we dealt with different kinds of motion. We didn't bother about their cause. This chapter is concerned about the cause of motion (i.e., force) and its effect (i.e., acceleration) and their relationship. We will read different kinds of force.

When force is applied on a body, some acceleration is produced in it. These two quantities-force and acceleration are closely related. The existence of one causes the existence of other.

The great physicist sir Isaac Newton formulated three laws which govern the motion and its cause. This chapter deals with all the three laws of motion given by Newton and their applications in day to day life.

The chapter also gives us detailed knowledge of linear momentum, its conservation, impulse and their practical applications.

FORCE

In our daily life we find that a body at rest can be moved and one that is moving can be stopped. The doors and windows of a house are pushed to open and pulled to shut them. In games like cricket, hockey and football, players on the ground sometimes stop a moving ball or manage to deflect it to some other direction. The boatsman pulls a boat with a rope in order to bring it to the shore or pushes it to get it into the water. *This push or pull is called the force in everyday language*.

Force is a vector quantity (it has both magnitude and direction). There must be a net force (unbalanced force) acting on an object for the object to change its velocity (either magnitude and/or direction), or to accelerate.

If we want the car to stop, we have to do something to it. That is what our brakes are for : to exert a force that decreases the car's velocity. In a basketball, a player launches a shot by pushing on the ball.

Thus it can be said that the force is a physical influence which can change the state of motion or state of rest of a body. A force can change the direction of motion of a body.

A force is that physical quantity which tries to change or changes the state of rest or of uniform motion of a body.

To obtain a complete information about the force acting on an object one should know

(i) the point of application of force

(ii) the magnitude of force

(iii) the direction in which the force acts.

Units of Force

(i) The **S.I. unit** of force is newton.
One newton is the force which when acts on a body of mass 1 kg, produces an acceleration of 1 ms^{-2}.
i.e., 1 newton (N) = 1 kg × 1 ms^{-2}

(ii) In **C.G.S. system**, the unit of force is dyne.
One dyne is the force which when acts on a body of mass 1 gramme, produces an acceleration of 1 cm s^{-2}.
i.e., 1 dyne = 1 g × 1 cm s^{-2}

(iii) Units of force in terms of force due to gravity
In MKS system, the unit of force is the kilogramme force (kgf).
One kilogramme force is the force due to gravity on 1 kilogramme mass.
Thus, 1 kgf = force due to gravity on 1 kg mass = 1 kg mass × acceleration due to gravity m s^{-2} = g newton
Since the average value of g is 9.8 ms^{-2}
∴ *1 kgf = 9.8 newton (or 9.8 N)*

CHECK Point

- **If an object experiences no acceleration, there is no force acting on it.**

Solution

If an object has no acceleration, you cannot conclude that no forces act on it. In this case, you can only say that the net force on the object is zero.

Some Common Mechanical Forces

(i) Contact forces: When a body *M* is in contact with body *N*, *M* can exert force on *N* and *N* can exert a force on *M*. These forces are called contact forces. Push or pull by a person, force by wind, force by a weight on the head of a porter, frictional force, normal reaction force, tension in strings, forces exerted during collision are the examples of contact forces.

Practical examples

(a) The pulling of a trolley by a coolie.

(b) The pulling of a cart by a horse.

(c) The pushing of a door to close it.

(d) The stretching of a spring by suspending a load.

(e) The squeezing of a gum or toothpaste tube to extract the gum or toothpaste.

(ii) Elastic recoil: The special property of solids, in contrast to liquids and gases, is that they resist any change in their shape. When we push on a solid, it pushes back. The plank exerts an upward force against the hand because the hand is changing the shape of the board. The floor pushes up when our weight pushes down on it, because our weight changes its shape.

(iii) Tension: This is a special type of elastic recoil, resulting from stretching something. In a tug-of-war, the rope is under great tension, so it pulls back on the teams at the opposite ends. The spring scale is actually a tension-measuring device. The harder you pull on both ends of the spring, the more it stretches, and the harder it pulls back.

(iv) Compression: This is the opposite of tension. Pushing on a window pole shortens the pole-a little-and the pole pushes back. Actually, the bending board exerts its recoil because its upper surface is compressed and its lower surface is tensed.

(v) Buoyancy: Anything submerged in a liquid or a gas experiences an upward force. A rock under water feels lighter than the same rock in the air. If buoyancy is greater than weight, as for a cork under water or a helium-filled balloon in the air, the object rises.

If we are under water, we can rise to the top until we emerge into the air. Then, buoyancy and gravity are equal, and we float.

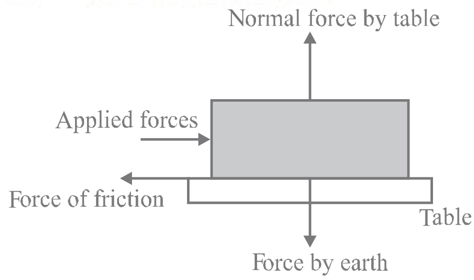
(vi) Viscous drag: Anything moving through a liquid or gas feels a retarding, friction-like force. That is why a boat needs an engine, why we cannot swim fast enough to get into the Olympics, why automobiles are streamlined, and why a parachute is advisable if we plan to fall out of an airplane.

(vii) Normal force: If contact force between the bodies is perpendicular to the surface in contact, the force is known as normal force. Let us consider a block on a table. The table pushes the block upwards and block pushes the table downwards. Then forces are perpendicular to the surfaces of block and table. Thus the table applies a normal force on block in the upward direction and block applies a normal force on table in

downward direction.

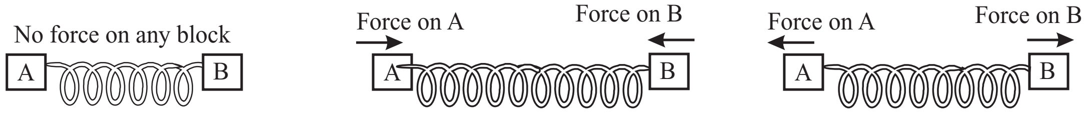
(viii) Friction: Two bodies placed in contact can also exert forces parallel to the surfaces in contact, such a force is called force of friction or simply friction. Suppose a body is placed on the table. Following three forces acts on it.

(1) Force by earth in downward direction
(2) Normal force due to table in upward direction.
(3) Applied force towards right.



Body is not moving, so all the forces must be balanced, normal force due to table and force by earth balanced each other. To balance the applied force there must be an equal and opposite force. This force is known as force of friction. If we increase the applied force the body is still at rest. It means force of friction is also increased till it is balanced by the applied force. The force of friction is self adjusting force. On increasing the applied force, the force of friction will increase up to a limit . It is known as limiting friction. After it, on increasing the applied force, the body will start to move.

(ix) Spring force: A spring is made of a coiled metallic wire. A spring has a definite length when it has been neither pushed nor pulled. The length is called natural length. At natural length the spring does not exert any force on the objects attached to its ends. If the spring is pulled at the ends its length becomes larger than natural length. It is known as stretched or extended spring. Extended spring pulls objects attached to its ends.



If the spring is pushed at the ends, its length becomes less than natural length. It is known as compressed spring. A compressed spring pushes the block attached to its ends.

(x) **Weight:** The earth attracts all the bodies towards its centre.The force exerted by the earth on the body is known as the weight of the body. It acts in vertically downward direction. These forces are not contact forces. If mass of the body is m and gravitational acceleration due to earth is g, then the weight of body is mg, here $g = 9.8$ m/sec^2.

BALANCED AND UNBALANCED FORCES

One effect of a force is to alter the dimension or shape of a body on which the force acts; like by loading a spring, there occurs an increase in its length. By hammering a small piece of silver sheet, thin foil is made. The steam pushing out from a pressure cooker occupies a large volume. On pressing a piece of rubber, its shape changes. In a cycle pump, when the piston is lowered, the air is compressed to occupy less volume. Another is to alter the state of motion of the body. A player applies force with a hockey stick to change the speed and direction of motion of a ball. When more force is applied on the pedal by a cyclist, speed of the cycle increases. In many situations we may find that a body remains at rest or moves uniformly even if it is acted on by a force ! For example try to push a heavy stone or a heavy iron safe, probably we may not be able to move it. Does it mean that something is wrong with the definition of force ? What happens actually is that there exists another force which is acting on the body in a direction opposite to your push and exactly compensating it. In effect there is no net force acting on the body. This force which we have not taken into account is the force of friction.

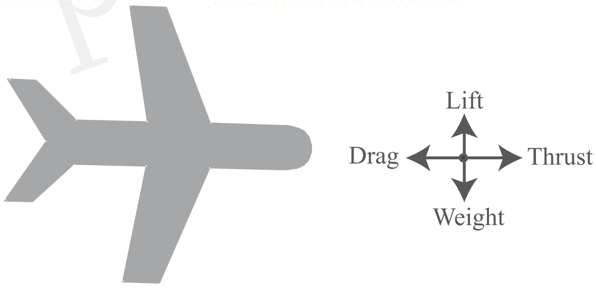
When several forces act on a body simultaneously, their effects can compensate one another with the result there is no change in the state of rest or motion. When this is the case, the body is said to be in **equilibrium**. Equivalently, one can say that the net force or resultant force acting on the body is zero for balanced forces. Balanced forces do not change the speed. This means that the body as a whole either remains at rest or moves in a straight line with constant rate.


Balanced forces

If a set of forces acting on a body produces no acceleration in it, the forces are said to be **balanced**.

An object in equilibrium may or may not be at rest. A parachutist, descending at constant speed, is in equilibrium. His weight is just balanced by the viscous drag on the parachute-which is why he put it on in the first place. A heavier parachutist falls a little faster, his speed increases until the viscous drag just balances his weight.

Balancing the vertical forces is not enough to produce equilibrium. An airplane traveling at constant speed, as in figure, is in equilibrium under the influence of four forces, two vertical and two horizontal.



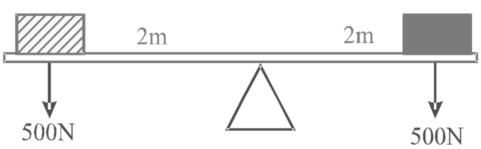
Vertical force: Weight or gravity (down) is just balanced by the lift produced by the flow of air across the wing.

Horizontal force: Viscous drag is just balanced by the thrust of the engines. Both the vertical and the horizontal velocities are constant.

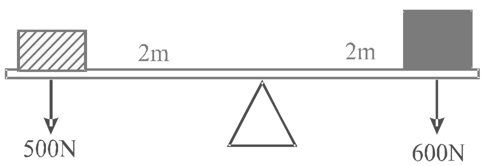
When the resultant force or net force acting on a body is not zero we say that it is acted on by an unbalanced force. A net or unbalanced force when acts on a body it changes its state of rest or uniform motion.

If a set of forces acting on a body produces non-zero acceleration in it, the forces are said to be **unbalanced.**

To imagine unbalanced force, imagine see saw. If the people on either side push down with the same force, they cancel each other. The see-saw is balanced and does not move [(fig. (a)]. In fig (b), there is an unbalanced force of 100N. This moves the see-saw down at one end.



(a)



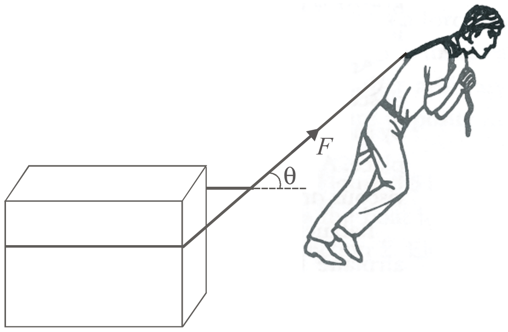
(b)

Resultant or Net force

If a single force acting on a body produces the same acceleration as produced by a number of forces then that single force is called the resultant force of these individual forces.

Components of a force

The carate in figure is being dragged along the floor by means of a rope, which is not horizontal. The rope makes an angle θ to the floor.



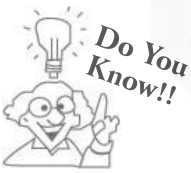
The tension in the rope, acting on the crate, does two things to it. First, it drags the crate across the floor. Second, it tends to lift the crate off the floor. The smaller the angle θ, the larger the effective force that is dragging the crate, and smaller the effective force that is lifting it. When θ = 0°, the entire force is dragging and there is no lifting at all. Conversely, when θ = 90°, the entire force is lifting the carate.

How can you find out how much force is being used to drag the crate ? Force is a vector, and it obeys the same mathematical rules as velocity vectors and displacement vectors. The dragging force is the component of the tension in the rope acting parallel to the floor, that is, the horizontal component

$$F_{horizontal} = F \cos \theta$$

Similarly, the component of the force tending to lift the carate is, the vertical component and is given by

$$F_{vertical} = F \sin \theta$$

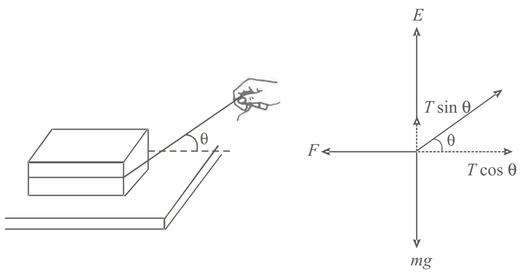


When a body is travelling at constant speed, the net force on it is zero.

EQUILIBRIUM OF A BODY WITH SEVERAL FORCES

For the purpose of making a complete analysis of the forces acting on an object, a vector diagram is a useful device.

A vector diagram showing the forces on a brick (figure) being dragged along a tabletop. Four forces act: gravity (weight), friction, elastic recoil of the tabletop, and tension in the cord. Each force is represented by a vector, drawn at the correct angle and with its length proportional to the force. For the purpose of analysis, all vectors are represented by components along a pair of axes perpendicular to each other. In this case, we select to use horizontal and vertical axes, since three of the forces are already on these axes. To do the analysis, we have to resolve the tension vector into its components on the vertical and horizontal axes. Then we can write two equations: one says that there is no net vertical force, and the other says that there is no net horizontal force.



CHECK Point

- **At the moment an object that has been tossed upward into the air reaches its highest point, is it in equilibrium? Defend your answer.**

Solution

- When an object reaches at the highest point of its path, when it is tossed upwards in air, its velocity is zero. The force due to gravity acts in the downward direction which balances upward force with which it is thrown. So it is in equilibrium.

ILLUSTRATION : 1

The 2.5 kilogram brick of figure in above section is being pulled by a cord that makes an angle of 20° with the horizontal and has 7.0 N of tension in it. Find (a) the force of friction, (b) the elastic recoil of the table top.

SOLUTION :

(a) On the horizontal axis, the friction must be equal to the horizontal component of the tension, so

$$F = (7.0\text{ N})(\cos 20°) \simeq 6.6\text{ N}$$

(b) On the vertical axis, the downward force (weight) must equal the sum of upward forces, so

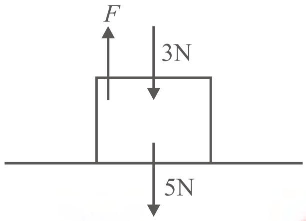
$$mg = E + T\sin\theta \quad \Rightarrow \quad E = mg - T\sin\theta$$

$$E = (2.5)(9.8) - (7.0)(\sin 20°) \qquad \text{[value are seen from trigonometric table]}$$

$$E = 24.5\text{ N} - 2.4\text{ N} = 22.1\text{ N}$$

ILLUSTRATION : 2

A block of weight 5N is placed on a horizontal table. A person push the block from top by exerting a downward force of 3N on it. Find the force exerted by the table on the block.



SOLUTION :

There are three forces on the body:
(i) 5N, downward by earth
(ii) 3N, downward by the person
(iii) F, upward by the table

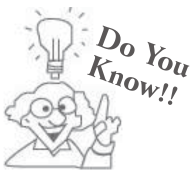
As the block is at rest, the resultant force on it must be zero. The total downward force is 5N + 3N = 8N.

Hence the upward force F should be 8N. So the force exerted by the table on the block is 8N in the upward direction.

NEWTON'S LAWS OF MOTION

Newton's First Law of Motion

According to this law, *an object continues in a state of rest or in a state of motion at a constant speed along a straight line, unless compelled to change that state by a net force*. In other words, if a body is in a state of rest, it will remain in the state of rest and if it is in the state of motion, it will remain moving in the same direction with the same velocity unless an external unbalanced force is applied on it. This law is also called **law of inertia.** It gives qualitative definition of force.



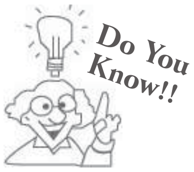
A common misconception about Newton's first law is that a force is required to keep an object in motion. This is not so. Experiments done on air tracks (where there is a negligible friction) show that no force is required to keep an object moving with constant velocity. We get this misconception because friction is always present in our everyday lives

Inertia and Mass

A greater net force is required to change the velocity of some objects than of others. The net force that is just enough to cause a bicycle to pick up speed will cause an imperceptible change in the motion of a freight train. In comparison to the bicycle, the train has a much greater tendency to remain at rest. Accordingly, we say that the train has more inertia than the bicycle. Quantitatively, the inertia of an object is measured by its mass. Inertia is the natural tendency of an object to remain at rest or in motion at a constant speed along a straight line. The mass of an object is a quantitative measure of inertia. *The greater the mass, the greater is the inertia of body*. The definition of inertia and mass indicates why Newton's first law is sometimes called the law of inertia.

Although the law of inertia was first clearly enunciated by Galileo, the English natural philosopher Issac Newton (1642-1727) incorporated the law into the solid logical basis on which he founded the science of mechanics in his great 1687 work, commonly called Principia.

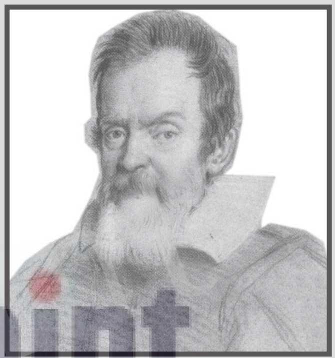
Too little force, too little time to overcome "inertia" of tableware.



According to Aristotle, an external force is required to keep a body in motion. His observation was based on common day experience. He was led to this wrong conclusion due to the reason that friction was not known in those days!

Galileo Galilei, born in Pisa, Italy in 1564 was a key figure in the Scientific Revolution in Europe about four centuries ago. Galileo invented the concept of acceleration. From experiments on motion of bodies on inclined planes or falling freely, he contradicted the Aristotelian notion that a force was required to keep a body in motion, and that heavier bodies fall down faster under gravity. The law of inertia he thus arrived at was the starting point of the subsequent epochal work of Isaac Newton.

With Galileo came a turning point in the method of scientific inquiry. Science was no longer merely observations of nature and logical inferences from them. Science meant devising and doing experiments to verify or refute theories. Science meant measurement of quantities and a search for mathematical relations between them. Not undeservedly, many regard Galileo as the father of modern science.



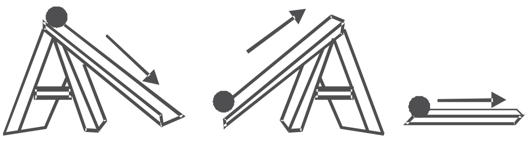
Galileo Galilei

Law of Inertia

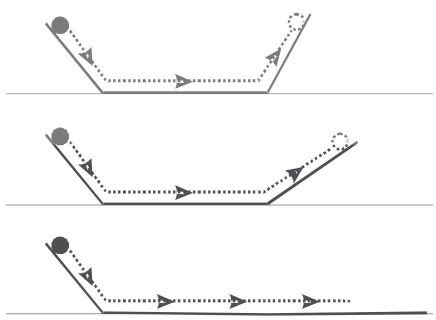
Galileo studied the motion of objects on an inclined plane. Objects moving down an inclined plane accelerate, while those going up the plane suffer retardation. Motion on a horizontal plane is an intermediate situation. Galileo concluded that an object moving on a horizontal plane must neither have acceleration nor retardation, i.e. it should move with constant velocity.

Another experiment of Galileo leading to the same conclusion involves a double inclined plane. A ball released from rest on one of the planes rolls down and climbs up the other. If the two planes are not very rough. the final height of the ball is nearly the same (a little less but never greater) as the initial height. In the ideal situation, when friction is completely eliminated, the final height of the ball should equal its initial height ?

If now the slope of the second plane is decreased and the experiment repeated, the ball will still reach the same height, but in doing so it will travel a longer distance. In the limiting case, when the slope of the second plane is zero (i.e. it is a horizontal plane) the ball travels an infinite distance. In other words, its motion will never cease. This is, of course, an idealised situation. In practice, the ball does come to a stop after in motion continues to move with uniform velocity. This property of every object in nature is called inertia. Inertia means 'resistance to change]. A body does not change its state of rest or of uniform motion, unless an external force compels it to change that state.



(a)



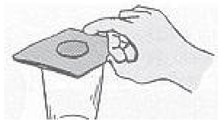
(b)

Types of Inertia

1. **Inertia of rest :** *The tendency of the body to continue in state of rest even when some external unbalanced force is applied on it is called inertia of rest.*

Examples: (i) When a carpet is suddenly jerked the dust fly off, because due to the sudden moment the carpet moves but the dust on account of inertia of rest is left behind.

(ii) The passenger standing in a bus tends to fall backwards when the bus suddenly starts, this is because his feet are in direct contact with the floor of the bus and the friction at the contact is high this friction does not allow the feet to slip on the floor, the feet therefore move forward with the floor and the upper part of the body is still at rest for a while thus the passenger gets a jerk.



(iii) Coin drops into the glass when sudden force is applied on the cardboard. It is because of the property of inertia of rest, the coin continues in the state of rest.

(iv) On shaking a tree, the fruits fall down. The reason is that when the stem or branches are shaken, they come in motion while the fruits remain in the state of rest due to the inertia of rest. Thus the fruits get detached from the branches and fall down due to the pull of gravity.

2 **Inertia of motion :** *The tendency of the body to continue in its state of motion even when some unbalanced force is applied on it is called the inertia of motion.*

Examples : (i) It is dangerous to jump out of a moving vehicle (bus/train), because the jumping man, who is moving with the high speed of the vehicle would tend to move with the high speed of the vehicle. On reaching the ground his feet come to rest but upper part of the body continues to move with the speed of vehicle and the person falls forward on the ground. It is dangerous to jump out of a moving train and it is better to come out when it halts. However if in case of some emergency if some person wants to jump safely from a moving vehicle he should run for quite a while in the direction of motion of the vehicle after the jump so that his entire body remains in motion for sometime.

(ii) When a running car stops suddenly, the passenger is jerked forward. The reason is that in a running car, the whole body of passenger is in the state of motion. As the car stops suddenly, the lower part of his body being in contact with the car, comes to rest but his upper part remains in the state of motion due to the inertia of motion. Thus he gets jerked forward.

(iii) In an event of long jump, an athlete runs fast before making the jump. Due to inertia of motion he is able to jump to a longer distance.

(iv) When we shake a wet piece of cloth, cloth as well as water in it comes to motion but when cloth comes to rest, the water is still moving due to the inertia of motion and falls forward on the front body of the person.

CHECK Point

- **Is it possible to have motion in the absence of a force? Is it possible to have force in the absence of motion?**

Solution

Motion requires no force. Newton's first law says an object in motion continues to move by itself in the absence of external forces. It is possible for forces to act on an abject with no resulting motion if the forces are balanced.

MOMENTUM

Most of us know from our experience of playing cricket that a cricket ball moving with a high speed could be fatal. Is it due to the velocity of the ball only ? Had it been so, a dust particle moving with the same velocity would have been equally fatal. On the other hand, it could not be only due to the mass of the ball as we can safely hold a stationary (or a slowly moving) cricket ball in our hand without any fear. From this and many such examples we conclude that a quantity consisting the product of mass and velocity (mv) is of much significance. This quantity is so important in physics that it is given its own name and symbol. We call this quantity as momentum and represent it by symbol p.

Momentum is a measure of the quantity of motion in a body.

Example : Consider, a truck and a rickshaw moving with same velocities, heading towards each other and eventually ending up in a head on collision. Needless to say, that the rickshaw might get deformed to such an extent, that it would be difficult for us to make it, were it was a rickshaw before ! However, the truck might get some minor damages. Why is it so ? The first thing that comes to mind, is because the truck has more mass. Exactly! But in this context, it is more wise to say that , it all happens because the truck has more quantity of motion (i.e., momentum).

The momentum of a moving body is defined as the product of its mass and velocity. If we represent the mass and velocity of a body by m and $\vec{v}$ respectively, then momentum is given by

$$\vec{p} = m\vec{v}$$

The direction of momentum of a body is same as that of its velocity. If only the magnitude is considered, then

$$p = mv$$

The **SI unit** of momentum is kilogram meter per second (kgm/s).

ILLUSTRATION : 3

A ball of mass 500g is thrown with a velocity of 72 km/h. What is its initial momentum ?

SOLUTION :

Given mass of the ball = 500g $= \frac{1}{2}$ kg and velocity (initial) of the ball = 72 km/h = 20 m/s

$\therefore$ Momentum of the ball $p = mv = \frac{1}{2} \times 20 = 10 \text{ kg m/s}$

NEWTON'S SECOND LAW OF MOTION

It states that the rate of change of momentum of a body is directly proportional to the applied unbalanced force.

i.e., Rate of change of momentum $\propto$ force applied

or, $F \propto \frac{\Delta p}{\Delta t}$

If a body is moving with initial velocity u and after applying a force F on it. Its velocity becomes v in time t, then

Initial momentum of the body $p_1 = mu$

Final momentum of body $p_2 = mv$

Change in momentum in time $t = mv - mu$

So, rate of change of momentum $= \frac{mv - mu}{t}$

But according to Newton's second law, $\frac{mv - mu}{t} \propto F$ or $F \propto \frac{m(v-u)}{t}$

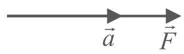
Here $\frac{v-u}{t} = a$ (acceleration)

So $F \propto$ ma   or $F = k\,ma$,   where k is proportionality constant.

If 1N force is applied on a body of mass 1 kg and the acceleration produced in the body is 1 m/sec² then $1 = k \times 1 \times 1$ or $k = 1$

Hence, $\boldsymbol{F = ma}$

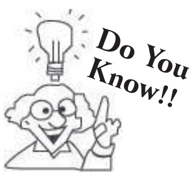
So, the magnitude of the resultant force acting on a body is equal to the product of mass of the body and the acceleration produced. Direction of the force is same as that of the acceleration.



The Logical Connection between Newton's First and Second Laws

Newton's first law asserts that a body tends to move with constant velocity. Newton's second law provides a quantitative measure of the force that will produce a given acceleration of the mass.

At first glance, it may seem that the Newton's first law is not an independent law of nature at all, but merely the special case of Newton's second law, $F = ma$, in which the net external force is zero. If $F = 0$, we have $a = F/m = 0$ and therefore v = constant, as the first law states. Is this all there is to Newton's first law ? No, because this interpretation of Newton's first law is based on an incomplete statement of the law.



F = ma equation was first written down by the Swiss mathematician Euler in 1747 after 20 years the death of Newton, to whom it is usually and falsely described, it was Euler not Newton, who first understood that this definition of force is useful in every case of motion.

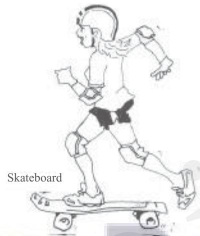
Impulse or Change in Momentum

From Newton's second law, $\vec{F} = \dfrac{\Delta \vec{p}}{\Delta t}$, we can derive the **impulse-momentum theorem**. This theorem states that impulse is equal to the change in momentum, or $\vec{F}\Delta t = \Delta\vec{p} = \vec{p} - \vec{p}_0$ where $\vec{F}\Delta t$ is called impulse, $\vec{F}$ is the average force and Δt is the time interval the force is in action).

Since impulse is product of force and time, it is measured in either newton or $kgms^{-1}$ the unit of momentum. Impulse-momentum is very useful in explaining some everyday phenomena like a cricket player while taking a catch always moves his hands backward, because the total change in momentum $(mv - mu)$ remains constant for a moving ball. The change in momentum is numerically equal to $F \times t$. Here F is force applied by the hands for time t. Now the player moves his hands backward so that the value of t will increase. With the result the value of applied force F will decrease.

Thus the hands of the player are not hurt while taking a catch. Similarly the idea of the impulse of a force is important when considering a hammer driving a nail into a block of wood, a boy kicking a football and a girl striking a hockey ball.

Examples :



(i) A girl standing at rest with her right foot on a skateboard thrusts backwards on the ground with her left foot. The push of the ground on her left foot will increase the forward momentum of the girl and skateboard.
From $Ft = mv - 0$, the velocity v acquired by the girl will depend on the force F which can be applied and the time t for which it acts.

(ii) A body throwing a javelin is shown in figure. For a javelin of a given mass, the distance it will travel depends upon the force exerted by the boy's arm and the time for which it is exerted.

(iii) During the collision of a truck with the wall, wall exerts great impulse on truck

Do You Know!!

The change in momentum of a body depends on the magnitude and direction of the applied force and the period of time over which it is applied; i.e. it depends on its impulse.

ILLUSTRATION : 4

A 650-kilogram rocket is to be speeded up from 440 meters per second to 520 meters per second in outer space. If the thrust of the engine is 1200 newtons, for how long must the engine be fired ?

SOLUTION :

The change in the momentum of the rocket $\Delta P = mv - mu$

$= (650)\ (520) - (650)\ (440) = 52000$ kg m/s.

This must be equal to the impulse, so $F\ \Delta t = (1200\ N)\ (\Delta t) = 52000$ kg m/s

$\therefore \quad \Delta t = 43 s$

ILLUSTRATION : 5

A 0.10 kilogram ball is dropped onto a table top. The speeds of the ball right before hitting the table top and right after hitting the table top are 5.0 m/s and 4.0 m/s, respectively. If the collision between the ball and the table top lasts 0.15 s, what is the average force exerted on the ball by the table top ?

SOLUTION :

Given : $m = 0.10$ kg, $v_0 = -5.0$ m/s (downward), $v = +4.0$ m/s (upward), $\Delta t = 0.15$s, average force $\vec{F} = ?$

The velocity of the ball before the collision is downward and the velocity of the ball after the collision is upward. Since these two velocities are opposite, we have to assign signs (+ or –) to them to find the change in velocity, Δv properly. Conventionally, we choose the upward direction as positive so the downward direction is negative.

From impulse-momentum theorem, we have $\vec{F}\Delta t = \Delta\vec{p} = \vec{p} - \vec{p}_0$

So, $$\vec{F} = \frac{\vec{p}-\vec{p}_0}{\Delta t} = \frac{mv - mv_0}{\Delta t} = \frac{(0.10\text{ kg})(4.0\text{ m/s}) - (0.10\text{ kg})(-5.0\text{ m/s})}{0.15\text{ s}} = +6.0\text{ N}$$

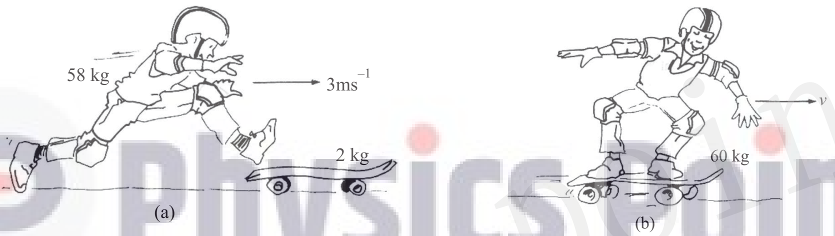
The positive sign indicates that the force on the ball by the table top is upward, which makes sense.

ILLUSTRATION : 6

A boy of mass 58 kg jumps with a horizontal velocity of 3 ms^{-1} onto a stationary skateboard of mass 2kg. What is his velocity as he moves off on the skateboard ?

SOLUTION :

Assume there is zero unbalanced horizontal force in the horizontal direction and that left to right is the positive (+) direction.



Momentum before interaction $= m_1u_1 + m_2u_2 = 58 \times 3 + 2 \times 0$

Momentum after interaction $= m_1v_1 + m_2v_2 = 58 \times v + 2 \times v = 60v$

Equating momenta $60v = 58 \times 3$

$$v = \frac{58 \times 3}{60} = +2.9\text{ ms}^{-1}.$$

ILLUSTRATION : 7

A force acts for 0.1s on a body of mass 1.2 kg initially at rest. The force then ceases to act and the body moves through 2m in the next one second. Find the magnitude of force.

SOLUTION :

When force ceases to act the body shall move with the constant velocity since it moves a distance 2m in 1s therefore its uniform velocity = 2 ms^{-1}. Thus under the influence of force the body acquires a velocity 2 ms^{-1} in 0.1s.

i.e. $u = 0$, $v = 2\ ms^{-1}$ and $t = 0.1$s

$$\text{Acceleration } a = \frac{\text{change in velocity}}{\text{time}} = \frac{v-u}{t} = \frac{2-0}{0.1} = 20\text{ ms}^{-2}$$

From the relation $F = ma$

Force = 1.2 × 20 = 24 N.

ILLUSTRATION : 8

A motorcar is moving with a velocity of 108 km/h and it takes 4s to stop after the brakes are applied. Calculate the force exerted by the brakes on the motorcar if its mass along with the passengers is 1000 kg.

SOLUTION :

The initial velocity of the motorcar

$$u = 108\text{ km/h} = 108 \times 1000\text{ m}/(60 \times 60\text{s}) = 30\text{ ms}^{-1}$$

and the final velocity of the motorcar $v = 0\ ms^{-1}$

The total mass of the motorcar along with its passengers = 1000 kg and the time taken to stop the motorcar, $t = 4$s.

We have the magnitude of the force (F) applied by the brakes as $\dfrac{m(v-u)}{t}$

On substituting the values, we get

$$F = 1000 \text{ kg} \times (0 - 30) \text{ ms}^{-1}/4\text{s} = -7500 \text{ kg ms}^{-2} \quad \text{or} \quad -7500 \text{ N}$$

The negative sign tells us that the force exerted by the brakes is opposite to the direction of motion of the motorcar.

ILLUSTRATION : 9

A force of 20N acts on a body of mass 5 kg for 5 sec. Find:
(i) the acceleration of the body` (ii) velocity at the end of 5 sec, and (iii) displacement at the end of 5 sec.

SOLUTION :

Given : $F = 20$ N, $m = 5$ kg and $t = 5$ sec.

(i) From $\vec{F} = m\vec{a} \Rightarrow 20 = 5 \times a$

Thus, acceleration $a = \dfrac{20}{5} = 4$ m/s²

(ii) $u = 0,\ a = 4,\ t = 5,\ v = ?$

Using $v = u + at$

$v = 0 + 5 \times 4 = 20$ m/s

(iii) $u = 0,\ a = 4,\ t = 5,\ s = ?$

From $s = ut + \dfrac{1}{2}at^2 = 0 + \dfrac{1}{2} \times 4 \times 25 = 50$m

ILLUSTRATION : 10

A ball of mass 10g is initially moving with a velocity of 50 ms^{-1}. On applying a constant force on ball for 2.0 s, it acquires a velocity of 70 ms^{-1}. Calculate :

(i) the initial momentum of ball (ii) the final momentum of ball
(iii) the rate of change of momentum (iv) the acceleration of ball, and (v) the magnitude of force applied

SOLUTION:

Given : $m = 10\text{g} = \dfrac{10}{1000}\text{kg} = 0.01$ kg

$u = 50 \text{ ms}^{-1},\ t = 2.0\text{s},\ v = 70 \text{ ms}^{-1}$.

(i) Initial momentum of ball = mass × initial velocity = $mu = 0.01 \text{ kg} \times 50 \text{ ms}^{-1} = 0.5 \text{ kg ms}^{-1}$.

(ii) Final momentum of the ball = mass × final velocity = $mv = 0.01 \text{ kg} \times 70 \text{ ms}^{-1} = 0.7 \text{ kg ms}^{-1}$.

(iii) Rate of change of momentum $= \dfrac{\text{final momentum} - \text{initial momentum}}{\text{time interval}} = \dfrac{(0.7 - 0.5) \text{ kg ms}^{-1}}{2.0\text{s}} = 0.1 \text{ kg ms}^{-2}$ (or 0.1 N)

(iv) Acceleration $a = \dfrac{v-u}{t} = \dfrac{70-50}{2} = 10 \text{ ms}^{-2}$

(v) Force = mass × acceleration = $ma = 0.01 \text{ kg} \times 10\text{ms}^{-2} = 0.1$N

NEWTON'S THIRD LAW OF MOTION

Newton's first two laws of motion describe what happens to a single object that has forces acting on it while Newton's third law deals with the relationship between the forces objects exert on each other.

If we push on a wall it pushes back. This doesn't hurt if you push gently, but if you punch a wall hard it hurts very much . Newton hypothesized that any time two objects interact in such a way that a force is exerted on one of them, there is always a force that is equal in magnitude exerted in the opposite direction on the other object. This hypothesis is called Newton's third law.

i.e., *To every action there is always an equal and opposite reaction.*

Newton's third law of motion states that 'if a body A exerts a force $+F$ on a body B, then body B exerts a force $-F$ on A, that is a force of the same size and along the same line of interaction but in the opposite direction'. This law says that forces always occur in pairs as the result of the interaction between two objects. Note that two objects are involved and the two forces each act on a different object. The two opposing forces are sometimes called the action and reaction forces.

Action and reaction are equal in magnitude but both act on the body in opposite direction.

Examples:

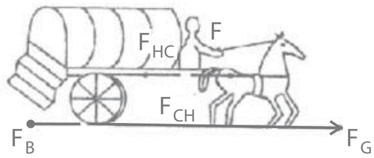
(i) Motor cars are able to move along a road because the reaction of the road pushes the car along in response to the action of the wheels pushing on the road.

(ii) A swimmer pushes (or applies force) the water with his hands and feet to move in the forward direction in water. It is the reaction to this force that pushes the swimmer forward.

(iii) The propellers of an aeroplane pushes the air backwards and the forward reaction of the air makes the aeroplane move forward.

(iv) When a bullet is fired from a gun, the force sending the bullet forward is equal to the force sending the gun backward. But due to the high mass of the gun, it moves only a little distance backward and gives a kick to the shoulder of the gunman. The gun is said to have recoiled.

(v) Consider, a horse cart being driven by a horse. The horse pulls forward, the cart with a force F_{CH}. The cart also pulls the horse with an equal force F_{HC}, but in opposite direction. But then how does the complete system (horse + cart) moves forward ? The reason is that there are yet some other forces active on the system, which lead to the movement of the horse-cart system.



The forward thrust, F_G offered by the frictional force between ground and horse and the frictional force acting backwards, between ground and cart F_B.

Thus, the net force on the system, acting forwards is given by

$$F = F_G - F_{HC} + F_{CH} - F_B$$

$$F = F_G - F_B \quad [\because F_{HC} = F_{CH}]$$

Now, if $F_G > F_B$, then $F > 0$

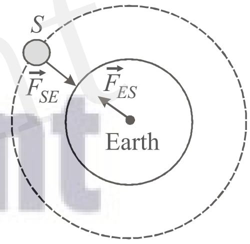
and hence the system would start moving with an acceleration

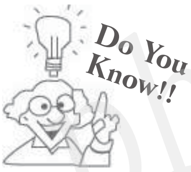
$$a = \frac{F}{m} = \frac{F_G - F_B}{m}$$ where m is the mass of the system.

But if $F_G < F_B$, no motion is possible

(vi) For an orbiting satellite action of earth on the satellite is the force exerted on the satellite by the gravitational pull of the earth $(\vec{F}_{SE})$

Reaction of the satellite on the earth. It is the gravitational pull of the satellite on the earth $(\vec{F}_{ES})$

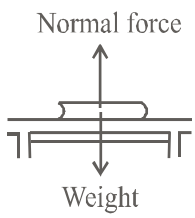




The paired forces (called action and reaction) always act on different bodies. There is no way one of them can balance the other one!

Misconcept

Consider a book on the table. The table pushes the book upward i.e. normal force. Student feels weight and normal force are action and reaction but they aren't action and reaction. **Reason:** The book does not fly up because there is another force on the book pulling it downward this is the force exerted by the earth i.e. the weight of the book as book does not accelerate so, we conclude that the forces acting on the book are balanced but they are not action and reaction pair because the action and reaction acts on different bodies.



There is **another misconception** concerning the third law. The third law states that the two forces action and reaction are always equal in magnitude but opposite in direction no matter what are the two objects.

For example, an egg and a stone collide with each other, the egg breaks and the stone is intact.

Since the egg breaks, we often conclude that the force by the stone on the egg is greater than the force by the egg on the stone. This is not so. The forces are always equal. The egg breaks because it is simply easier to break. It takes a smaller force to break the egg than to break the stone.

Think it Over!

What is the pair to each of these forces?

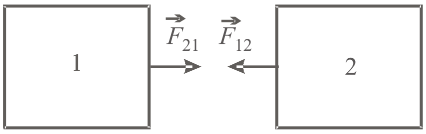
(a) The pull of earth's gravity on the moon. (b) The weight of a man. (c) The impact of a dart on a dart board.

Newton's Third Law from Newton's Second Law

Let us consider two bodies 1 and 2. Let F_{21} be the force exerted by second body on first and F_{12} the force exerted by the first body on second.

External force = 0

$\therefore \quad \vec{F}_{21} + \vec{F}_{12} = 0 \quad \Rightarrow \quad \vec{F}_{21} = -\vec{F}_{12}$



Experiment: Demonstrating action and reaction

The ring of a spring balance B is attached to the hook fixed in a wall and then the hook of another spring balance A is attached to the hook of the spring balance B. Now the ring of balance A is pulled. We find that both the balances represent the same reading. The reason is that the balance A pulls the balance B due to which we get some reading in B. But the same reading in balance A shows that the balance B also pulls the balance A by the same force of reaction. This concludes that "to every action, there is an equal and opposite reaction" (i.e., $F_{AB} = F_{BA}$ but in opposite direction).

Guidelines to solve problems based on Newton's laws of motion.

(i) Draw a free-body diagram for each object involved in the analysis.

(ii) Select a rectangular coordinate system. The solutions will be much easier if you select the +x-axis in the direction of acceleration and the +y-axis perpendicular to the x-axis.

(iii) Resolve all forces not pointing in the x or y directions to their x and y components, respectively.

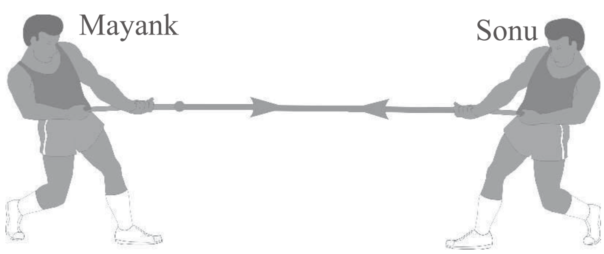
(iv) Add algebraically, all the x components and y components of the forces, respectively.

(v) Set $\Sigma F_x = ma$ and $\Sigma F_y = 0$ and solve for the unknown quantities.

Since you have choosen the +x-axis in the direction of acceleration, the object will not accelerate in the y-direction. So its acceleration in the y direction is zero and $\Sigma F_y = ma_y = 0$.

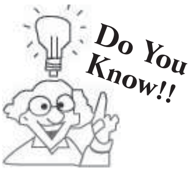
CHECK Point

- **Mayank strongman and Sonu small pull on opposite ends of a rope in a tug of war. Who exerts the greater force on the rope?**

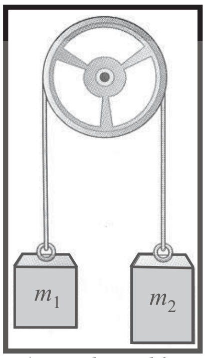


Solution

Both of them would exert the same force on the rope. Mayank can pull no harder on the rope than Sonu. Rope tension is the same all along the rope, including the ends. Just as a wheel on ice can exert no more force on the ice than the ice exerts on the wheel and just as one cannot punch an empty paper bag with any more force than the bag can exert on the puncher. Mayank can exert no more force on his end of the rope than Sonu exerts on his end.



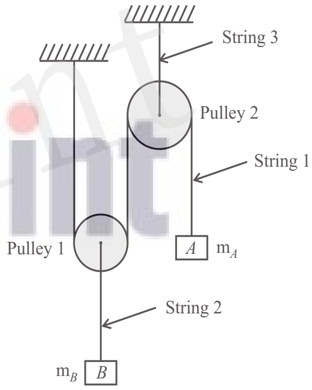
Atwood's machine gives us a direct way of demonstrating Newton's laws of motion in the laboratory.



Atwood machine

CONNECTING TOPIC

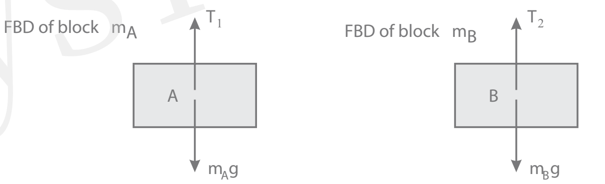
Free Body Diagram (FBD)



To study the motion of an individual body of a system, a diagram is drawn which represents different types of interactions with surrounding in terms of forces. This diagram is said to be the **free body diagram (FBD)** of that body. To draw the free body diagram, the object is first made isolated from its surroundings and then all forces acting on it are represented by arrows (⟶). To make it clear, let us see the following example.

Two blocks of mass m_A and m_B are arranged in the diagram as shown in Fig. The free body diagram of

1. Block m_A
2. Block m_B
3. Pulley 1
4. Pulley 2



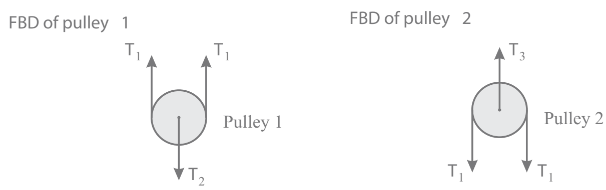


ILLUSTRATION : 11

A water skier is towed by a motorboat at a constant velocity of magnitude 15 km/h. The boat speeds up, and after a short interval the skier is towed at a new constant velocity of magnitude 20 km/h. What is the net force on the skier when she is (i)moving at 15 km/h and (ii) at 20 km/h?

SOLUTION :

(i) Two major forces act on the water skier, one of these forces is that exerted on her hands by the towrope. The other is the resistance of the water (and to a lesser extent the air). When the skier is moving in a straight line at a constant 15 km/h, her velocity is constant. According to Newton's first law, the net force on the skier (and the skis) must be zero. Indeed, the law itself is the basis for asserting that the force exerted by the water and the air on the skier is exactly equal in magnitude, and opposite in direction, to that exerted on her by the towrope.

(ii) When the skier is moving at 20 km/h, the force exerted on her by the towrope is greater in magnitude than that at 15 km/h. But so is the resistive force. Again, the net force on the skier must be zero because her velocity is constant.

ILLUSTRATION : 12

What thrust is needed to fire a 350-kilogram rocket straight up with an acceleration of 8.0 m/s² ?

SOLUTION :

The net force needed to produce this acceleration is

$F = ma = (350 \text{ kg})\,(8.0 \text{ m/s}^2) = 2800 \text{ N}$

However, there is a downward force acting on it as well, equal to

$W = \text{mg} = (350 \text{ kg})\,(9.8 \text{ m/s}^2) = 3430 \text{ N}$

The net force is the thrust minus the weight, i.e., thrust – 3430 = 2800 N

So the rocket engine must produce a thrust of 6230 N.

ILLUSTRATION : 13

A pile driver of mass 150 kg falls from a height of 5m above the pile and is brought to rest in 0.5s. Ignoring the motion of the pile, calculate the average force exerted on the pile.

SOLUTION :

Here, $m = 150$ kg, $h = 5$m, $v = ?$, $t = 0.5$ s, $v_1 = 0$ m/s, $u = 0$ m/s

$v^2 = u^2 + 2gh$

$v^2 = 2 \times 10 \times 5 = 100 \Rightarrow v = 10$ m/s

$Ft = mv_1 - mv$

$F \times 0.5 = 0 - 150 \times 10$

$\Rightarrow \quad F = -\dfrac{1500}{0.5} = -3000 \text{ N}$

F is the force exerted by the pile on the driver. Thus the force the driver exerts on the pile is +3000 N.

ILLUSTRATION : 14

A force of 20 N acting on a mass m_1, produces an acceleration of 4 m/sec². The same force is applied on mass m_2 then the acceleration produced is 0.5 m/sec². What acceleration would the same force produce, when both masses are tied together.

SOLUTION :

For mass m_1 : $F = 20$ N, $a = 4$ m/sec² then $m_1 = \dfrac{F}{a} = \dfrac{20}{4} = 5$ kg

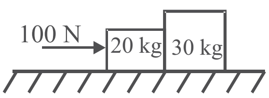
For mass m_2 : $F = 20$ N, $a = 0.5$ m/sec² then $m_2 = \dfrac{F}{a} = \dfrac{20}{0.5} = 40$ kg

When m_1 and m_2 tied together : mass = $m_1 + m_2 = 45$ kg, F = 20 N

$\therefore \quad$ Acceleration, $a = \dfrac{F}{(m_1 + m_2)} = \dfrac{20}{45} = 0.44$ m/sec²

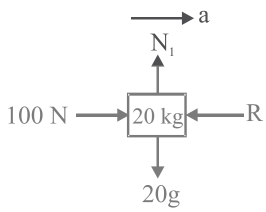
ILLUSTRATION : 15

Find acceleration and contact force between the two bodies masses 20 kg and 30 kg.

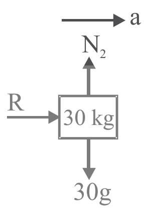


SOLUTION :

FBD of mass 20 kg



FBD of mass 30 kg



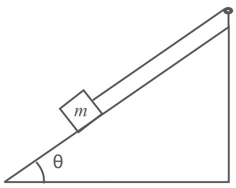
$100 - R = 20a$ ... (i) $\quad R = 30a$ ... (ii)

From eqs. (i) and (ii)

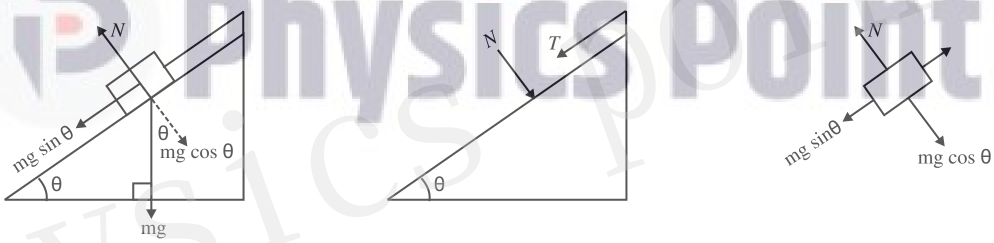
$a = 2 \text{ m/s}^2$, $R = 60$ N

ILLUSTRATION : 16

Draw F.B. D. of the system shown.



SOLUTION :



CONSERVATION OF MOMENTUM

The greatest value of the concept of momentum is in the calculation of what happens when two or more objects interact. It turns out that we can often find out the results of a collision, for example, without knowing anything at all about the forces involved or how long they persist.

Think of two objects, *A* and *B*, colliding. Object *A* exerts a force *F* on object *B*, and *B* exerts a force –*F* on *A*. The negative sign indicates that the two forces are in opposite directions. They have equal magnitudes. Since the duration of the impact, Δt is the same for both, the impulse that *B* exerts on *A* is the negative of the impulse *A* exerts on *B*.

Now, impulse is equal to change in momentum. It follows that, in the collision, the change in the momentum of *A* is the negative of the change in the momentum of *B*. The sum of the two changes is thus zero. In other words, during the collision, the total momentum does not change. This gives us the very fundamental and important law of nature called the law of conservation of momentum: *In an isolated system, the total momentum does not change*.

Think of using this rule whenever dealing with an interaction between two objects. For example, a little girl is standing on a wagon, and jumps off to the rear of it. To do so, she has to kick the wagon so that it moves forward. Before she jumped, the total momentum of the system was zero, so it must continue to be zero afterward. Her momentum backward must equal the wagon's momentum forward.

In other words, 4.5 The principle of conservation of momentum states that "if there is a direction in which there is zero unbalanced force acting on a system then the total momentum of that system in that direction is constant even if the bodies act on each other'. e.g. the pull of the Earth, do act on the bodies, but the result can still be used if there is a direction in which the external forces are balanced.

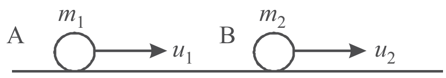
Law of Conservation of Linear Momentum

Suppose two objects *A* and *B* of mass m_1 and m_2 are moving in the same direction with velocity u_1 and u_2 respectively ($u_1 > u_2$). Object *A* collides with object *B* and after time *t* both moves in the original direction with velocity v_1 and v_2 respectively.

The change in momentum of object A is $m_1v_1 - m_1u_1$

The force on B by A is

$$F_1 = \frac{\text{change in momentum}}{\text{time}}$$



Before collision ($u_1 > u_2$)

or $$F_1 = \frac{m_1v_1 - m_1u_1}{t} \quad \text{...(i)}$$

After collision

The change in momentum of object B is $m_2v_2 - m_2u_2$

The force on A by B is $$F_2 = \frac{\text{change in momentum}}{\text{time}}$$

$$F_2 = \frac{m_2v_2 - m_2u_2}{t} \quad \text{...(ii)}$$

By Newton's third law $F_1 = -F_2$

$$\frac{m_1v_1 - m_1u_1}{t} = -\left(\frac{m_2v_2 - m_2u_2}{t}\right) \Rightarrow m_1v_1 - m_1u_1 = m_2u_2 - m_2v_2$$

or $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

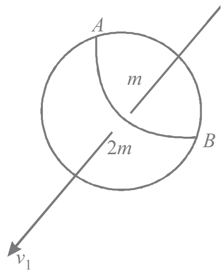
or Initial momentum = final momentum

IDEA BOX

Guns recoil when fired, because of the law of conservation of momentum. The positive momentum gained by the bullet is equal to negative recoil momentum of the gun and so the total momentum before and after the firing of the gun is zero.

Examples of conservation of linear momentum

(1) Bomb shell explosion: A bomb shell initially at rest is shown in figure. Now the bomb shell suddenly explodes in two fragments having masses in the ratio 2 : 1. As there is no external force involved in the process of explosion the momentum of the system should be conserved. In this case as the bomb is initially at rest the initial momentum of system is zero. The conservation of momentum demands that the final momentum should also be zero. The two fragments should carry equal and opposite momentum to make total momentum zero. Thus the two fragments will move along the same straight line but opposite in direction.



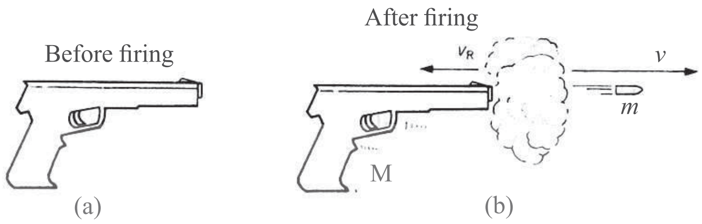
Let the masses of the two fragments be $2m$ and m and their velocities be v_1 and v_2 respectively. According to the law of conservation of momentum. $2\,(m)\,v_1 + m\,(v_2) = 0$

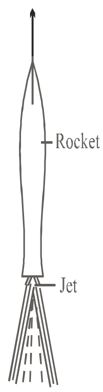
or $v_2 = -2v_1$ [in magnitude $v_2 = 2v_1$]

Thus the smaller fragment of mass m will move with double the speed of larger fragment i.e. with a velocity of magnitude $2v_1$ in the direction opposite to that of heavier fragment of mass $2m$.

(2) Recoil of the gun : Initially, before a bullet is fired from the gun, both, the bullet and the gun are at rest. So, before a bullet is fired, the initial momentum of the system (gun + bullet) is zero.

Now, when a bullet is fired it leaves the barrel with some velocity, that is it has some momentum. According to the law of conservation of momentum, the final momentum should be equal to the initial momentum which is in this case is zero. The final momentum could be zero only if the gun has a momentum equal and opposite to that of the bullet. The gun is said to be recoiled. Although their momenta are equal but as the mass of the gun is much higher than that of the bullet the recoil velocity of the gun is much smaller compared to the velocity of the bullet.

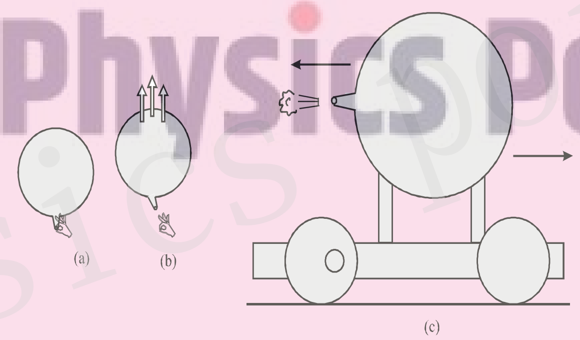




(3) Jet engines and Rockets : The working of jet engines and rockets can also be described on the basis of the law of conservation of momentum. A rocket standing at the launching pad has zero momentum. When the chemicals inside the rocket burn a high velocity blast of hot gases is produced. Theses gases pass out through the tail nozzle of the rocket in downward direction in the form of jet with tremendous velocity. Therefore the rocket moves up with such a velocity so as to make the momentum of the system (rocket + emitted gases) zero again.

Think it Over!

Tie the mouth of a balloon tightly to a small piece of discarded ballpoint refill or a plastic tube having a narrow bore and inflate it. Close the opening of the tube with your finger to prevent the air to escape (fig. (a)). Now let the air escape from the opening of the tube (fig. (b)). In which direction does the balloon move ? Why does it move in a direction opposite to the direction in which the air escapes ?



In doing this activity you may also fix the inflated balloon on the top of a toy car or a trolley before you let the air escape (fig. c). You will notice that the toy car moves in the direction opposite to the direction in which the air escapes. This is the basic principle involved in the working of jet engines and rockets. In jet engines, a large volume of gases produced by the combustion of fuel is allowed to escape through a jet and as a result, it moves the forward.

ILLUSTRATION : 17

A bullet of mass m is fired from a gun of mass M with a horizontal velocity of v. What is the recoil velocity of the gun ?

SOLUTION :

Choose a suitable direction (the horizontal direction) in which the effect of the external forces is zero. Choose a sign convention for the direction of the velocities : here left to right is positive (+). Let v_R be the recoil velocity of the gun.

Momentum before gun is fired = $m \times 0 + M \times 0 = 0$ (gun at rest)

Momentum after gun is fired = $m \times (+v) + M \times (-v_R)$

Momentum before firing = momentum after firing

$$0 = mv - Mv_R \quad \Rightarrow \quad Mv_R = mv$$

or $$v_R = \frac{mv}{M}$$

In hand-held guns the recoil can be absorbed by the person holding the gun. In very large guns which fire massive shells at high velocity the recoil could rip the gun from its mounting unless special provision is made to absorb the force.

ILLUSTRATION : 18

Two particles of mass 200g and 500g are released from rest at some mutual separation. If the velocity of the smaller mass be 2.5 ms^{-1} at any instant, then what is the velocity of the larger mass?

SOLUTION :

Since the particles are released from rest, so initial momentum of the system of particles is zero. Further, since the interaction between the particles is mutual, so the forces acting on them are purely internal, and hence the momentum of the system remains constant (and is zero) with time.

$\therefore \quad \vec{p} = m_1\vec{v}_1 + m_2\vec{v}_2 = 0$

$\Rightarrow \quad 200 \times 10^{-3} \times 2.5 + 500 \times 10^{-3} \; \vec{v}_2 = 0$

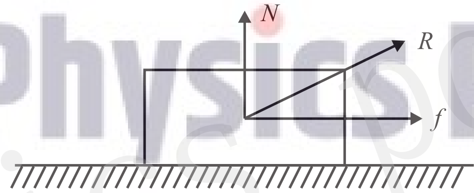
$\therefore \quad \vec{v}_2 = -1.0 \text{ ms}^{-1}$

The negative sign indicates that the direction of velocity of larger mass is opposite to that of the smaller mass.

CONNECTING TOPIC

FRICTION

Friction is a resistance to the relative motion between two objects in contact (in case of solid objects) or the body and its surroundings (in case object is moving in a fluid). Actually, when two objects are kept in contact, a reaction force R acts between the two objects as shown in the figure.



This reaction force R has two components $-f$, along the surface and N, perpendicular to the surface. The force f which acts along the surface is called the force of friction.

It is not appreciated that the existence of friction between surfaces does not depend on their roughness or smoothness in the everyday sense of these words. In fact there can be very large frictional force between two highly polished flat metal surfaces. Consequently, it is important, in mechanics, to interpret smooth as frictionless rather than free from projections.

The results of experimental investigation into the behaviour of frictional forces confirm that:

(i) frictional force opposes the movement of an object across the surface of another with which it is in rough contact.

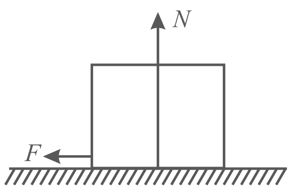
(ii) the direction of the frictional force is opposite to the potential direction of motion.

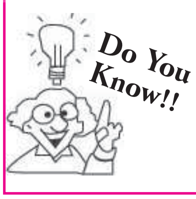
(iii) the magnitude of the frictional force is only just sufficient to prevent movement and increases as the tendency to move increases, up to a limiting value. When the limiting value is reached, the frictional force cannot increase any further and motion is about to begin (limiting equilibrium). When the frictional force F reaches its limit, its value then is related to the normal reaction N in the following way $\quad F = \mu N$

The **constant μ is called the coefficient of friction** and each pair of surfaces has its own value for this constant.

In limiting equilibrium $\quad F = \mu N$ or, $\mu = \dfrac{F}{N}$

In general $\quad F \le \mu N$



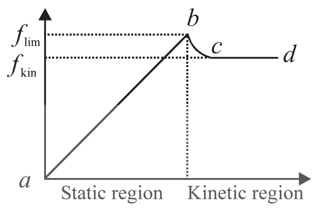


The atomic and molecular forces of attraction between the two surfaces at the point of contact give rise to friction between the surfaces.

Static Frictional Force

When there is no relative motion between the contact surfaces, frictional force is called static frictional force. It is a self-adjusting force, it adjusts its value according to requirement.

In the example static frictional force is equal to applied force. Hence one can say that the portion of graph *ab* will have a slope of 45°. $(f_s \le \mu_s N)$



Limiting Frictional Force

This frictional force acts when body is about to move. This is the maximum frictional force that can exist at the contact surface. We calculate its value using laws of friction.

Kinetic Frictional Force

Once relative motion starts between the surfaces in contact, the frictional force is called as kinetic frictional force. The magnitude of kinetic frictional force is also proportional to normal force.

$$f_k = \mu_k N$$

From the previous observation we can say that $\mu_k < \mu_s$.

Although the coefficient of kinetic friction varies with speed, we shall neglect any variation i.e., one relative motion starts a constant frictional force starts opposing its motion.

idea box

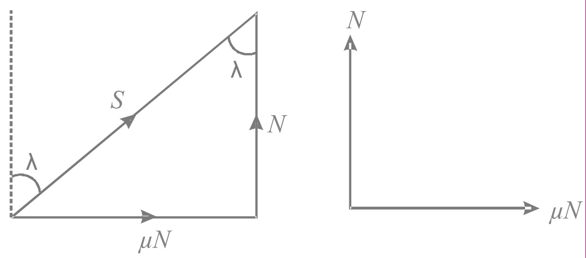
The friction between two surfaces increases (rather than to decrease), when the surfaces are made highly smooth.

ANGLE OF FRICTION

At a point of rough contact, where slipping is about to occur, the two forces acting on each object are the normal reaction *N* and frictional force μN.

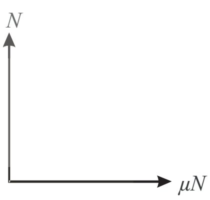
The resultant of these two forces is *S*, and *S* makes an angle λ with the normal, where

$$\tan \lambda = \frac{\mu N}{N} = \mu$$

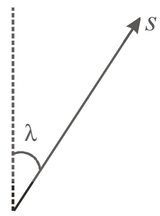


The angle λ is called the angle of friction.

At a point of rough contact when slipping is about to occur we can, therefore, use either.



Components *N* and μN at right angles to each other



S at an angle λ to the normal where *S* is the resultant contact force or total reaction and λ is the angle of friction.

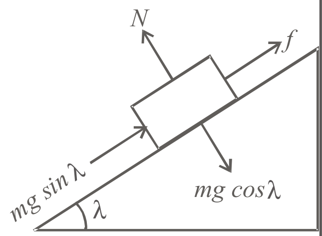
The use of *S* instead of *N* and μN reduces the number of forces in a problem and can often lead to a three force problem.

1. When the surfaces of two objects in rough contact tend to move relative to each other, equal and opposite frictional forces act on the objects, opposing the potential movement.

2. Up to a limiting value, the magnitude of a frictional force, F, is just sufficient to prevent motion.
3. When the limit is reached $F = \mu N$ where N is the normal reaction and μ is the coefficient of friction for the two surfaces in contact.
4. At all times $F \le \mu N$
5. The resultant of N and μN makes an angle λ with the normal, where $\tan\lambda = \mu$ and λ is the angle of friction.

ANGLE OF REPOSE

Suppose a body is kept above a rough inclined surface, whose angle of inclination is slowly increased such that at angle 'λ' it is about to slide down, then this angle is called angle of repose.



$f = mg\sin\lambda$ but $f = \mu N$, $N = mg\cos\lambda$

so, $\mu mg\cos\lambda = mg\sin\lambda$

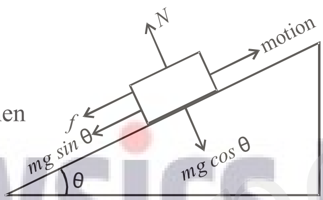
$\mu = \tan\lambda$ i.e., angle of repose = angle of friction

In case $\theta > \lambda$, body will slide down and its acceleration can be found as under

$N = mg\cos\theta$

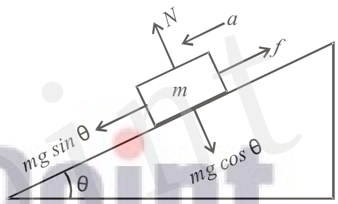
$f = \mu N = \mu mg\cos\theta$ but $mg\sin\theta - f = ma$

so, $a = g\sin\theta - \mu g\cos\theta$



If body is projected upward on the inclined plane, then

$a = -(g\sin\theta + \mu g\cos\theta)$



MISCELLANEOUS SOLVED EXAMPLES

1. A force, F_1 acting on a 2.0 kg body produces an acceleration 2.5 m/s^2. Another force F_2 acting on a 5.0 kg body produces an acceleration of 2.0 m/s^2. Find the ratio F_2/F_1.

Sol. We have F = kma.

Thus, F_1 = k(2.0 kg) (2.5 m/s^2), and F_2 = k(5.0 kg) (2.0 m/s^2).

These equations give $\frac{F_2}{F_1} = \frac{5.0 \times 2.0}{2.0 \times 2.5} = 2$.

2. A force acts on a particle of mass 200 g. The velocity of the particle changes from 15 m/s to 25 m/s in 2.5 s. Assuming the force to be constant, find its magnitude.

Sol. The acceleration of the particle is

$$a = \frac{v-u}{t} = \frac{(25\text{ m/s}) - (15\text{ m/s})}{2.5\text{ s}} = \frac{10}{2.5}\text{ m/s}^2 = 4.0\text{ m/s}^2.$$

The force is F = ma

$$= (200\text{ g}) \times (4.0\text{ m/s}^2) = \left(\frac{200}{1000}\text{ kg}\right) \times (4.0\text{ ms}^2) = 0.8\text{ N}.$$

3. A bullet of mass 20 g moving with a speed of 120 m/s hits a thick muddy wall and penetrates into it. It takes 0.03 s to stop in the wall. Find (a) the acceleration of the bullet in the wall, (b) the force exerted by the wall on the bullet, (c) the force exerted by the bullet on the wall, and (d) the distance covered by the bullet in the wall.

Sol. (a) The velocity of the bullet as it hits the wall is u = 120 m/s. The velocity after 0.03 s is v = 0. So, using v = u + at,

0 = (120 m/s) + a(0.03 s)

or $a = -\frac{120}{0.03}\text{ m/s}^2 = -4000\text{ m/s}^2$.

(b) The force exerted by the wall on the bullet is

F = ma

= (20 g) (–4000 m/s^2)

$= \left(\frac{20}{1000}\text{kg}\right)(-4000\text{ m/s}^2) = -80\text{ N}.$

The negative sign shows that the force by the wall on the bullet is in a direction opposite to that of the velocity.

(c) From Newton's third law, the force exerted by the bullet on the wall is also 80 N, in direction of the velocity.

(d) The distance covered by the bullet in the wall is

$s = ut + \frac{1}{2}at^2$

$= (120\text{ m/s})(0.03\text{ s}) + \frac{1}{2}(-4000\text{ m/s}^2)(0.0009\text{ s}^2)$

= 3.6 m – 1.8 m = 1.8 m

4. Two particles A and B of masses 20 g and 30 g respectively are at rest at a certain time. Because of the forces exerted by them on each other, the particles start moving. At a given instant, particle A is found to move towards the east with a velocty of 6 cm/s. What is the velocity of particle B at this instant?

Sol. As the particles are at rest initially, the linear momentum (mv) of each is zero. Taking the two particles together as a system, the linear momentum of the system is also zero. The forces exerted by the particles on each other are internal forces for the system and cannot change the linear momentum of the system, which remains zero during this time.

At the given instant, the linear momentum of particle A is

$p_1 = m_1v_1$ = (20 g) (6 cm/s) = 120 g cm/s.

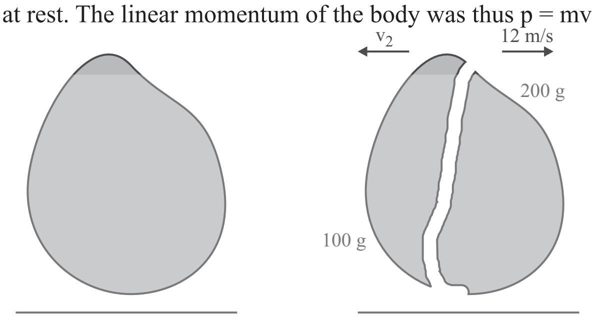
The linear momentum of particle B at this instant must be –120 g cm/s to make the momentum of the system zero. Thus, particle B moves towards the west with a velocity V_2, where

m_2v_2 = 120 g cm/s

or $v_2 = \frac{120\text{ g cm/s}}{m_2} = \frac{120\text{ g cm/s}}{30\text{ g}} = 4\text{ cm/s}.$

5. A body of mass 300 g kept at rest breaks into two parts due to internal forces. One part of mass 200 g is found to move at a speed of 12 m/s towards the east. What will be the velocity of the other part?

Sol. Before it broke, the body was at rest. The linear momentum of the body was thus p = mv = 0.



Before breaking (a) After breaking (b)

The body breaks due to internal forces. As the external force acting on it is zero, its linear momentum will remain constant, i.e., zero

The linear momentum of the first part is

$p_1 = m_1v_1 = (200\text{ g}) \times (12\text{ m/s})$, towards the east.

For the total momentum to remain zero, the linear momentum of the other part must have the same magnitude and should be opposite in direction. It therefore moves towards the west. If its speed is v_2, its linear momentum is

$p_2 = m_2v_2 = (100\text{ g}) \times v_2$.

Thus, $(200\text{ g}) \times (12\text{ m/s}) = (100\text{ g}) \times v_2$

or $v_2 = 24$ m/s.

The velocity of the other part is 24 m/s towards the west.

6. A force of 12 N starts acting on a body kept at rest. Find the momentum of the body at 1 s, 2 s and 5 s after the force starts acting.

Sol. We have $F = \dfrac{p_2 - p_1}{t_2 - t_1}$.

At t = 0, the momentum is $p_1 = 0$. So, at a time t, its value is given by

$F = \dfrac{p_2 - p_1}{t - 0}$

or $P_2 = p_1 + Ft = Ft$.

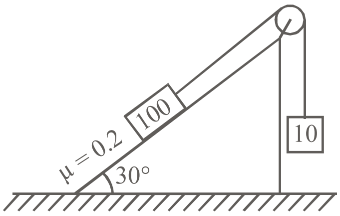
The momentum at t = 1 s is $p_2 = Ft = (12\text{ N}) \times (1\text{ s}) = 12$ N s.

At t = 2 s, it is $p_2 = (12\text{ N}) \times (2\text{ s}) = 24$ N s.

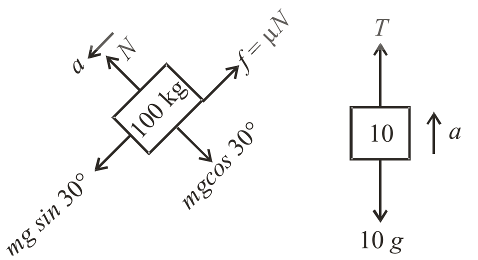
At t = 5 s, it is $p_2 = (12\text{ N}) \times (5\text{ s}) = 60$ N s.

SOLVED EXAMPLES BASED ON CONNECTING TOPICS

7. Find acceleration of masses and tension in the string for the system shown in figure ($g = 10\text{ m/s}^2$).



Sol. Drawing F.B.D of two masses



$N = mg\cos\theta$ and $f = \mu N$

$100\,g\sin 30° - T - f = 100\,a$

$100 \times 10 \times \frac{1}{2} - T - 0.2 \times 100 \times 10 \times \frac{\sqrt{3}}{2} = 100\,a$ … (i)

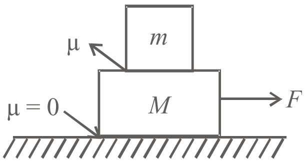
$10\,g - T = 10\,a$ … (ii)

Adding eqs. (i) and (ii)

$500 + 100 - 173 = 110a \Rightarrow a = 3.88\ \text{m/s}^2$

and $T = 10\,g - 10 \times 3.88 = 61.2$ N

8. Find maximum force applied on the lower mass so that the two masses move together, for the system of masses shown.

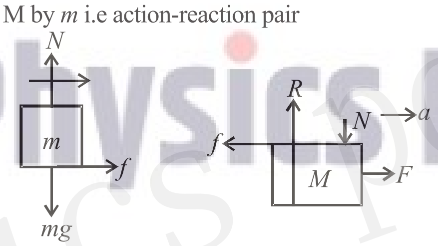


Sol. In order to find the direction of friction on the mass m let us initially assume that friction is absent, when lower mass is pulled by F the upper mass will fall backwards hence friction on the upper mass has to be in forward direction.

(As friction on the upper body is in forward direction so on the lower body it will be in backward direction i.e. action -reaction pair)

R = Reaction force of the ground on M

N = Normal reaction on m by M and on M by m i.e action-reaction pair



$f = \mu N : N = mg$

$f = \mu mg$ … (1) and $f = ma$

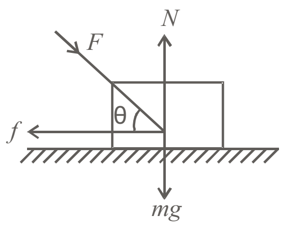
Also $F - f = Ma \Rightarrow F - ma = Ma \Rightarrow a = \frac{F}{M+m}$

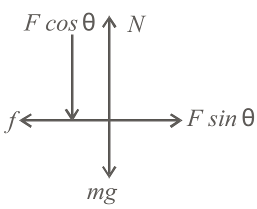
$F = f + ma$

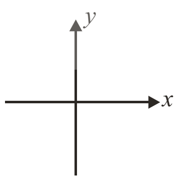
$$F = \mu mg + M\frac{F}{M+m} \Rightarrow F - \frac{MF}{m+M} = \mu mg \Rightarrow F = \mu g(M+m)$$

9. Explain why it is easier to pull than push a box on a rough surface?

Sol. Pushing a box





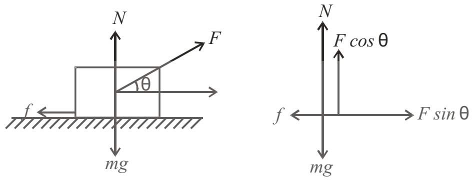


Along y-axis

$N = mg + F\cos\theta$

$f = \mu N = \mu(mg + F\cos\theta)$ … (i)

Pulling a box



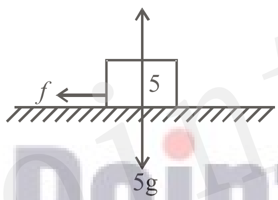
Along y-axis, $N + F\cos\theta = mg$; $N = mg - F\cos\theta$

So, $f = \mu\,(mg - F\cos\theta)$ …(ii)

It is quite evident from equation (i) and (ii) that in pushing friction force is greater due to larger normal reaction, that is why pulling is easier than pushing. But if friction is absent pulling and pushing would require the same effort.

10. A mass of 5kg having initial velocity of 20 m/s comes to rest after travelling a distance of 200 m. Find coefficient of friction and friction force acting on the body.

Sol. Here $u = 20$ m/s : $v = 0$; $s = 200$ m



Using $v^2 = u^2 + 2as$

$0 = (20)^2 + 2a \times 200 \Rightarrow a = -1\ \text{m/s}^2$

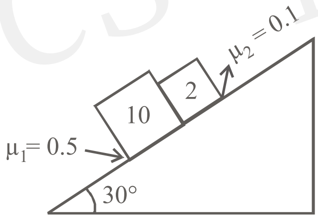
But $f = ma$; $f = \mu N = \mu mg$

$\mu mg = ma, \Rightarrow a = \mu g$

So , $\mu \times 10 = 1 \Rightarrow \mu = 0.1$

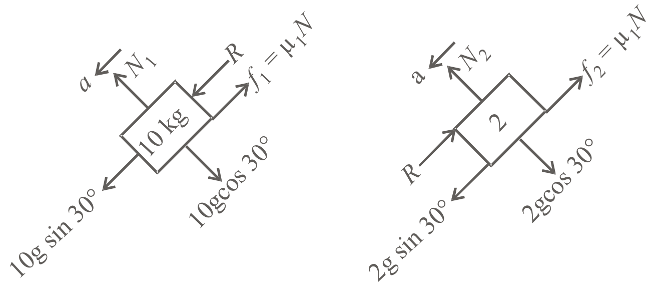
Friction force, $f = 0.1 \times 5 \times 10 = 5$N

11. Find acceleration and contact force between two masses in the given figure (Take $g = 10$ m/s^2).



Sol. Drawing F.B.D (Two masses will slide down together as $\mu_1 < \mu_2$)

R - contact force



$f_1 = 0.5 \times 10 \times g \cos 30°$

$f_2 = 0.1 \times 2 \times g \times \cos 30°$

$10g \sin 30° + R - f_1 = 10\,a$ …(i)

and $2g \sin 30° - R - f_2 = 2a$ … (ii)

Adding eqs, (i) and (ii)

$$100 \times \frac{1}{2} - 0.5 \times 10 \times 10 \times \frac{\sqrt{3}}{2} + 2 \times 10 \times \frac{1}{2} - 0.1 \times 2 \times 10 \times \frac{\sqrt{3}}{2} = 12a$$

$60 - 26\sqrt{3} = 12a \Rightarrow a = 1.25\,\text{m/s}^2$ **(acceleration)** and From eqn. (i) $R = -50 + 25\sqrt{3} + 10 \times 1.25 \Rightarrow R = 5.75$ N

12. When a force of 50 N is applied on a body of mass 10 kg it is about to move and once the body is set into motion the same force of 50N produces an acceleration of 0.5m/s^2. Find coefficient of static and kinetic friction (g = 10 m/s^2).

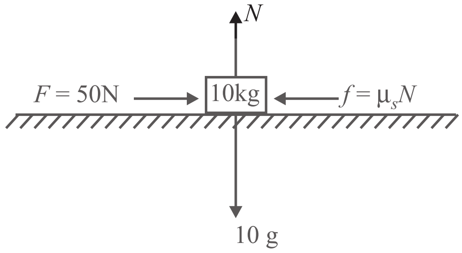
Sol. As on the application of 50 N force, body does not move, the applied force is equal to the limiting force of friction.

$f = 50$ N

$N = 10 \times 10 = 100\ N$

$\mu_s \times N = f$

$\therefore \quad \mu_s = \frac{f}{N} = \frac{50}{100} = 0.5$

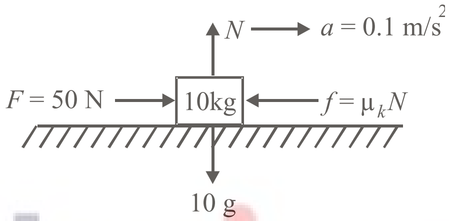


When the body is set into motion then kinetic friction will come into play, that is why the same force will produce acceleration

$50 - \mu_k \times 100 = 10 \times 0.1$

$\Rightarrow 50 - 1 = \mu_k \times 100$

or, $\quad \mu_k = 0.49$



1 EXERCISE

Fill in the Blanks :

DIRECTIONS : *Complete the following statements with an appropriate word / term to be filled in the blank space(s).*

1. When a running car stops suddenly, the passengers are jerked ....

2. .... is a measure of the inertia of a body.

3. To every action, there is an............. and................. reaction.

4. Application of a force changes the .... of an object.

5. An object moving at constant speed is in a state of ....

6. Impulse is the product of force and ....

7. The change in the momentum of an object is equal to the .... applied to it.

8. The SI unit of force is the ....

9. If a force of 200 newtons is required to move a wagon up a frictionless hill at constant speed, the force needed to left the wagon roll downhill at constant speed is ....

10. In any interaction between two or more isolated objects, the total .... does not change.

11. The change in the velocity of an object is proportional to the ....applied to it.

12. If there are several forces on an object, its acceleration depends on its mass and the .... force.

13. ....is equal to change in momentum.

True/False :

DIRECTIONS : *Read the following statements and write your answer as true or false.*

1. When we push our foot against the ground backwards (action), the ground exerts an equal and opposite force (reaction) on our foot which causes us to more forward.

2. It is easier to start motion in a lighter body than a heavier body.

3. Action and reaction force acts on the same object.

4. Particle is at rest, if force is zero.

5. Particle moves in the direction of force.

6. If particle is initially at rest then it moves in direction of net force.

7. No net force acts on a rain drop falling vertically with a constant speed.

8. If net force acting on the body is zero, momentum of the body remains constant.

9. A body can be in equilibrium under the action of three coplaner forces.

10. Momentum is never created nor destroyed

11. The product of the mass of a body and its velocity is called inertia.

12. Action and reaction act on the same body.

Match the Columns :

DIRECTIONS : *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D, E) in Column I have to be matched with statements (p, q, r, s, t) in Column II.*

1.

	Column I		Column II
(A)	To every action there is equal and opposite	(p)	Momentum
(B)	During collision, there is conservation of	(q)	Force
(C)	Rate of change of velocity is	(r)	Reaction
(D)	Rate of change of momentum is	(s)	Acceleration
(E)	Force that opposes motion	(t)	Force of Friction

2.

	Column I		Column II
(A)	Force	(p)	kg ms–1
(B)	Momentum	(q)	newton
(C)	Impulse	(r)	kg
(D)	Mass	(s)	ms–2
(E)	Acceleration	(t)	Force × time

Very Short Answer Questions :

DIRECTIONS : *Give answer in one word or one sentence.*

1. The length of an ideal spring increases by 0.1cm when a body of 1kg is suspended from it. If this spring is laid on a frictionless horizontal table and bodies of 1 kg each are suspended from its ends, then what will be the increase in its length?

2. The two ends of a spring-balance are pulled each by a force of 10kg-wt. What will be the reading of the balance?

3. A retarding force is applied to stop a motorcar. If the speed of the motorcar is doubled, how much more distance will it cover before stopping under the same retarding force?

4. A ball of 0.5kg mass moving with a speed of 10m/s rebounds after striking normally a perfectly elastic wall. Find the change in momentum of the ball.

5. A body of mass 2kg is suspended on a spring balance hung vertically in a lift. If the lift is falling downward under acceleration due to gravity g, then what will be the reading of the balance? If going upward with the same acceleration, then?

6. Under what circumstances is Newton's first law same as Newton's second law.

7. Write the equation of Newton's second law when the body is moving opposite to the direction in which force is applied.

8. It is easier to roll a barrel than to pull it along the road. Why?

Short Answer Questions :

DIRECTIONS : *Give answer in 2-3 sentences.*

1. Vehicles stop on applying brakes. Does this phenomenon violate the principle of conservation of momentum?

2. A bird is sitting on the floor of a closed glass cage in the hands of girls. Will the girl experience any change in the weight of the cage when the bird (i) starts flying in the cage with a constant velocity ? (ii) flies upwards with acceleration ? (iii) flies downwards with acceleration ?

3. The driver of a truck travelling with a velocity v suddenly notices a brick wall in front of him at a distance d. It is better for him to apply brakes or to make a circular turn without applying brakes in order to just avoid crashing into the wall? Why?

4. What is inertia ? Why do we call the Newton's first law as the law of inertia ? Explain.

5. Show that the Newton's first law of motion gives the qualitative definition of force and the second law gives the measure (or quantitative definition) of force.

6. State Newton's second law of motion. Show that it gives the measure of force.

7. Prove that Newton's second law of motion is the only real law of motion.

8. What is force ? What are its absolute and gravitational units ? How are these related to each other?

9. Define the terms momentum and impulse. Obtain the relation between impulse and momentum.

10. State the principle of conservation of linear momentum. Explain, how you will prove this law. Explain one example, where we make use of this law.

Long Answer Questions :

DIRECTIONS : *Give answer in four to five sentences.*

1. Water skier is towed by a motorboat at a constant velocity of magnitude 15 km/h. The boat speeds up, and after a short interval the skier is towed at a new constant velocity of magnitude 20 km/h. What is the net force on the skier when she is moving at 15 km/h and at 20 km/h?

2. State Newton's second law of motion. Hence, derive the equation of motion $F = m.a$. From it obtain the unit of force in SI. Show that Newton's second law of motion is the real law of motion.

3. State Newton's laws of motion and the principle of conservation of momentum. Prove the principle of conservation of momentum from third law of motion.

2 EXERCISE

Text-Book Questions :

1. Which of the following has more inertia :
   (a) a rubber ball and a stone of the same size?
   (b) a bicycle and a train?
   (c) a five-rupees coin and a one-rupee coin?
2. In the following example, try to identify the number of times the velocity of the ball changes:
   "A football player kicks a football to another player of his team who kicks the football towards the goal. The goalkeeper of the opposite team collects the football and kicks it towards a player of his own team".
   Also identify the agent supplying the force in each case.
3. Explain why some of the leaves may get detached from a tree if we vigorously shake its branch.
4. Why do you fall in the forward direction when a moving bus brakes to a stop and fall backwards when it accelerates from rest?
5. If action is always equal to the reaction, explain how a horse can pull a cart.
6. Explain, why it is difficult for a fireman to hold a hose, which ejects large amounts of water at a high velocity.
7. From a rifle of mass 4 kg, a bullet of mass 50 g is fired with an initial velocity of 35 m s^{-1}. Calculate the initial recoil velocity of the rifle.
8. Two objects of masses 100 g and 200 g are moving along the same line and direction with velocities of 2 m s^{-1} and 1 m s^{-1}, respectively. They collide and after the collision, the first object moves at a velocity of 1.67 m s^{-1}. Determine the velocity of the second object.

Text-Book Exercise :

1. An object experiences a net zero external unbalanced force. Is it possible for the object to be travelling with a non-zero velocity? If yes, state the conditions that must be placed on the magnitude and direction of the velocity. If no, provide a reason.
2. When a carpet is beaten with a stick, dust comes out . Explain why ?
3. Why is it advised to tie any luggage kept on the roof of a bus with a rope?
4. A batsman hits a cricket ball which then rolls on a level ground. After covering a short distance, the ball comes to rest. The ball slows to a stop because
   (a) the batsman did not hit the ball hard enough.
   (b) velocity is proportional to the force exerted on the ball.
   (c) there is a force on the ball opposing the motion.
   (d) there is no unbalanced force on the ball, so the ball would want to come to rest.
5. A truck starts from rest and rolls down a hill with a constant acceleration. It travels a distance of 400 m in 20 s. Find its acceleration. Find the force acting on it if its mass is 7 metric tonnes (Hint: 1 metric tonne = 1000 kg.)
6. A stone of 1 kg is thrown with a velocity of 20 m s^{-1} across the frozen surface of a lake and comes to rest after travelling a distance of 50 m. What is the force of friction between the stone and the ice?
7. A 8000 kg engine pulls a train of 5 wagons, each of 2000 kg, along a horizontal track. If the engine exerts a force of 40000 N and the track offers a friction force of 5000 N, then calculate:
   (a) the net accelerating force;
   (b) the acceleration of the train; and
   (c) the force of wagon 1 on wagon 2.
8. An automobile vehicle has a mass of 1500 kg. What must be the force between the vehicle and road if the vehicle is to be stopped with a negative acceleration of 1.7 m s^{-2}?
9. What is the momentum of an object of mass m, moving with a velocity v?
   (a) $(mv)^2$ **(b)** mv^2
   (c) $\frac{1}{2}mv^2$ **(d)** mv
10. Using a horizontal force of 200 N, we intend to move a wooden cabinet across a floor at a constant velocity. What is the friction force that will be exerted on the cabinet?
11. Two objects, each of mass 1.5 kg, are moving in the same straight line but in opposite directions. The velocity of each object is 2.5 m s^{-1} before the collision during which they stick together. What will be the velocity of the combined object after collision?
12. According to the third law of motion when we push on an object, the object pushes back on us with an equal and opposite force. If the object is a massive truck parked along the roadside, it will probably not move. A student justifies this by answering that the two opposite and equal forces cancel each other. Comment on this logic and explain why the truck does not move.
13. A hockey ball of mass 200 g travelling at 10 m s^{-1} is struck by a hockey stick so as to return it along its original path with a velocity at 5 m s^{-1}. Calculate the change of momentum occurred in the motion of the hockey ball by the force applied by the hockey stick.

14. A bullet of mass 10 g travelling horizontally with a velocity of 150 m s^{-1} strikes a stationary wooden block and comes to rest in 0.03 s. Calculate the distance of penetration of the bullet into the block. Also calculate the magnitude of the force exerted by the wooden block on the bullet.

15. An object of mass 1 kg travelling in a straight line with a velocity of 10 m s^{-1} collides with, and sticks to, a stationary wooden block of mass 5 kg. Then they both move off together in the same straight line. Calculate the total momentum just before the impact and just after the impact. Also, calculate the velocity of the combined object.

16. An object of mass 100 kg is accelerated uniformly from a velocity of 5 m s^{-1} to 8 m s^{-1} in 6 s. Calculate the initial and final momentum of the object. Also, find the magnitude of the force exerted on the object.

17. Akhtar, Kiran and Rahul were riding in a motorcar that was moving with a high velocity on an expressway when an insect hit the windshield and got stuck on the windscreen. Akhtar and Kiran started pondering over the situation. Kiran suggested that the insect suffered a greater change in momentum as compared to the change in momentum of the motorcar (because the change in the velocity of the insect was much more than that of the motorcar). Akhtar said that since the motorcar was moving with a larger velocity, it exerted a larger force on the insect. And as a result the insect died. Rahul while putting an entirely new explanation said that both the motorcar and the insect experienced the same force and a change in their momentum. Comment on these suggestions.

18. How much momentum will a dumb-bell of mass 10 kg transfer to the floor if it falls from a height of 80 cm? Take its downward acceleration to be 10 m s^{-2}.

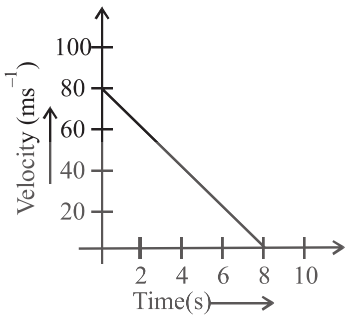
Exemplar Questions:

1. Two balls of the same size but of different materials, rubber and iron are kept on the smooth floor of a moving train. The brakes are applied suddenly to stop the train. Will the ball start rolling? If so, in which direction? Will they move with the same speed? Give reasons for your answer.

2. Two identical bullets are fired-one by a light rifle and another by a heavy rifle with the same force. Which rifle will hurt the shoulder more and why?

3. Suppose a ball of mass m is thrown vertically upward with an initial speed v, its speed decreases continuously till it becomes zero. Thereafter, the ball begins to fall downward and attains the speed v again before striking the ground. It implies that the magnitude of initial and final momentums of the ball are same. Yet, it is not an example of conservation of momentum. Explain why?

4. Velocity versus time graph of a ball of mass 50 g rolling on a concrete floor is shown in Fig. Calculate the acceleration and frictional force of the floor on the ball.



5. Two friends on roller-skates are standing 5 m apart facing each other. One of them throws a ball of 2 kg towards the other, who catches it. How will this activity affect the position of the two? Explain your answer.

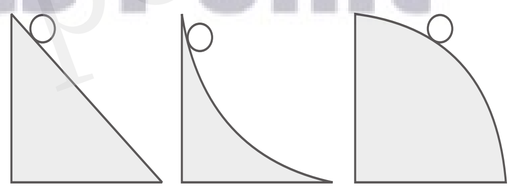
6. Using second law of motion, derive the relation between force and acceleration. A bullet of mass 10 g strikes a sand-bag at a speed of 10^3 ms^{-1} and gets embedded after travelling 5 cm. Calculate

(i) the resistive force exerted by the sand on the bullet

(ii) the time taken by the bullet to come to rest.

Hots Questions:

1. On which of these hills does the ball roll down with increasing speed and decreasing acceleration along the path ? (Use this example if you wish to explain to someone the difference between speed and acceleration.)



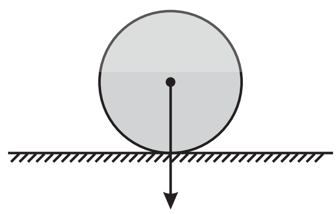
2. If you drop an object, its acceleration toward the ground is 10 m/s^2. If you throw it down instead, would its acceleration after throwing be greater than 10 m/s^2 ? Why or why not?

3. When your hand turns the handle of a faucet, water comes out. Do your push on the handle and the water coming out comprise an action-reaction pair ? Defend your answer.

4. A stone is shown at rest on the ground.

(a) The vector shows the weight of the stone. Complete the vector diagram showing another vector that results in zero net force on the stone.

(b) What is the conventional name of the vector you have drawn ?



5. What is the net force on the stone in the preceding exercise when it is at the top of its path ? What is its instantaneous velocity ? Its acceleration?

3 EXERCISE

Single Option Correct :

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

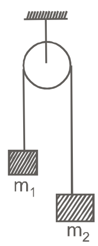
1. A force F_1 acting on a body of 2 kg produces an acceleration of 2.5 m/sec². An other force F_2 acting on the another body of mass 5 kg produces an acceleration of 2 m/sec². Find the ratio of F_1/F_2.
(a) 2 : 1 (b) 4 : 1
(c) 1 : 2 (d) 1 : 8

2. A field gun of mass 1.5 t fires a shell of mass 15 kg with a velocity of 150 m/s. Calculate the velocity of the recoil of the gun.
(a) 1 m/sec (b) 1.5 m/sec
(b) 3 m/sec (d) 5 m/sec

3. A feather of 20 grams is dropped from a height. It is found to fail down arts a constant velocity. The net force acting on it is
(a) 200 N (b) 0.2 N
(c) 0.02 N (d) zero

4. A body of mass 1 kg is kept at rest. A constant force of 6.0N acting on it, the time taken by the body to move through a distance of 12m
(a) 2 sec. (b) 3 sec.
(c) 4 sec. (d) 5 sec.

5. Two masses $m_1 = 5$ kg and $m_2 = 4.8$ kg tied to a string are hanging over a light frictionless pulley. What is the acceleration of the masses when left free to move ? (g = 9.8 m/s^2)



(a) 5 m/s^2
(b) 9.8 m/s^2
(c) 0.2 m/s^2
(d) 4.8 m/s^2

6. By applying a force of one Newton, one can hold a body of mass
(a) 102 gram (b) 102 kg
(c) 102 mg (d) None of these

7. The speed of a falling body increases continuously, this is because
(a) no force acts on it
(b) it is very light
(c) the air exert the frictional force
(d) the earth attract it

8. A gun of mass 4.5 kg fires a bullet of mass 20g. with a velocity of 108 km/hr. the recoil velocity of the gun is
(a) 1.33 m/sec (b) 0.133 m/sec
(c) 13.3 m/s (d) 133 m/sec

9. If an object is in a state of equilibrium
(a) it is at rest
(b) it is in motion at constant velocity
(c) it is in free fall
(d) may be more than one of the above

10. If a boat is moving along a constant speed, it may be assumed that
(a) a net force is pushing it forward
(b) the sum of only vertical forces is zero
(c) the buoyant force is greater than gravity
(d) the sum of all forces is zero

11. To slow down a car, a braking force of 1200 newtons is applied for 10 seconds. How much force would be needed to produce the same change in velocity in 6 seconds –
(a) 2000 N (b) 3000 N
(c) 2500 N (d) 1500 N

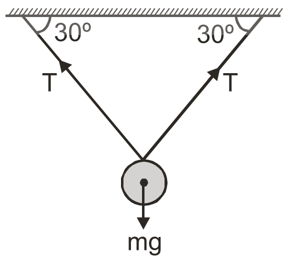
12. A frictionless wagon is pushed from rest, with a force of 60 newtons for 14 seconds. If it then strikes a wall and comes to rest in 0.15 second, how much average force does the wall exert on it ?
(a) 6000 N (b) 5600 N
(c) 4500 N (d) 4000 N

13. What force is needed to a accelerate a 60-kilogram wagon from rest to 5.0 meters per second in 2.0 seconds
(a) 100 N (b) 120 N
(c) 150 N (d) 130 N

14. A frictionless wagon going at 2.5 meters per second is pushed with a force of 380 N, and its speed increases to 6.2 meters per second in 4.0 seconds. What is its mass –
(a) 410 kg (b) 420 kg
(c) 480 kg (d) 310 kg

15. A solid sphere of 2 kg is suspended from a horizontal beam by two supporting wires as shown in fig. Tension in each wire is approximately (g = 10 ms^{-2})

(a) 30 N
(b) 20 N
(c) 10 N
(d) 5 N



16. What braking force is needed to bring a 2200 kilogram car going 18 meters per second to rest in 6.0 seconds ?
(a) 6600 N (b) 6500 N
(c) 6000 N (d) 6200 N

17. A 500 kg rocket is fired straight up from the earth, the engines providing 7500 newtons of thrust. Its acceleration is
(a) 4.5 m/s square (b) 5.2 m/s square
(c) 9.8 m/s square (d) 15 m/s square

18. If a jet engine provides a thrust of 45000 newtons, how long must it fire to produce 1 million newton-seconds of impulse ?
(a) 22s (b) 18s
(c) 25s (d) 15s

19. What force is needed to speed up a frictionless 60 kg cart from 4.0 meters per second to 6.5 meters per second in 3.0 seconds ?
(a) 50 N (b) 100 N
(c) 5N (d) 20 N

20. What force must the brakes and tires apply to a 2800 kg truck going 30 meters per second to bring it to rest in 8.0 seconds
(a) 12000 N (b) 13000 N
(c) 11000 N (d) 12500 N

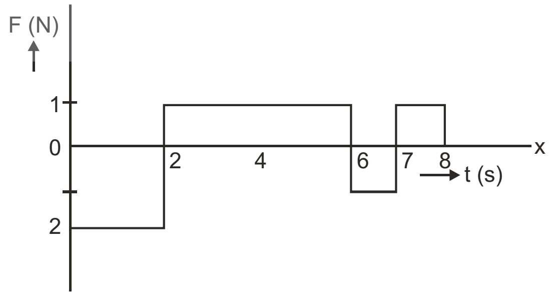
21. A 35 kg girl on roller skates, standing still, throws a 6 kg medicine ball forward at 3.5 metes per second. How much is her recoil velocity (the backward speed she acquires as a result of the throw)
(a) – 0.6 m/s (b) – 1.6 m/s
(c) – 2.6 m/s (d) – 5.6 m/s

22. The recoil velocity of a 7.5 kg rifle if it fires an 8.0 gram bullet with a muzzle velocity of 640 meters per second is
(a) 0.12 m/s (b) 0.68 m/s
(c) 2.68 m/s (d) 6.8 m/s

23. When a body is stationary
(a) There is no force acting on it
(b) The force acting on it not in contact with it
(c) The combination of forces acting on it balances each other
(d) The body is in vacuum

24. A rider on horse falls back when horse starts running, all of a sudden because
(a) rider is taken back
(b) rider is suddenly afraid of falling
(c) inertia of rest keeps the upper part of body at rest while lower part of the body moves forward with the horse
(d) none of the above

25. A force time graph for the motion of a body is shown in Fig. Change in linear momentum between 0 and 8s is



(a) zero (b) 4 N-s
(c) 8 Ns (d) None of these

26. A man getting down a running bus, falls forward because
(a) due to inertia of rest, road is left behind and man reaches forward
(b) due to inertia of motion upper part of body continues to be in motion in forward direction while feet come to rest as soon as they touch the road
(c) he leans forward as a matter of habit
(d) of the combined effect of all the three factors stated in (a), (b) and (c)

27. The heart is pumping blood at x kg per unit time, with constant velocity v. The force needed is
(a) $x\,v$ (b) $v\dfrac{dx}{dt}$
(c) $x\dfrac{dv}{dt}$ (d) zero

28. A force of 50 dynes is acted on a body of mass 5 g which is at rest for an interval of 3 sec., then impulse is
(a) 0.15×10^{-13} Ns (b) 0.98×10^{-3} Ns
(c) 1.5×10^{-3} Ns (d) 2.5×10^{-3} Ns

29. A force 10 N acts on a body of mass 20 kg for 10 sec. Change in its momentum is
(a) 5 kg m/s (b) 100 kg m/s
(c) 200 kg m/s (d) 1000 kg m/s

30. Swimming is possible on account of
(a) First law of motion
(b) Second law of motion
(c) Third law of motion
(d) Newton's law of gravitation

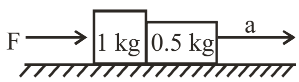
31. The distance x covered in time t by a body having velocity v_0 and having a constant acceleration a is given by $x = v_0 t + \frac{1}{2}at^2$. This result follows from
(a) Newton's first law (b) Newton's second law
(c) Newton's third law (d) none of these

32. The average force necessary to stop a hammer having momentum 25 N-s in 0.05 second is
(a) 25 N (b) 50 N
(c) 1.25 N (d) 500 N

33. A force of 100 dynes acts on mass of 5gm for 10 sec. The velocity produced is
(a) 2 cm/sec (b) 20 cm/sec
(c) 200 cm/sec (d) 2000 cm/sec

34. The Newton's laws of motion are valid in
(a) inertial frames (b) non-inertial frames
(c) rotating frames (d) accelerated frames

35. A 1 kg block and a 0.5 kg block move together on a horizontal frictionless surface . Each block exerts a force of 6 N on the other. The block move with a uniform acceleration of



(a) 3 ms^{-2} (b) 6 ms^{-2}
(c) 9 ms^{-2} (d) 12 ms^{-2}

36. Newton's third law is equivalent to the
(a) law of conservation of linear momentum
(b) law of conservation of angular momentum
(c) law of conservation of energy
(d) law of conservation of energy and mass

37. When we jump out a boat standing in water it moves
(a) forward (b) backward
(c) side ways (d) none of the above

38. A man is at rest in the middle of a pond on perfectly smooth ice. He can get himself to the shore by making use of Newton's
(a) first law (b) second law
(c) third law (d) all the laws

39. A parrot is sitting on the floor of a closed glass cage which is in a boy's hand. If the parrot starts flying with a constant speed, the boy will feel the weight of the cage as
(a) unchanged (b) reduced
(c) increased (d) nothing can be said

40. A cannon after firing recoils due to
(a) conservation of energy
(b) backward thrust of gases produced
(c) Newton's third law of motion
(d) Newton's first law of motion

More than One Option Correct :

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

1. Acceleration of a body can be calculated using
(a) $a = F/m$ (b) $a = \frac{v-u}{t}$
(c) $a = P/m$ (d) $a = s/t$

2. Which physical quantity has its unit as 'Newton' ?
(a) Friction (b) Acceleration
(c) Force (d) Momentum

3. The acceleration produced by a force of 5 N on a mass of 10 kg is
(a) 5 m/s^2 (b) 2 m/s^2
(c) 0.5 ms^{-2} (d) 50 cm s^{-2}

4. What force would be required to produce an acceleration of $4 m/s^2$ in a ball of mass 6 kg ?
(a) 24 N (b) 42 N
(c) 42 kg m/s^2 (d) 24 kg m/s^2

5. A body is in translatory equilibrium if
(a) it is in uniform motion
(b) resultant force on it is zero
(c) it is at rest
(d) it is in accelerated motion

6. If a particle under the action of a force $\vec{F}$ has potential energy U, then in equilibrium
(a) $\vec{F} = 0$ but $U \neq 0$ (b) $\vec{F} \neq 0$ and $U \neq 0$
(c) $\vec{F} \neq 0$ but $U = 0$ (d) $\vec{F} = 0$ but $U = 0$

7. If the tension in the cable supporting a lift is equal to the weight of the lift, the lift may be
(a) going up with uniform speed
(b) going up with increasing speed
(c) going down with uniform speed
(d) going down with increasing speed

8. In which of the following cases, the net force is zero?
(a) A car moving with constant speed of 30 km/hr on a rough road.
(b) A drop of train falling down with terminal velocity
(c) A cork of mass 10 g floating on water.
(d) A charged particle moving parallel to the magnetic field.

Multiple Matching Questions :

DIRECTIONS : *Following question has four statements (A, B, C and D) given in Column I and 6 statements (p, q, r , s, t, u) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in Column I with entries in Column II.*

1. | Column I | Column II |
|---|---|
| (A) Newton's first law | (p) Dust coming out of carpet beaten with stick |
| (B) Newton's second law | (q) Recoil of gun |
| (C) Newton's third law | (r) Law of inertia |
| (D) Law of conservation of momentum | (s) Catching a cricket ball |
| | (t) Use of seat belts in car |
| | (u) Flying of rockets and aeroplanes |

	A	B	C	D
(a)	p, r	s, r	q, u	q, u
(b)	p, q, r, s	q	p, q, r, s	p, q, r, s
(c)	r, t	u, s	p, r	p, r
(d)	p,	q, r	r	s

Passage Based Questions :

DIRECTIONS : *Study the given paragraph(s) and answer the following questions.*

PASSAGE

Two toy cars A and B are moving towards each other on a horizontal surface. The car A has mass of 60 g and moves towards right with a speed of 120 cms^{-1}. The car B has a mass of 100 g and moves towards the left with a speed of 50 cms^{-1}. The two cars collide and get stuck to each other.


A


B

1. Momentum of the cars A and B before collision in g cms^{-1} is:
(a) 7200, –5000 (b) 3600, 2000
(c) 3600, 1000 (d) 3200, 1600

2. Velocity of the cars with which they move after collision is:
(a) 20 m/s (b) 15 m/s
(c) 10 m/s (d) 5 m/s

3. After collision the cars will move towards:
(a) right (b) left
(c) either right or left (d) None of these

Assertion & Reason :

DIRECTIONS : *Each of these questions contains an Assertion followed by Reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.
(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.
(c) If **Assertion** is **correct** but **Reason** is **incorrect**.
(d) If **Assertion** is **incorrect** but **Reason** is **correct**.

1. **Assertion:** When a bullet is fired from a gun, there is a forward force on the bullet and recoil of gun.
Reason: Every action has an equal and opposite reaction.
2. **Assertion:** When astronauts throw something in space, that object would continue moving in the same direction and with the same speed.
Reason: The acceleration of an object produced by a net applied force is directly related to the magnitude of the force, and inversely related to the mass of the object.
3. When we sit on a chair, our body exerts a force downward and
Assertion: That chair needs to exert an equal force upward or the chair will collapse.
Reason: The third law says that for every action there is an equal and opposite reaction.
4. **Assertion:** The wings of a bird push air upwards and the air must be pushing the bird downwards.
Reason: For every action there is an equal and opposite reaction.
5. **Assertion:** When a firefly hits a bus, each of them exerts the same force.
Reason: Firefly has more mass as compared to the windshield.
6. **Assertion:** Force exerted by the ground on the man moves him forward.
Reason: It is a reactional force.
7. **Assertion:** A quick collision between two bodies is more violent than a slow collision, even when the initial and the final velocities are identical.
Reason: Because the rate of change of momentum which determines the force is greater in the first case.
8. **Assertion:** Change in momentum is impulse.
Reason: Impulse is the area between (F – t) graph and time axis.
9. **Assertion:** A body is momentarily at rest when it reverses the direction.
Reason: A body cannot have acceleration if its velocity is zero at a given instant of time.
10. **Assertion:** While walking on ice, one should take small steps to avoid slipping.
Reason: This is because smaller steps ensure smaller friction.
11. **Assertion:** Force required to accelerate a mass in two perpendicular directions is never same.
Reason: The presence of g will not influence the acceleration.

Integer/Numeric type Questions :

DIRECTIONS : *Following are integer based/Numeric based questions. Each question, when worked out will result in one integer or numeric value.*

1. A motor car of mass 1200 kg is moving along a straight line with a uniform velocity of 90 km/h. Its velocity is slowed down to 18 km/h in 4s by an unbalanced external force. Calculate the acceleration and change in momentum. Also calculate the magnitude of the force required.
2. By how much does the momentum of a body of mass 5 kg change when its speed
(i) decreases from 20 m/s to 0.20 m/s; and
(ii) increases from 30 m/s to 40 m/s?
3. Calculate the mass of the body, when a force of 525 N, produces an acceleration of 3.5 m/s^2.
4. Which would require a greater force accelerating a 2 kg mass at 5 ms^{-2} or a 4 kg mass at 2 ms^{-2}
5. A man pushes a box of mass 50 kg with a force of 80 N. What will be the acceleration of the box due to this force? What would be the acceleration if the mass were halved?
6. For how long should a force of 100 N act on a body of mass 20 kg so that it acquires a velocity of 100 m/s?
7. A car of mass 800 kg travelling at a speed of 10 ms^{-1} is brought to rest in 20 seconds by applying brakes. Calculate the average braking force acting on the wheels.
8. A bullet of mass 10 g is fired at a speed of 400 ms^{-1} from the gun of mass 4 kg. What is the recoil of the gun ?
9. A 40 kg shell is lying at a speed of 72 km/h. It explodes into two pieces, one piece of mass 15 kg stops. Calculate the velocity of the other piece.
10. A bullet of mass 10g is fired with a rifle. The bullet takes 0.003 s to move through its barrel and leaves it with a velocity of 300 m/s. What is the force exerted on the bullet by the rifle ?
11. A force produces an acceleration of 16 m/s^2 in a body of mass 0.5 kg, and an acceleration of 4 m/s^2 in another body. If both the bodies are fastened together, then what is the acceleration produced by that force?
12. A 20 gm bullet moving at 300 m/s stops after penetrating 3 cm of bone. Calculate the average force exerted by the bullet.
13. A force of 50 N is inclined to the vertical at an angle of 30°. Find the aceeleration it produces in a body of mass 2 kg which moves in the horizontal direction.
14. A gun weighing 10 kg fires a bullet of 30 g with a velocity of 330 m/s. With what velocity does the gun recoil? What is the resultant momentum of the gun and the bullet before and after firing?

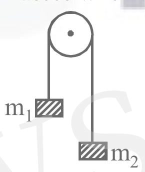
4 ADVANCED EXERCISE
BASED ON CONNECTING TOPICS

DIRECTIONS (Qs. 1–7) : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. A body is imparted motion from rest to move in a straight line. If it is then obstructed by an opposite force, then
(a) the body will necessarily change direction
(b) the body is sure to slow down
(c) the body will necessarily continue to move in the same direction at the same speed
(d) None of these

2. A block weighs W is held against a vertical wall by applying a horizontal force F. The minimum value of F needed to hold the block is
(a) Less than W (b) Equal to W
(c) Greater than W (d) Data is insufficient

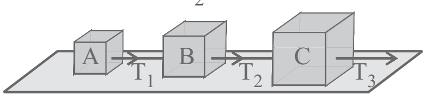
3. Two blocks $m_1 = 5$ gm and $m_2 = 10$ gm are hung vertically over a light frictionless pulley as shown here. What is the acceleration of the masses when they are left free?



(a) g / 3 (b) g / 2 (c) g (d) g / 5
(where g is acceleration due to gravity)

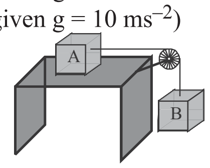
4. A man weighing 80 kg, stands on a weighing scale in a lift which is moving upwards with a uniform acceleration of $5 m/s^2$. What would be the reading on the scale ? ($g = 10 m/s^2$)
(a) 1200 N (b) Zero (c) 400 N (d) 800 N

5. Three blocks A, B and C weighing 1, 8 and 27 kg respectively are connected as shown in the figure with an inextensible string and are moving on a smooth surface. T_3 is equal to 36 N. Then T_2 is



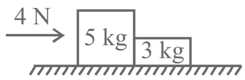
(a) 18 N (b) 9 N (c) 3.375 N (d) 1.25 N

6. A block A of mass 7 kg is placed on a frictionless table. A thread tied to it passes over a frictionless pulley and carries a body B of mass 3 kg at the other end. The acceleration of the system is (given $g = 10 ms^{-2}$)



(a) $100 ms^{-2}$ (b) $3 ms^{-2}$
(c) $10 ms^{-2}$ (d) $30 ms^{-2}$

7. Two blocks of masses 5 kg and 3 kg are placed in contact on a horizonrtal frictionless surface as shown in the figure. A force of 4N is applied on mass 5 kg. The acceleration of the mass 3 kg will be

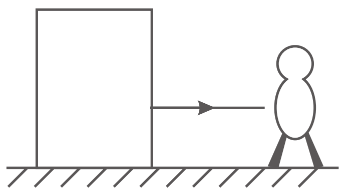


(a) $\frac{4}{5} m/s^2$ (b) $\frac{4}{3} m/s^2$
(c) $2 m/s^2$ (d) $0.5 m/s^2$

DIRECTIONS (Qs. 8–11) : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

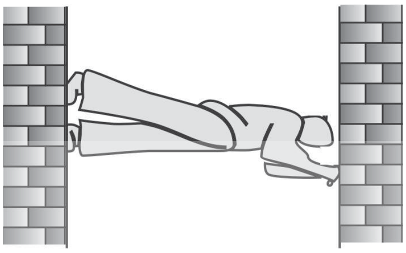
8. A monkey of mass *m* kg slides down a light rope attached to a fixed spring balance with an acceleration *a*. The reading of the spring is W kg (g = acceleration due to gravity). Then:
(a) the tension in the slope is W_g N
(b) the force of friction exerted by the rope on the monkey is $m(g - a)N$.
(c) $m = \frac{W_g}{g-a}$ (d) $m = W\left(1 + \frac{a}{g}\right)$

9. A man pulls a block heavier than himself with a light rope.



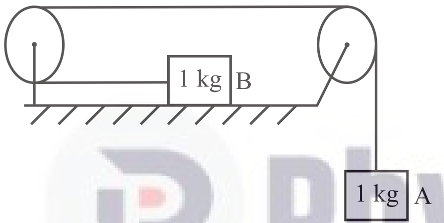
The magnitude of friction is the same between the man and the ground and between the block and the ground. Then
(a) if both move, the acceleration of the man is greater than the acceleration of the block.
(b) the block will not move unless the man also moves
(c) the man can move even when the block is stationary.
(d) none of these assertions is correct.

10. A man tries to remain in equilibrium by pushing with his hands and feet against two parallel walls. For equilibrium:



(a) the magnitude of friction must be the same between both walls and the man
(b) he must exert equal forces on the two walls
(c) the forces of friction at the two walls must be equal
(d) friction must be present on both walls

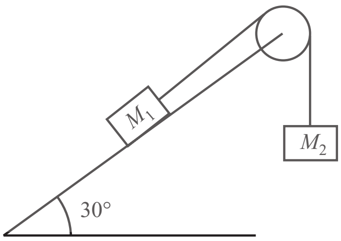
11. Refer to the figure, the pulleys and the string are light. T is the tension in the string. Take $g = 10\ ms^{-2}$. Which of the following is are correct?



(a) The acceleration of the system is 5 ms^{-2}
(b) T = 0N
(c) The acceleration of the system is 10 ms^{-2}
(d) T = 5 N

DIRECTIONS (Qs. 12) : *Following question has four statements (A, B, C and D) given in Column I and four statements (p, q, r , s...) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in Column I with entries in Column II.*

12. For the system shown in the figure, the incline is frictionless and the string is massless and inextensible pulley is light and frictionless. As the system is released from rest.



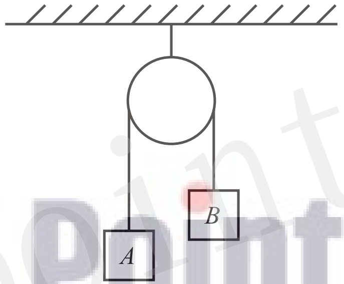
Column I		Column II
(A) $M_1 > M_2$	(p)	M_2 accelerates down
(B) $M_2 > M_1$	(q)	M_2 accelerates up
(C) $M_1 = M_2$	(r)	M_1, M_2 in equilibrium
(D) $M_1 >> M_2$	(s)	Tension in string equals the weight of either block

	A	B	C	D
(a)	p,q	q	r, s	q, r
(b)	p, q, r, s	p, s	p	q
(c)	p, s	q	r, s, t	r
(d)	p, q	s, p	q	r, s

DIRECTIONS (Qs. 13–15) : *Study the given paragraph(s) and answer the following questions.*

PASSAGE

Two blocks A and B of mass 2 kg and 3 kg respectively are connected with the help of a massless, inextensible string passing over a smooth pulley as shown, The system is released from rest at $t = 0$, then: (take $g = 10\ ms^{-2}$)



13. Acceleration of blocks is
(a) 5 ms^{-2} (b) 2 ms^{-2}
(c) 20/3 ms^{-2} (d) 6 ms^{-2}

14. If at t = 1s, block B is stopped momentarily and released, after how much time will the string become taut again?
(a) 0.2s (b) 0.4s
(c) 0.5s (d) 1s

15. What will be the velocity of the blocks, just after the string becomes taut?
(a) 0.2 ms^{-1} (b) 0.4 ms^{-1}
(c) 1 ms^{-1} (d) 2 ms^{-1}

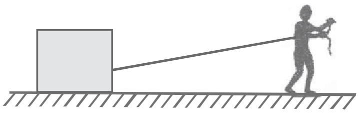
DIRECTIONS (Qs. 16–18): *Each of these questions contains an Assertion followed by Reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.
(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.
(c) If **Assertion** is **correct** but **Reason** is **incorrect**.
(d) If **Assertion** is **incorrect** but **Reason** is **correct**.

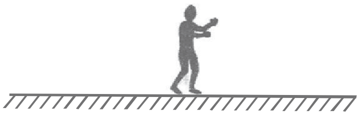
16. Assertion : A block placed on a table is at rest, because action force cancels the reaction force on the block.
Reason : The net force on the block is zero.

17. **Assertion :** It is easier to pull a heavy object than to push it on a level ground.
Reason : The magnitude of frictional force depends on the nature of the two surfaces in contact.

18. **Assertion ::** A man and a block rest on smooth horizontal surface. The man holds a rope which is connected to block. The man cannot move on the horizontal surface.



Reason : A man standing at rest on smooth horizontal surface cannot start walking due to absence of friction (The man is only in contact with floor as shown).

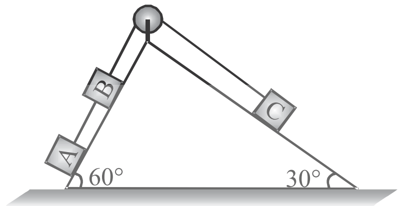


Integer/Numeric type Questions :

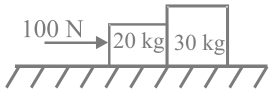
DIRECTIONS (Qs. 19–23) : *Following are integer based/ Numeric based questions. Each question, when worked out will result in one integer or numeric value.*

19. A 12 kg monkey climbs a lift rope as shown in figure. The rope passes over a pulley and is attached to a 16 kg bunch of bananas. Mass and friction in the pulley are negligible so that the pulley's only effect is to reverse the direction of the rope. What is the maximum acceleration the monkey can have without lifting the bananas ?

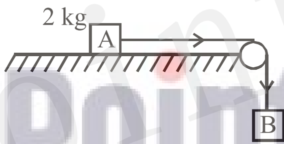
20. In the figure shown, masses of A, B and C are 1 kg, 3 kg and 2 kg respectively. Find (a) the acceleration of the system and (b) tensions in the string. Neglect friction and take $g = 10 \text{ m/s}^2$.



21. Find acceleration and contact force between the two bodies masses 20 kg and 30 kg.



22. The coefficient of static friction, μ_s, between block A of mass 2 kg and the table as shown in the figure is 0.2. What would be the maximum mass value of block B so that the two blocks do not move? The string and the pulley are assumed to be smooth and massless. ($g = 10 \text{ m/s}^2$)



23. A conveyor belt is moving at a constant speed of 2m/s. A box is gently dropped on it. The coefficient of friction between them is $\mu = 0.5$. Find the distance that the box will move relative to belt before coming to rest on it. (Take, $g = 10 \text{ ms}^{-2}$)

SOLUTIONS

Brief Explanations of Selected Questions

1 EXERCISE

FILL IN THE BLANKS

1. forward
2. mass
3. equal, opposite
4. velocity
5. equilibrium
6. time
7. impulse
8. newton
9. 200 newtons
10. momentum
11. impulse
12. net unbalanced
13. impulse

TRUE/FALSE :

1. True 2. True 3. False 4. False 5. False
6. True 7. True 8. True 9. True 10. True
11. False 12. False

MATCH THE COLUMNS

1. (A) → (r); (B) → (p); (C) → (s); (D) → (t)
2. (A) → q; (B) → p; (C) → t; (D) → r; (E) → s

VERY SHORT ANSWER QUESTIONS

1. It would increase in length by 0.1cm only.
2. 10kg-wt.
3. Since distance ∝ (velocity) ^{2}, motorcar will cover the distance four times longer than before.
4. Change in linear momentum $= -mv - mv = -2mv$
   $= -2 \times 0.5 \times 10 = -10$ kgms^{-1}.
5. Physical balance will show no change whereas, spring balance will show higher reading.
6. When the force applied on a body is zero, Newton's first law becomes a special case of Newton's second law.
7. $F = -m \times a$ or $-F = m \times a$
8. The rolling friction is lesser as compared to the sliding friction.

SHORT ANSWER QUESTIONS

1. Vehicle stops on applying brakes, which is in accordance with the law of conservation of momentum. When brakes are applied, opposition force acts on the vehicle. In pursuit, car will be at rest. Loss of momentum of the vehicle is exactly equal to the impulse of the applied force.
2. In closed glass cage, air inside is bound with the cage. Therefore, (i) no change in weight of the cage, when bird flies with constant velocity, (ii) cage will be heavier, when bird flies with upward acceleration , (iii) cage will be lighter, when bird flies with downward acceleration.
3. While applying brakes, let F_B be the force required to stop the truck in distance d.
   $\therefore F_B \times d = \frac{1}{2} mv^2$ or, $F_B = mv^2/2d$
   For taking a turn radius d, the force required, $F_T = mv^2/d$ $= 2F_B$ or, $F_B = \frac{1}{2} F_T$, which means it is better to apply brakes.

LONG ANWER QUESTIONS

1. Two major forces act on the water skier, one of these forces is that exerted on her hands by the towrope. The other is the resistance of the water (and to a lesser extent the air). When the skier is moving in a straight line at a constant 15 km/h, her velocity is constant. According to Newton's first law, the net force on the skier must be zero. Indeed, the law itself is the basis for asserting that the force exerted by the water and the air on the skier is exactly equal in magnitude, and opposite in direction, to that exerted on her by the towrope.
   When the skier is moving at 20 km/h, the force exerted on her by the towrope is greater in magnitude than that at 15 km/h.
   But so is the resistive force. Again, the net force on the skier must be zero because her velocity is constant.

2 EXERCISE

TEXT-BOOK QUESTIONS

1. (a) a stone
   (b) a train
   (c) a five rupee coin
   Reason for all three cases is the same. Greater the mass greater is the inertia.
2. The velocity of ball changes three times.
   First time, the velocity changes when the ***football player of one team*** kicks the ball.
   Second time, the velocity changes when ***another player of same team*** kicks the football.
   Third time, the velocity changes when the ***goalkeeper of the opposite team*** kicks the football.
   The agents supplying the force is the first and second.
3. Before shaking the branches, leaves are at rest. When branches are shaken, they come in motion while the leaves tend to remain at rest due to inertia of rest. As a result leaves get detached from the branches and fall down.
4. When a moving bus brakes to a stop, the lower part of our body in contact with the bus comes to rest while the upper part of our body tends to keep moving due to inertia of motion. Hence we fall forwards. When the bus accelerates from rest, the lower part of our body comes into motion alongwith the bus while the upper part of body tends to remain at rest due to inertia of motion. Hence we fall backwards.
5. The horse pulls the cart with a force (action) in the forward

direction. The cart also pulls the horse with an equal force (reaction) in the backward direction. While pulling the cart, the horse pushes the ground with its feet in the backward direction, the reaction of the earth makes it move in the forward direction alongwith the cart . Here action is on the ground while the reaction on the horse.

6. It is difficult for a fireman to hold a hose because the water is ejected out in the forward direction with a large force due to which same force is developed in the hose in the opposite direction and therefore hose is pushed in the backward direction. Thus, it becomes difficult for a firemean to hold a hose.

7. Mass of bullet, m_1 = 50 g = 0.05 kg
Mass of rifle, m_2 = 4 kg
Initial velocity of bullet, u_1 = 0
Initial velocity of rifle, u_2 = 0
Final velocity of bullet, v_1 = 35 ms^{-1}
Recoil velocity of rifle, v_2 = ?
According to the law of conservation of momentum,
Total momenta after firing = Total momenta before firing

$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$$
$$0.05 \times 35 + 4v_2 = 0 + 0$$
$$v_2 = -\frac{0.05 \times 35}{4} = -\frac{7}{16} = -0.44 \text{ ms}^{-1}$$

The negative sign indicates that the direction in which the rifle would recoil is opposite to that of the bullet.

8. Here, $m_1 = 100$ g = 0.1 kg, $m_2 = 200$ g = 0.2 kg, $u_1 = 2\text{ms}^{-1}$, $u_2 = 1ms^{-1}$, $v_1 = 1.67 \text{ ms}^{-1}$, v_2 = ?
According to the law of conservation of momentum.
Total momenta before collision = Total momenta after collision

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
or $$0.1 \times 2 + 0.2 \times 1 = 0.1 \times 1.67 + 0.2v_2$$
or $$0.4 = 0.167 + 0.2\, v_2$$
or $$v_2 = \frac{0.4 - 0.167}{0.2} = 1.165 \text{ ms}^{-1}$$

TEXT-BOOK EXERCISE

1. Yes, an object may travel with a non-zero velocity even when the net external force on it is zero. A rain drop falls down with a constant velocity. The weight of the drop is balanced by the upthrust and the viscosity of air. The net force on the drop is zero.
2. When we beat the carpet with a stick, it comes into motion. But the dust particles continue to be at rest due to inertia and get detached from the carpet.
3. Due to sudden jerks or due to the taking sharp turn on the road, the luggage may fall down from the roof because of its tendency to continue moving in the original direction. To avoid this, the luggage is tied with a roap on the roof.
4. (c) Force of friction acts on the ball in the opposite direction hence it comes to rest.
5. Here, u = 0, s = 400m, t = 20s

$\therefore$ $$s = ut + \frac{1}{2}at^2$$
or $$400 = 0 + \frac{1}{2}a(20)^2$$
or $$a = \frac{400 \times 2}{400} = 2 \text{ m/s}^2$$

Now m = 7 metric tonne = 7000 kg, a = 2 m/s^2
$\therefore$ Force, F = ma = 7000 × 2 = 14,000 N

6. Here, m = 1kg, $u = 20 \text{ ms}^{-1}$, $v = 0$, s = 50m
As $$v^2 - u^2 = 2as$$
$\therefore$ $$0^2 - 20^2 = 2a \times 50$$
or $$a = -\frac{400}{100} = -4 \text{ ms}^{-2}$$
Force of friction, F = ma = 1 × (–4) = –4 N

7. Total mass of engine and 5 wagons,
m = 8,000 + 5 × 2,000 = 18,000 kg
(a) The net accelerating force,
F = force exerted by engine – friction force
= 40,000 – 5,000 = 35000 N.
(b) The acceleration of the train.
$$a = \frac{F}{m} = \frac{35,000}{18,000} = \frac{35}{18} = 1.94 \text{ ms}^{-2}$$
(c) The force of wagon 1 on wagon 2
= The net accelerating force – mass of wagon × acceleration
$= 35,000 - 2,000 \times \frac{35}{18}$
= 35,000 – 3888.8
= 31, 111.2 N.

8. Here m = 1,500 kg, a = –1.7 ms^{-2}
F = ma = 1,500 × (–1.7) = –2,550 N
The force between the vehicle and the road is 2550 N, in a direction opposite to the direction of the motion of the vehicle.

9. (d) mv

10. The cabinet will move with constant velocity only when the net force on it zero.
$\therefore$ Force of friction on the cabinet = 200 N, in a direction opposite to the direction of motion of the cabinet.

11. Here, $m_1 = m_2 = 1.5$ kg, $u_1 = 2.5ms^{-1}$, $u_2 = -2.5 \text{ ms}^{-1}$
Let $\boldsymbol{v}$ be the velocity of the combined object after the collision. By conservation of momentum,
Total momenta after collision = Total momenta before collision.
$$(m_1 + m_2)v = m_1u_1 + m_2u_2$$
$$(1.5 + 1.5)\, v = 1.5 \times 2.5 + 1.5 \times (-2.5)$$
$$3.0\, v = 0$$
or $$v = 0 \text{ ms}^{-1}$$

12. Action and reaction always act on different bodies, so they do not cancel each other. When we push a massive truck, the force of friction between its tyres and the road is very large and the force exerted by push is not sufficient to over come that force so the truck does not move.

13. Here, m = 200 g = 0.2 kg, u = 10ms^{-1} , v = – 5ms^{-1}
Change in momentum = $m(v - u)$
= 0.2(–5 – 10) = –3 kg ms^{-1}

14. Here, m = 10 g = 0.01 kg, u = 150 ms^{-1}, v = 0, t = 0.03 s

$$a = \frac{v-u}{t} = \frac{0-150}{0.03} = -5000 ms^{-2}$$

The distance of penetration of the bullet into the block,

$$s = ut + \frac{1}{2}at^2$$

$$= 150 \times 0.03 + \frac{1}{2} \times (-5000) \times (0.03)^2$$

$$= 4.5 - 2.25 = 2.25 \text{ m}$$

The magnitude of the force exerted by the wooden block on the bullet.

= ma = 0.01 × 5000 = 50 N.

15. Here, m_1 = 1 kg, u_1 = 10 ms^{-1}, m_2 = 5 kg, u_2 = 0

Let v be the velocity of the combined object after the collision.

Total momentum just before the impact

$= m_1u_1 + m_2u_2$

= 1 × 10 + 5 × 0 = 10 kg ms^{-1}

Total momentum of the combined object just after the impact.

$= (m_1 + m_2)v = (1 + 5)v = 6v$ kg ms^{-1}

By conservation of momentum,

Final momentum = Initial momentum

$6v = 10$

or $v = \frac{10}{6} = \frac{5}{3}$ ms^{-1}

∴ Total momentum just after the impact

$$= 6 \times \frac{5}{3} = 10 \text{ kg ms}^{-1}$$

16. Here, m = 100 kg, u = 5 ms^{-1}, v = 8ms^{-1}, t = 6s

Initial momentum,

p_1 = mu = 100 × 5 = 500 kg ms^{-1}

Final momentum,

p_2 = mv = 100 × 8 = 800 kg ms^{-1}

The magnitude of the force exerted on the object,

$$F = \frac{p_2 - p_1}{t} = \frac{800-500}{6} \; \frac{300}{6} = 50\text{N}$$

17. Rahul gave the correct explanation. According to Newton's 3rd law both the motorcar and insect experience the equal force and hence a same change in their momentum. Due to smaller mass or inertia, the insect dies. The mass of motor car is much larger so there is little change in its velocity.

18. Here, m = 10 kg, u = 0, h = 80 cm = 0.80 m, a = 10ms^{-2}

Let v be the velocity gained by the dumb-bell as it reaches the floor.

As $v^2 - u^2 = 2ah$

∴ $v^2 - 0^2 = 2 \times 10 \times 0.80 = 16$

or $v = 4$ ms^{-1}

Momentum transferred by the dumb-bell to the floor

p = mv = 10 × 4

= 40 kg ms^{-1}

EXEMPLAR QUESTIONS

1. Yes, the ball will start rolling in the direction in which the train was moving. Due to the application of the brakes, the train comes to rest but due to inertia the balls remain in motion, therefore, they begin to roll. Since the masses of the balls are not the same, therefore, the inertial forces are not same on both the balls. Thus, the balls will move with different speeds.

2. From the light rifle, according to law of conservation of momentum or explanation of Newton's laws of motion.

3. Law of conservation of momentum is applicable to isolated system (on external force is applied). In this case, the change in velocity is due to the gravitational force of earth.

4. Acceleration

$$a = \frac{v-u}{t} = \frac{80}{8} ms^{-2} = 10 \text{ ms}^{-2}$$

$$\text{Force} = \text{ma} = \frac{50}{1000} \times 10 = 0.5 \text{ N}$$

5. Seperation between them will increase. Initially the momentum of both of them are zero as they are at rest. In order to conserve the momentum the one who throws the ball would move backward. The second will experience a net force after catching the ball and therefore will move backwards that is in the direction of the force.

6. (i) $m = 10 \text{ g} = \frac{10}{1000}$ kg

$v = 10^3$ m/s

v = 0

$s = \frac{5}{100}$ m

$v^2 - u^2 = 2as$

$$0 - (10^3)^2 = 2.a.\frac{5}{100}$$

$$a = \frac{-1000 \times 1000}{10} \times 100 = -10^7 \text{ ms}^{-2}$$

F = m.a = 10^5 N

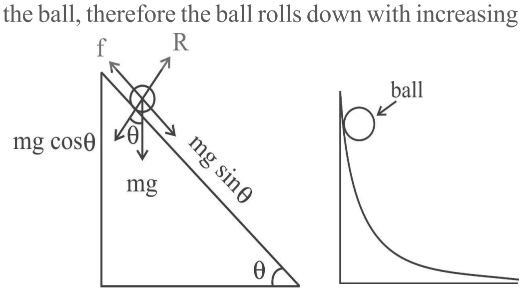
(ii) v = u + at

$0 = 10^3 - 10^7$t

$$10^7 t = 10^3 \Rightarrow t = \frac{10^3}{10^7} = 10^{-4}\text{s}$$

HOTS QUESTIONS

1. When the ball rolls down the hill as shown in the figure, a component of weight of the ball acts downwards along the inclined plane. It is in the opposite direction of the force of friction. The net force along the inclined plane remains constant throughout the motion and hence the acceleration is also constant. Since a constant acceleration is acting on the ball, therefore the ball rolls down with increasing speed.

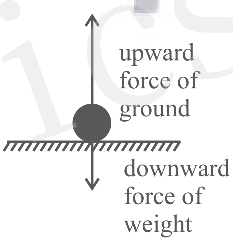


When the ball rolls down the hill as shown in the figure, its acceleration starts decreasing after a certain height because the slope of the hill is not constant. When the ball rolls down from the top of the hill its acceleration increases but as soon as it reaches the point from where the slope is lesser, the acceleration starts decreasing.

2. If an object is dropped from rest i.e. its initial speed is zero, it falls under gravity with an acceleration 10 ms^{-2}. On the other hand if the object is thrown down instead then we impart some initial velocity (non zero) to it. As soon as it leaves our hand it moves under the force of gravity and hence its acceleration would be the same as 10 ms^{-2}. This is because the acceleration due to gravity remains constant in both the cases whether an object is dropped from rest or thrown downwards with some initial velocity.

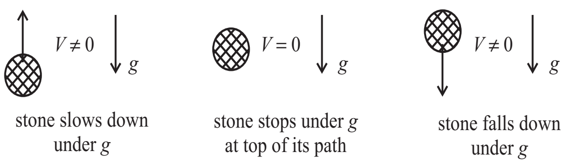
3. When our hand turns the handle of a faucet, the faucet in turn applies a pressure on our fingers. These two forces i.e, the push of our hand on the handle of the tap and the push of the tap on our hand make the action-reaction pair.
When force is applied by our hand turns the faucet which moves the screw inside the tap and releases the water which flows out of the nozzle of the tap. The force of screw of the tap on the water and force of water on it make another action-reaction pair. But this means that our push on the handle and the water coming out of the tap do not make an action-reaction pair since our hand and water is not touching each other.

4. (a) The figure shows a stone resting on the ground. The net force on the stone is zero as its acceleration is zero. One of the force that acts on stone is the weight force of the stone, due to earth's gravitational pull, acting in the downward direction. This force acts on the ground and pushes it down. As a reaction to this force the ground also exerts an equal and opposite force on the stone in the upward direction. As a result the two forces cancel and no net force acts on the stone. The force vectors can be represented as shown in figure 1.



(b) The downward weight force is balanced by an upward force. This upward force is conventionally called reaction force of the ground that prevents the ground from sinking and keeps the stone at rest. It acts normal to the surface of the ground.

5. At the top of the path the stone is at rest. This happens because the upward acceleration of the stone is balanced by the downward acceleration due to gravity. The net force on the stone at the top is the force due to gravity



At that moment, its instantaneous velocity Vinst is zero (rest point). The acceleration of the stone is not zero but it is equal to 'g' i.e. acceleration due to gravity. This is because the upward moving stone slows down under the effect of gravity which acts at a retarding force and brings it to rest after which the stone falls down. At all times gravitational force is acting on the stone and it changes its velocity by decreasing it to zero and then increasing it in reverse direction.

3 EXERCISE

SINGLE OPTION CORRECT :

1. (c) **2.** (b) **3.** (b) **4.** (a)

5. (c) Acceleration = $\left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)g$

$$= \frac{(5-4.8)\times 9.8}{(5+4.8)} m/s^2 = 0.2 \text{ m/s}^2$$

6. (a) **7.** (d) **8.** (a) **9.** (d) **10.** (d) **11.** (a)
12. (b) **13.** (c) **14.** (a)
15. (b) $2T\cos 60° = mg$
or $T = mg = 2\times 10 = 20$ N.
16. (a) **17.** (d) **18.** (a) **19.** (a) **20.** (c)
21. (a) **22.** (b) **23.** (c) **24.** (c)
25. (a) Change in momentum = Force × time = Area which the force-time curve encloses with time axis.
26. (b) **27.** (a) **28.** (c) **29.** (b) **30.** (c) **31.** (b)
32. (d) **33.** (c) **34.** (a)
35. (d) For 0.5 kg block, 6 = 0.5 a
36. (a) **37.** (b) **38.** (c) **39.** (a) **40.** (c)

MORE THAN ONE OPTION CORRECT

1. (a, b) **2.** (a, c) **3.** (c, d) **4.** (a, d)
5. (a, b, c)
For translatory equilibrium $F_{net} = 0$
$\therefore$ $a = 0$ or v = const
6. (a, d) For equilibrium, $f = 0$ while U may or may not be zero.
7. (a. c) If T = mg, a = 0
8. (a, b, c, d)

MULTIPLE MATCHING QUESTIONS

1. (a) (A) → (p, r); (B) → (s, r); (C) → (q, u); (D) → (q, u)

PASSAGE BASED QUESTIONS

1. **(a)** Let us take the direction towards the right as positive direction. The linear momentum of a before the collision is
$P_1 = m_1v_1 = 60 \times 120 = 7200$ g cms^{-1} and that of B is
$P_2 = m_2v_2 = 100 \times (-50) = -5000$ $gcms^{-1}$
The total momentum of A and B before collision is
$P = P_1 + P_2 = 7200 - 5000 = 2200$ $gcms^{-1}$

2. (b) Let the velocity just after collision be v. The total momentum
$m_1v + m_2v = (160\text{ g})\,v$
By the law of conservation of momentum
$2200\text{ gcms}^{-1} = (160\text{ g})v$
$v = 13.75\text{ ms}^{-1}$

3. (a) As v comes out to be positive, the cars move together towards the right.

ASSERTION & REASON

1. (a) 2. (b) 3. (a) 4. (d) 5. (c) 6. (b)
7. (a) 8. (b) 9. (c) 10. (a) 11. (c)

INTEGER/NUMERIC QUESTIONS

1. -5ms^{-2}, -24000 kgms^{-1}, -6000 N
2. (i) -99 kg m/s (ii) 50 kg m/s
3. 150 kg
4. 2 kg mass at 5 ms^{-2}
5. 3.2 m/s^2
6. 20 s
7. Average braking force = – 400 N.
8. The gun recoils with a velocity of 1 ms^{-1} in the direction opposite to that of the bullet.
9. 32 m/s
10. 10^3 N
11. 3.2 m/s^2
$a_1 = 16\text{ m/s}^2$, $m_1 = 0.5$ kg,
$F = m_1a_1 = 16 \times 0.5\text{ kg} = 8$ N

$$ma_2 = \frac{F}{a_2} = \frac{4}{8} = 2\text{ kg}\ ;\ m_1 + m_2 = 0.5 + 2 = 2.5\text{ kg},$$

$$F = 8\ N;\quad a = \frac{F}{m_1 + m_2} = \frac{8}{2.5} = 3.2\ \text{m/s}^2$$

12. -3×10^4 N

Hints: $m = \frac{20}{1000}$ kg, $u = 300$ m/s, $v = 0$, $s = 3$ cm
$= 3 \times 10^{-2}$ m

$$\therefore\quad a = \frac{v^2 - u^2}{2s} = \frac{0-(300)^2}{2\times3\times10^{-2}} = \frac{-300\times300}{2\times3\times10^{-2}}$$
$= -1.5 \times 10^6\text{ m/s}^2$

$$\therefore\quad F = ma = \frac{20}{1000}\times(-1.5\times10^6) = -3\times10^4\text{ N}$$

13. 12.5 m/s^2
Hints: Horizontal component of force = F sin q

$$\Rightarrow a = \frac{F\sin\theta}{m} = \frac{50\sin30°}{2} = 12.5\text{m/s}^2$$

14. – 0.99 m/s, zero before and after firing

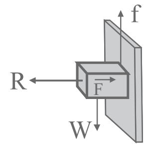
Hints: M = 10 kg, $m = \frac{30}{1000}$ kg., V = ?, v = 330 m/s
From conservation of momentum, MV + mv = 0

$$\Rightarrow 10\times V = -\frac{30}{1000}\times330 \Rightarrow V = -0.99\text{ m/s}$$

Resultant momentum before and after firing is zero as no external froce is acting on it.

4 ADVANCED EXERCISE
BASED ON CONNECTING TOPICS

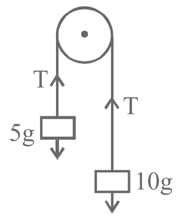
1. **(b)** Opposite force causes retardation.
2. **(c)** Here applied horizontal force F acts as normal reaction.
For holding the block
Force of friction = Weight of block
$f = W \Rightarrow \mu R = W \Rightarrow \mu F = W$



$$\Rightarrow F = \frac{W}{\mu}$$

As $\mu < 1\ \therefore F > W$

3. **(a)** Let T be the tension in the string.
$\therefore\quad 10g - T = 10a$ ....(i)
$T - 5g = 5a$ ....(ii)



Adding (i) and (ii),
$$5g = 15a \Rightarrow a = \frac{g}{3}\ m/s^2$$

4. **(a)** Reading of the scale
= Apparent wt. of the mass = m(g + a)
= 80 (10 + 5) = 1200 N

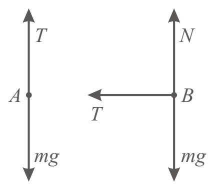
5. **(b)** $$T_2 = (m_A + m_B)\times\frac{T_3}{m_A + m_B + m_C}$$
$$= (1+8)\times\frac{36}{(1+8+27)} = 9\text{ N}$$

6. **(b)** $$a = \frac{m_2}{m_1 + m_2}g = \frac{3}{7+3}\times10 = 3\text{ m/s}^2$$

7. **(d)** Both blocks will move with same acceleration (a) given by
$$a = \frac{F}{m_1 + m_2} = \frac{4}{5+3} = \frac{4}{8} = 0.5\text{ m/s}^2$$

8. (a, b, c) 9. (a, b, c) 10. (b, c, d)

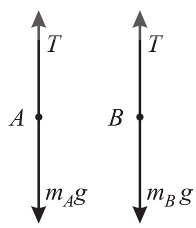
11. (a, c) For motion of block A
$mg - T = ma$ ...(1)
For motion of block B
$T = ma$ ...(2)
On solving



$$a = \frac{g}{2} = 5\text{ms}^{-2}\ \&\ T = \frac{1}{2}mg = 5\text{N}$$

12. (b) (A) → (p, q, r, s); (B) → (p, s); (C) → (p); (D) → (q)

13. (b) *FBD* is of A and B are shown in figure, as $m_B > m_A$, A moves up and B moves down with same acceleration Q.
Here,
$m_Bg - T = m_Ba$ ...(1)
& $T - m_Ag = m_Aa$ ...(2)
On solving



$$a = \frac{m_B - m_A}{m_B + m_A}g = \frac{g}{5} = 2\text{ ms}^{-2}$$

14. (a) Speed of A at $t = 1$s,
$v = u + at = 2\text{ ms}^{-1}$

The string will become taut again when distance covered by both the blocks is equal, i.e.

$s_A = s_B$

$2t - \frac{1}{2}gt^2 = \frac{1}{2}gt^2$

or $t = 0.2$s.

15. (b) Let u_A and u_B be their velocities, just before the string become taut, then

$u_A = 2 - gt = 0$, $u_B = gt = 2\ ms^{-1}$

Let v be their common speed just after the string becomes taut, then

impulse of A on B = – impulse of B on A

$m_A(v - 0) = - m_B(v - 2)$

$2v = -3(v - 2)$

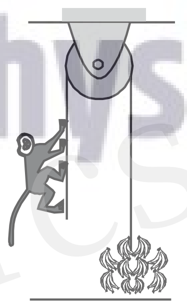
or $v = 0.4\ ms^{-1}$

16. (d) The net force on the block is zero, but action cannot cancel the reaction because these two act on different bodies.

17. (b)

18. (d) The man can exert force on block by pulling the rope. The tension in rope will make the man move.

19. Effective weight of monkey



$W_m = M_m(g + a)$

As per given condition

$W_m = M_b$

$\Rightarrow M_m(g + a) = M_b g$

$\Rightarrow a = \frac{(M_b - M_m)g}{M_m}$

$= \left(\frac{16-12}{12}\right) \times 9.8$

$\Rightarrow a = \frac{9.8}{3} = 3.26\ m/s^2$

20. (a) In this case net pulling force,

$F = m_A g \sin 60° + m_B g \sin 60° - m_C g \sin 30°$

$= (m_A + m_B) g \sin 60° - m_C g \sin 30°$

$= (1 + 3) \times 10 \times \frac{\sqrt{3}}{2} - 2 \times 10 \times \frac{1}{2}$

$= 20\sqrt{3} - 10 = 20 \times 1.732 - 10 = 24.64$ N

Total mass being pulled = 1 + 3 + 2 = 6 kg

$\therefore$ Acceleration of the system,

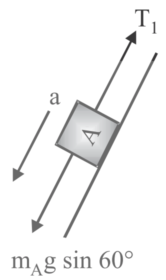
$a = \frac{24.64}{6} = 4.1\ m/s^2$

(b) For the tension in the string between A and B

$m_A g \sin 60° - T_1 = m_A a$

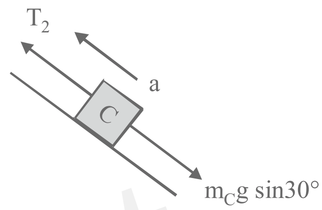
$\therefore T_1 = m_A g \sin 60° - m_A a = m_A (g \sin 60° - a)$

$\therefore T_1 = (1)\left(10 \times \frac{\sqrt{3}}{2} - 4.1\right) = 4.56$ N



F.B.D. of A

For the tension in the string between B and C



F.B.D. of C

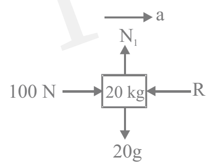
$T_2 - m_C g \sin 30° = m_C A$

$T_2 = mC(a + g \sin 30°)$

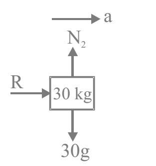
$= 2\left[4.1 + 10\left(\frac{1}{2}\right)\right] = 18.2$ N

21.

FBD of mass 20 kg



FBD of mass 30 kg



$100 - R = 20a$ … (i)

$R = 30a$ … (ii)

From eqs. (i) and (ii)

$a = 2\ m/s^2$, $R = 60$ N

22. $m_B g = \mu_s m_A g$ $\{\because m_A g = \mu_s m_A g\}$

$\Rightarrow m_B = \mu_s m_A$

or $m_B = 0.2 \times 2 = 0.4$ kg

23. Frictional force on the box $f = \mu mg$

$\therefore$ Acceleration in the box

$a = \mu g = 5\ ms^{-2}$

$v^2 = u^2 + 2as$

$\Rightarrow 0 = 2^2 + 2 \times (5)s$

$\Rightarrow s = -\frac{2}{5}$ w.r.t. belt

$\Rightarrow$ distance = 0.4 m

Chapter 3

GRAVITATION

INTRODUCTION

Throw a ball vertically upwards in air. It rises up and again falls back downwards. The question arises here that why does the ball fall down; Why does it not go upwards ? This question got an answer in the year 1665 when Newton provided the theory of gravitation to the world. According to his theory, earth attracts every body towards itself with a force known as 'gravity'. Due to the force of gravity the ball doesn't go upwards but it falls downwards after covering some vertical distance.

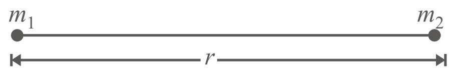
Actually, every object attracts every other object towards itself with a force. This force is called the gravitational force. Gravitational force is one among the four fundamental forces. It is always attractive in nature. This chapter is basically an outline of the theory of gravitation, acceleration due to gravity (the acceleration produced in an object due to the force of gravity), motion of a satellite and the Kepler's laws (laws governing the motion of a planet).

NEWTON'S UNIVERSAL LAW OF GRAVITATION

Newton came to the conclusion that any two objects in the Universe exert gravitational attraction on each other, with the force having a universal form:

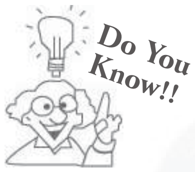
Any two particles of matter anywhere in the universe attract each other with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them, the direction of the force being along the line joining the particles, i.e. $F \propto \dfrac{m_1 m_2}{r^2}$

or $F = \dfrac{G m_1 m_2}{r^2}$



Here, the constant of proportionality G is known as the universal gravitational constant. It is termed a "universal constant" because it is thought to be the same at all places and all times, and thus universally characterizes the intrinsic strength of the gravitational force. If $m_1 = m_2 = 1$kg, $r = 1$m then $F = G$.

Gravitational constant is defined as the force of attraction acting between two unit masses placed at unit distance apart. G is a scalar quantity. $G = 6.673 \times 10^{-11}$ Nm^2/kg^2.



G is very small-hence gravitational force is very small, unless one (or both) of the masses is huge.

Important Characteristics of Gravitational Force

(a) Gravitational forces are always attractive and always acts along the line joining the two masses.

(b) Gravitational force is a mutual force hence it is action-reaction force, i.e., $\vec{F}_{12} = -\vec{F}_{21}$.

(c) Value of G is small, therefore, gravitational force is weaker than electrostatic and nuclear forces.

(d) Gravitational forces are conservative forces. Therefore, the work done by the gravitational force on a particle does not depend on the path described by the particle. It depends upon the initial and final position of the particle. Therefore, no work is done by the gravity if a particle moves in a closed path.

(e) Gravitational force is a central force because $F \propto \dfrac{1}{r^2}$.

(f) The gravitational force between two masses is independent of the presence of other objects and medium between the two masses.

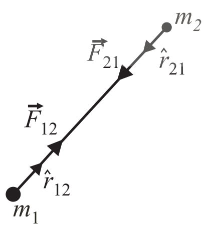
(g) If a particle 1 is acted on by n particles say, the net force $\vec{F}_1$ exerted on it, must be equal to the vector sum of the forces due to surrounding particles

$$\vec{F}_1 = \sum_{i=1}^{i=n} \vec{F}_i \text{, where } \vec{F}_1 = \text{force acted on the particle 1, by the } i\text{th particle.}$$

Hence, gravitational force between any two particles does not depend upon the presence or absence of other particles (bodies).

(h) Vector form of gravitational force is $\vec{F}_{21} = -\dfrac{G m_1 m_2}{r_{12}^2}\hat{r}_{12}$ force on m_2 due to m_1.

$\vec{F}_{12} = -\dfrac{G m_1 m_2}{r_{21}^2}\hat{r}_{21}$ force on m_1 due to m_2.



(i) Range of gravitational force is very large and acts up to interstellar separation.

CHECK Point

- **We can shield a charge from electrical forces by putting it inside a hollow conductor. Can we shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means?**

Solution

No, a body can not be shielded from the gravitational influence of nearby matter. Only the mass of the hollow sphere, surrounding it from all sides may not attract.

Principle of Superposition of Gravitational Force

It states that the resultant gravitational force $\vec{F}$ acting on a particle due to a number of point masses is equal to the vector sum of the forces exerted by the individual masses on the given particle

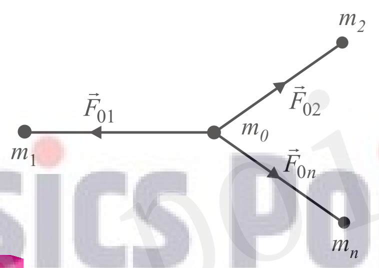
i.e., $\vec{F} = \vec{F}_{01} + \vec{F}_{02} + \cdots + \vec{F}_{0n} = \sum_{i=1}^{n} \vec{F}_{0i}$

where $\vec{F}_{01}, \vec{F}_{02}, ..., \vec{F}_{0n}$ are the gravitational forces on a particle of mass m_0 due to particles of masses m_1, m_2, ... m_n, respectively.

$$\vec{F}_{01} = -\frac{Gm_0M_1}{r^2}\hat{r}_{10} = \frac{Gm_0m_1}{|\vec{r}_{01}|^2}\hat{r}_{01}$$

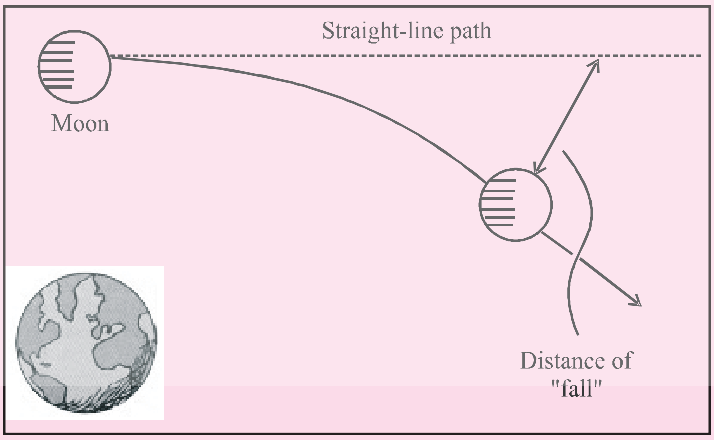
$$\vec{F}_{02} = \frac{Gm_0M_2}{|\vec{r}_{02}|^2}\hat{r}_{02} \text{ and } \vec{F}_{0n} = \frac{Gm_0m_n}{|\vec{r}_{0n}|^2}\hat{r}_{0n}$$

$$\vec{F}_0 = Gm_0\left[\frac{m_1\hat{r}_{01}}{|\hat{r}_{01}|^2} + \frac{m_2\hat{r}_{02}}{|\hat{r}_{02}|^2} + \cdots + \frac{m_n\hat{r}_{0n}}{|\vec{r}_{0n}|^2}\right]$$



idea box

Consider the moon moving in a direction shown by the solid arrow in the figure.



The moon would continue in a straight line if no force were applied to it. In fact, a force of gravity is exerted on it by the earth, so instead of going in a straight line, the moon's path is bent by the force. In 1 second, the moon "falls" from a straight-line path by just the amount predicted by Newton's law of gravity, $1/60^2$ of 4.9 meters. If you do the calculations, you will find that this distance is 0.00136 meters, or 0.136 centimeters. Isaac Newton did this calculation and realized that it confirmed his law of gravitation. It was the first application of a law of nature to both heavenly objects and earthly ones.

Importance of the Universal Law of Gravitation

The universal law of gravitation successfully explained several phenomena which were belived to be unconnected:

(i) the force that binds us to the earth;
(ii) the motion of the moon around the earth;
(iii) the motion of planets around the Sun; and
(iv) the tides due to the moon and the Sun.

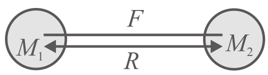
CHECK Point

- **If the attraction force between two bodies of mass M_1 and M_2 and situated at a distance R is F, then find the force F' between them at distance $(R + d)$.**

Solution

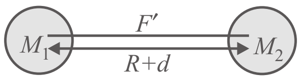
According to Newton's law of gravitation

Case - I :



$$F = \frac{GM_1M_2}{R^2}$$

Case - II :



$$F' = \frac{GM_1M_2}{(R+d)^2}$$

$$\therefore \quad \frac{F'}{F} = \frac{R^2}{(R+d)^2} \quad \text{or} \quad F' = \frac{R^2 F}{(R+d)^2}$$

ILLUSTRATION : 1

Two bodies of different masses are placed at a distance, if the distance is increased by 10%, then find the percentage change in gravitational force between the bodies.

SOLUTION :

Let the masses of the bodies are m_1 and m_2 respectively and the distance between them is r, then

the gravitational force between the bodies $F = \dfrac{Gm_1m_2}{r^2}$ .... (1)

Now distance is decreased by 10%, then new distance will be

$$r' = r - 10\% \text{ of r} \quad \Rightarrow \quad r' = r - \frac{10r}{100} = \frac{9r}{10}$$

New force $F' = \dfrac{Gm_1m_2}{(9r/10)^2} = \dfrac{100\, Gm_1m_2}{81r^2}$ .... (2)

Dividing eq. (2) by (1)

$$\frac{F'}{F} - 1 = \frac{100}{81} - 1 \quad \text{or} \quad \frac{F'-F}{F} = \frac{100-81}{81}$$

$$\% \text{ change } \frac{F'-F}{F} \times 100 = \frac{19 \times 100}{81} = 23.46\%$$

ILLUSTRATION : 2

Two persons having mass 50 kg each, are standing such that the centre of gravity are 1m apart. Calculate the force of gravitation and also calculate the force of gravity on each. (Take G = 6.67×10^{-11} Nm^2 kg^{-2}, mass of earth M = 6×10^{24} kg, radius of earth R = 6.4×10^6 m)

SOLUTION :

Here, $m_1 = m_2 = 50$ kg, $r = 1$m, G = 6.67×10^{-11} Nm^2 kg^{-2}

Force of gravitation $F = \frac{Gm_1m_2}{r^2} = \frac{6.67\times10^{-11}\times50\times50}{(1)^2} = 1.67\times10^{-7}\,N$

Force of gravity $F' = \frac{GMm}{r^2}$, here $r = R$ radius of earth and $m_1 = m_2 = m$

$$F' = \frac{6.67\times10^{-11}\times6\times10^{24}\times50}{(6.4\times10^6)^2} = 0.48\times10^3\,N$$

F' is much greater than F so the persons will not move towards each other but each of them moves towards the earth.

ILLUSTRATION : 3

What is the gravitational attraction between a 70 kg boy and a 60 kg girl who are 3 meters apart ?

SOLUTION :

Apply the Newton's Universal law of gravitation

$$F_{grav} = \frac{Gm_1m_2}{r^2}$$

$$F_{grav} = \frac{(6.67\times10^{-11})(70)(60)}{(3)^2} = 3\times10^{-8}\,\text{N}$$

MASS AND WEIGHT

The quantity of matter in a body is known as the mass of the body. Mass is quantitative measure of inertia. Mass is an intrinsic property of matter and does not change as an object is moved from one location to another. Weight, in contrast, is the gravitational force that the earth exerts on the object and can vary, depending on how far the object is above the earth's surface or whether it is located near another body such as the moon.

The relation between weight W and mass m can be written in one of two ways :

$W = \frac{GM_Em}{r^2}$; $W = mg$

As acceleration due to gravity on the surface of the moon is $\frac{1}{6}$ of acceleration due to gravity on the surface of the earth.

i.e., $g_{moon} = \frac{1}{6}\,g_{earth}$ therefore, weight of a body on the surface of the moon is $\frac{1}{6}$ the weight on the surface of the earth.

i.e., $w_{moon} = \frac{1}{6}\,w_{earth}$

According to the Newton's second law

Force = mass × acceleration or mass = $\frac{\text{force}}{\text{acceleration}}$

Inertial and Gravitaional Mass

The mass of a body is the quantity of matter possessed by a body. There are two different concepts about the mass of body as discussed below.

Inertial Mass

Inertial mass of a body is related to its inertia of linear motion, and is defined by Newton's second law of motion.

Let a body of mass m_i moves with acceleration a under the action of an external force F. According to Newton's second law of motion

$$F = m_ia \text{ or } m_i = \frac{F}{a}$$

The mass m_i of the body in this sense is the inertial mass of the body. If $a = 1$, then from above eqn. $m_i = F$

Thus *inertial mass of a body is equal to the magnitude of external force required to produce unit acceleration in the body.*

Infact inertial mass of a body is the measure of the ability of the body to oppose the production of acceleration in its motion by an external force.

Properties of inertial mass

1. It is proportional to the quantity of matter contained in the body.
2. It is independent of size, shape and state of the body.
3. It does not depend upon the temperature of the body.
4. It is not affected by the presence or absence of other bodies near it.
5. Inertial mass is conserved when the two bodies combine physically or chemically.
6. It can be added by simple laws of algebra, irrespective of the materials of the bodies.
7. Inertial mass of a body increases with the speed of the body. When a body moves with a velocity v, its inertial mass m is given by

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

where m_0 is the rest mass of the body, c is the velocity of light in vacuum. The mass m is affected only when the velocity of the body is comparable with the velocity of light.

Gravitational Mass

Gravitational mass of a body is related to gravtitational pull on the body and is defined by Newton's law of gravitation.

If a body of mass m_G is placed on the surface of earth of radius R and mass M, then gravitational pull on the body is given by

$$F = \frac{GMm_G}{R^2} \quad \Rightarrow \quad m_G = \frac{F}{\left(GM/R^2\right)} = \frac{F}{J}$$

The mass m_G of the body in this sense is the gravitational mass of the body. The inertia of the body has no effect on the gravitational mass of the body. If $J = 1$ then from eqn. $m_G = F$.

Thus gravitational mass of a body is defined as the magnitude of gravitational pull experienced by the body in a gravitational field of unit intensity.

The **properties of gravitational mass** are the same as those of inertial mass except those mentioned below in the comparison of inertial mass and gravtitational mass.

CONNECTING TOPIC

ACCELERATION DUE TO GRAVITY OF THE EARTH

When a body is dropped from a certain height above the ground, it begins to fall towards the earth under gravity. The acceleration produced in the body due to gravity is called the acceleration due to gravity. It is denoted by g. Its value close to the Earth's surface is 9.8 m/s^2.

Suppose that the mass of the Earth is M, its radius R, then the force of attraction acting on a body of mass m close to the surface of Earth is $F = \frac{GMm}{R^2}$

According to Newton's second law, the acceleration due to gravity

$$g = \frac{F}{m} \quad \text{or} \quad g = \frac{GM}{R^2} \text{ at the surface of the Earth}$$

This is the relation between acceleration due to gravity (g) and universal gravitational constant (G).

This expression is free from mass of the body, m. If two bodies of different masses are allowed to fall freely they will have the same acceleration, i.e., if they are allowed to fall from the same height, they will reach the earth simultaneously.

Acceleration due to gravity, g in terms of density of planet

Let ρ = density of the planet

Mass of the planet, $M = \frac{4}{3}\pi R^3 \rho$

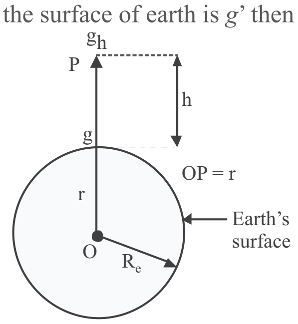
Aceeleration due to gravity, $g = \frac{GM}{R^2} = \frac{G \times \frac{4}{3}\pi R^3 \rho}{R^2}$ or $g = \frac{4}{3}\pi G R \rho$

VARIATION IN ACCELERATION DUE TO GRAVITY

The value of 'g' acceleration due to gravity, varies from place to place on the surface of earth. It also varies as we go above or below the surface of the Earth.

Variation in acceleration due to gravity with the height (altitude) above the earth's surface

If the value of acceleration due to gravity at the surface of earth is g and at a height 'h' above the surface of earth is g' then



$$g = \frac{GM_e}{R_e^2} \quad \ldots (i)$$

$$g' = \frac{GM_e}{(R_e + h)^2} \quad \ldots (ii)$$

From eq^{n}s (i) and (ii)

$$g' = \frac{g}{\left(1 + \frac{h}{R_e}\right)^2} \quad \text{or,} \quad g' = g\left(1 - \frac{2h}{R_e}\right) \quad (\text{if } h << R_e)$$

i.e., The decrease in the value of g on going up a height 'h' above the surface of earth by a factor $\left(1 - \frac{2h}{R_e}\right)$

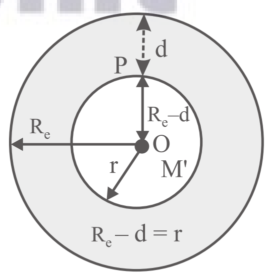
Change in aceeleration due to gravity, $\Delta g = \frac{2gh}{R_e}$

Variation in acceleration due to gravity with the depth below the earth's surface

If the value of acceleration due to gravity at the surface of earth is g and at a depth h from the earth's surface is g' then,

$$g = \frac{GM_e}{R_e^2} \quad \ldots (i)$$

$$g' = \frac{GM_e}{R_e^3}(R_e - d) \quad \ldots (ii)$$

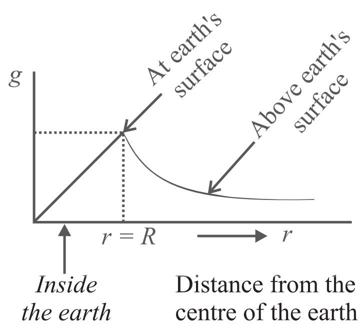


From eq^n s (i) and (ii)

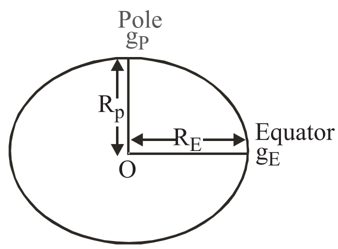
$$g' = g\frac{R_e - d}{R_e} \quad \text{or,} \quad g' = g\left(1 - \frac{d}{R_e}\right)$$

Thus the value of g decreases by a factor $\left(1 - \frac{d}{R_e}\right)$ as we go down below the surface of the earth.

The graph showing variation of acceleration due to gravity (g) with (i) height above the earth's surface and (ii) depth below the earth's surface.



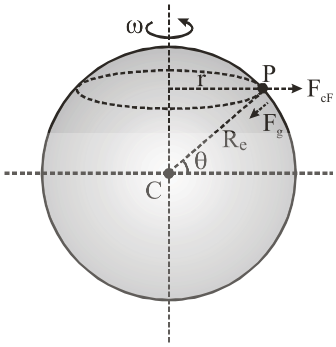
Variation in acceleration due to gravity due to shape of the earth



The earth is elliptical in shape. It is flatter at the poles and bulged out at the equator. Now, we know that

$g \propto 1/R^2$, $R_P < R_e$ $\quad \therefore \quad$ $g_E < g_P$

Therefore the value of g at the equator is minimum and the value of g at the poles is maximum.



Variation in acceleration due to gravity due to axial rotation (latitude) of the earth

If the observed value of g at the latitude λ is represented by g_λ ,then

$$g_\lambda = g_e - R_e\omega^2 \cos^2 \lambda$$

where ω is the angular velocity of the earth.

At equator, $\lambda = 0^o$; $\cos\lambda = 1$

$\therefore \quad g_\lambda = g_e - R_e\,\omega^2$

At poles, $\lambda = 90^o$; $\cos\lambda = 0$

$\therefore \quad g_\lambda = g_e$ i.e., $g_p > g_e$

ILLUSTRATION : 4

If the radius of the earth were to shrink by one per cent, its mass remaining the same, the value of *g* on the earth's surface would

(a) increase by 0.5% **(b) increase by 2%** **(c) decrease by 0.5%** **(d) decrease by 2%.**

SOLUTION :

(b) As we know, $g = \dfrac{GM}{R^2} \Rightarrow \dfrac{dg}{g} = -2\dfrac{dR}{R}$

$\because \quad \dfrac{dR}{R} = -1\% \quad \therefore \quad \dfrac{dg}{g} = 2\%$ i.e., the value of g increases by 2%

ILLUSTRATION : 5

What will be the value of *g* at the bottom of sea 7 km deep? Diameter of Earth is 12800 km and *g* on the surface of Earth is $9.8 ms^{-2}$.

SOLUTION :

Depth of sea, $d = 7$ km, $g = 9.8\ ms^{-2}$

Diameter of Earth, $D = 12800$ km

Let g_d = value of g at the bottom of sea

Then , $g_d = g\left(1-\dfrac{d}{R}\right) = 9.8\left(1-\dfrac{7}{6400}\right)ms^{-2} = \dfrac{9.8\times 6393}{6400}ms^{-2} = 9.789ms^{-2}$

GRAVITATIONAL FIELD

The gravitational field is the space around a mass or an assembly of masses over which it can exert gravitational forces on other masses. It is characterised by (a) gravitational field intensity and (b) gravitational potential.

Gravitational Field Intensity or Gravitational Field Strength

Gravitational field intensity at a point in the gravitational field is the force experienced by a unit mass placed at that point. It is directed towards the particle producing the field.

The gravitational field intensity is given by $\vec{E} = \dfrac{\vec{F}}{m}$ $\vec{E}$ has the same direction as that of $\vec{F}$.

Acceleration due to gravity $\vec{g}$ is also $\dfrac{\vec{F}}{m}$. Hence, for the Earth's gravitational field, $\vec{g}$ and $\vec{E}$ are same.

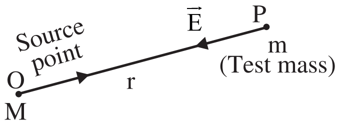
The E versus r (the distance from the centre of Earth) graph are same as that of g' versus r graph.

The **SI unit** of gravitational field intensity is N kg^{-1}

Gravitational Field Intensity due to a Point Mass

Suppose, a particle of mass M is placed at point O. We want to find the intensity of gravitational field $\vec{E}$ at a point P, at a distance r from O. Magnitude of force F acting on a particle of mass m placed at P is,

$$\vec{F} = \frac{GMm}{r^2}\hat{r}$$



$$\therefore \quad \vec{E} = \frac{\vec{F}}{m} = -\frac{GM}{r^2}\hat{r}$$

or $$\vec{E} = -\frac{GM}{r^2}\hat{r}$$

The direction of the force F and hence of E is from P to O as shown in fig.

Gravitational Field Due to a Uniform Solid Sphere

Field at an external point :

For $r \geq R$ $\quad \vec{E}(r) = -\frac{GM}{r^2}\hat{r}$ or $E(r) \propto \frac{1}{r^2}$

For $r = R$ at surface $\quad \vec{\ } = -\frac{GM}{R^2}\hat{r}$

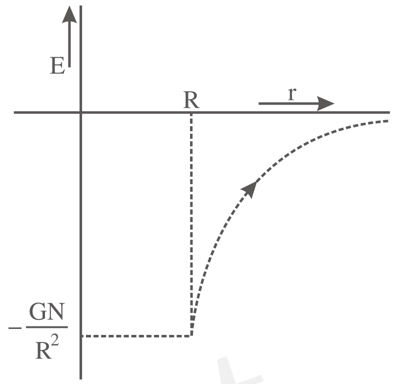
Field at an internal point :

At the centre $E = 0$

For $r \leq R$ $\quad \vec{E}(r) = -\frac{GM}{R^3}\cdot\vec{r}$

or $\quad E(r) \propto r$

Hence, E versus r graph is as shown in fig.



CHECK Point

- **The earth's gravitational field at a certain point out in space accelerates a 1 kg mass at 5 ms^{-2}. How much will it accelerate a 3 kg mass?**

Solution

The gravitational force on a body is proportional to its mass. Hence, if the mass increases by a factor of 3 (from 1 kg to 3 kg), the gravitational force in the second case will also become three times. However, the acceleration due to gravity will remain the same. Thus, at a point in space, where the earth's gravitational field accelerates a 1 kg mass at 5 ms^{-2}, a 3 kg mass will also be accelerated 5 ms^{-2}.

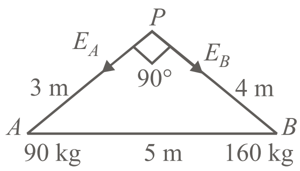
ILLUSTRATION : 6

Two masses 90 kg and 160 kg are at a distance 5 m apart. Compute the magnitude of intensity of the gravitational field at a point distance 3 m from the 90 kg and 4 m from the 160 kg mass. $G = 6.67 \times 10^{-11}$ Nm^{2}kg^{-2}.

SOLUTION :

Let A and B be the positions of the masses and P be the point at which the gravitational intensity is to be computed.

Gravitational intensity at P due to mass at A will be



$$E_A = G\frac{90}{3^2} = 10G, \text{ along } PA.$$

Gravitational intensity at P due to mass at B will be

$$E_B = G\frac{160}{4^2} = 10G, \text{ along } PB.$$

In ΔAPB, $AB^2 = AP^2 + PB^2$ $\quad (\therefore \angle APB = 90°)$

Therefore, the magnitude of resultant gravitational intensity at P will be

$$E = \sqrt{E_A^2 + E_B^2} = \sqrt{(10)^2 + (10)^2} = 10\sqrt{2}\,G.$$

$$= 10\sqrt{2} \times 6.67 \times 10^{-11} = 9.43 \times 10^{-10} \text{ N kg}^{-1}$$

GRAVITATIONAL POTENTIAL

At a point in a gravitational field, gravitational potential V is defined as negative of the work done per unit mass in shifting a rest mass from some reference point (usually at infinity) to the given point.

i.e., Gravitational potential, $V = -\frac{W}{m}$

As by definition, potential energy $U = -W$

So, $V = \frac{U}{m}$, i.e., $U = mV$

i.e., Physically potential at a point represents potential energy of a unit point mass at that point.

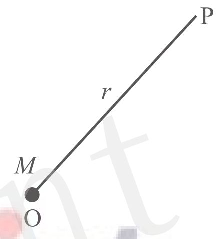
As by definition of work $W = \int \vec{F} \cdot \vec{dr}$

So, $V = -\frac{1}{m}\int \vec{F} \cdot \vec{dr} = -\int \vec{E} \cdot \vec{dr}$ $\left[\text{as } \frac{\vec{F}}{m} = \vec{E}\right]$

i.e., $dV = -E\,dr$ or $E = -\frac{dV}{dr}$

The **SI unit** of gravitational potential is joule/kg.

Gravitational Potential Due to a Point Mass



Suppose a point mass M is situated at a point O

The gravitational potential due to this mass at point P at a distance r from O

$$V = -\int_{\infty}^{r} \vec{F} \cdot \vec{dr} = -\int_{\infty}^{r} -\frac{GM}{r^2}\hat{r} \cdot \vec{dr} = \int_{\infty}^{r} \frac{GM}{r^2}\, dr = -\frac{GM}{r}$$

Gravitational Potential due to a Uniform Solid Sphere

Potential at an external point :

The gravitational potential due to a uniform sphere at an external point is same as that due to a single particle of same mass placed at its centre. Thus,

For $r \geq R$ $\quad V(r) = -\frac{GM}{r}$

For $r = R$ at the surface, $V = -\frac{GM}{R}$

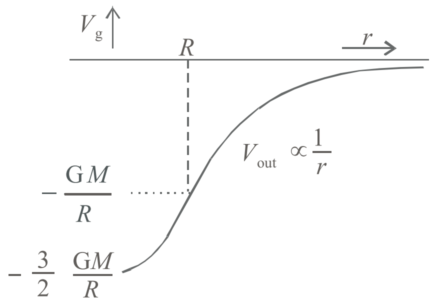
Potential at an internal point:

Potential at a point distant r from the centre of the sphere ($r < R$) inside,

$$V = \frac{-GM(3R^2 - r^2)}{2R^3}$$

At $r = R$, $V = -\frac{GM}{R}$

while at $r = 0$, $V = -\frac{3GM}{2R}$



Relation Between Gravitational Field and Potential

The work done by an external agent to move unit mass from a point to another point in the direction of the field E, slowly through an infinitesimal distance dr = force by external agent × distance moved = $-Edr$.

Thus, $dV = -Edr \Rightarrow E = \frac{-dV}{dr}$

Therefore, gravitational field at any point is equal to the negative gradient of potential at that point.

CHECK Point

- **Why is gravitational potential at a point negative?**

Solution

In gravitation, bodies attract each other. Gravitational potential increases as test mass is taken away from field mass and decreases as test mass is brought nearer to field mass. Since potential is zero at infinity and decreases as test mass is brought in the field nearer to the field mass, potential becomes negative.

GRAVITATIONAL POTENTIAL ENERGY

Change in potential energy (dU) of a system corresponding to a conservative internal force is given by

$$dU = -\vec{F}\cdot d\vec{r}$$

or $\int_i^f dU = -\int_i^f \vec{F}\cdot d\vec{r}$ or $U_f - U_i = -\int_i^f \vec{F}\cdot d\vec{r}$

We generally choose the reference point at infinity and assume potential energy to be zero there,
i.e., if we take $r_i = \infty$ (infinite) and $U_i = 0$ then we can write

Potential energy at point P $U = -\int_\infty^P \vec{F}\cdot d\vec{r} = -W$

i.e., *Gravitational potential energy of a body or system is negative of work done by the conservative forces in bringing it from infinity to the present position.*

Gravitational Potential Energy of a Two Particle System

The gravitational potential energy of two particles of masses m_1 and m_2 separated by a distance r is given by,

$$U = -\frac{Gm_1m_2}{r}$$

This is actually the negative of work done in bringing those masses from infinity to a distance r by the gravitational forces between them.

Gravitational potential energy of a body on earth's surface : The gravitational potential energy of a body of mass m in the gravitational field of mass M at a distance r from it is, (r is greater than radius of the Earth i.e. $r > R$)

$$U_p = -\int_\infty^r \left(-\frac{GMm}{r^2}\hat{r}\right)\cdot \vec{dr} = GMm\int_\infty^r r^{-2}dr = GMm\left(\frac{r^{-2+1}}{-2+1}\right)_\infty^r \quad \text{or} \quad U_P = -\frac{GMm}{r}$$

The Earth behaves for all external points as if its mass M were concentrated at its centre. Therefore, a mass m near Earth's surface may be considered at a distance R (the radius of earth) from M.

The potential energy of m at the surface of the Earth is $U = -\frac{GMm}{R}$

The gravitational potential energy of mass m at a height h above the surface of Earth is given by

$$U = -\frac{GMm}{R+h} \quad [\because \text{ The distance between the mass m and the centre of Earth is } (R+h)]$$

$$\therefore \quad U = -\frac{GMm}{R\left(1+\frac{h}{R}\right)} \text{ (for any height } h) \quad \text{or} \quad U = -\frac{GMm}{R}\left(1+\frac{h}{R}\right)^{-1}$$

So, expanding the right hand side of the above equation by Binomial theorem and neglecting squares and higher powers of $\frac{h}{R}$, we get

$$U = -\frac{GMm}{R}\left(1-\frac{h}{R}\right) \quad \text{for } h << R$$

or $$U = -\frac{GMm}{R} + \frac{GMmh}{R^2}$$

But $\frac{GM}{R^2}$ = g (acceleration due to gravity)

$$\therefore \quad U = -\frac{GMm}{R} + mgh$$

But $-\frac{GMm}{R}$ = gravitational potential energy of mass m at the surface of Earth.

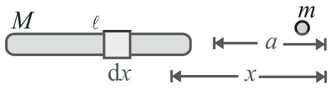
According to convention, the gravitational potential energy at the surface of Earth is taken to be zero.
In that case $U = mgh$ (If reference point is at the Earth's surface where $U = 0$)

ILLUSTRATION : 7

Find the potential energy of gravitational interaction of a point mass *m* and a thin uniform rod of mass *M* and length *l*, if they are located along a straight line at a distance *a* from each other.

SOLUTION :

Consider small element dx of the rod whose mass $dm = \frac{M}{\ell}dx$



Gravitational potential energy, $U = \int dU = -\frac{GmM}{\ell}\int_a^{a+\ell}\frac{dx}{x} = -\frac{GmM}{\ell}[\ln x]_a^{a+\ell}$

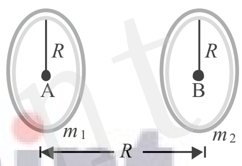
or, $U = -\frac{GmM}{\ell}\log_e\left(\frac{a+\ell}{a}\right)$.

ILLUSTRATION : 8

Two thin rings each of radius R are coaxially placed at a distance R. The rings have a uniform mass distribution and have mass m_1 and m_2 respectively. Then the work done in moving a mass m from centre of one ring to that of the other is

(a) zero **(b)** $\frac{Gm(m_1-m_2)(\sqrt{2}-1)}{\sqrt{2}R}$ **(c)** $\frac{Gm(\sqrt{2})(m_1-m_2)}{R}$ **(d)** $\frac{Gmm_1(\sqrt{2}+1)}{m_2R}$

SOLUTION :



(b) $V_A = \begin{pmatrix}\text{Potential at}\\ \text{A due to A}\end{pmatrix} + \begin{pmatrix}\text{Potential at}\\ \text{A due to B}\end{pmatrix} \Rightarrow V_A = -\frac{Gm_1}{R} - \frac{Gm_2}{\sqrt{2}R}$ and

$V_B = \begin{pmatrix}\text{Potential at}\\ \text{B due to A}\end{pmatrix} + \begin{pmatrix}\text{Potential at}\\ \text{B due to B}\end{pmatrix} \Rightarrow V_B = -\frac{Gm_2}{R} - \frac{Gm_1}{\sqrt{2}R}$

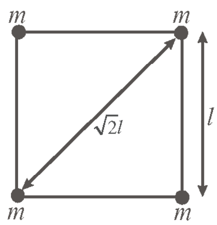
Since $W_{A\to B} = m(V_B - V_A) \Rightarrow W_{A\to B} = \frac{Gm(m_1-m_2)(\sqrt{2}-1)}{\sqrt{2}R}$

ILLUSTRATION : 9

Find the gravitational potential energy of a system of four particles, each having mass m, placed at the vertices of a square of side l. Also obtain the gravitational potential at the centre of the square.

SOLUTION :

The system has four pairs with distance l and two diagonal pairs with distance $\sqrt{2}\,\ell$. Therefore, gravitational potential energy of the system



$$U = -4\frac{Gm^2}{\ell} - 2\frac{Gm^2}{\sqrt{2}\ell} = -\frac{2Gm^2}{\ell}\left(2+\frac{1}{\sqrt{2}}\right) = -5.41\frac{Gm^2}{\ell}$$

Gravitational potential at the centre of the square is $(r = \sqrt{2}l/2)$

V = Algebraic sum of potential due to each particle

$$\Rightarrow V = -\frac{4\sqrt{2}Gm}{l}$$

ESCAPE SPEED

Escape speed is the minimum speed that should be given to the body to enable it to escape away from the gravitational field of earth.

If the mass of the planet is M and its radius is R, then the escape speed from its surface will be

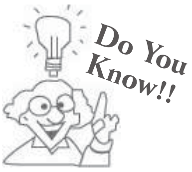
$$V_e = \sqrt{(2GM/R)}$$

or $V_e = \sqrt{(2gR)}$ [since $GM = gR^2$]

Escape speed from the surface of earth is **11.2 km/sec**.

- The value of escape velocity does not depend upon the mass of the projected body, instead it depends on the mass and radius of the planet from which it is being projected.
- The value of escape velocity does not depend on the angle and direction of projection.

- The minimum energy needed to escape a body from the surface of the planet = GMm/R.
- If the velocity of a satellite orbiting the earth is increased by 41.4% , then it will escape away from the gravitational field of the earth.



The escape velocity of a body from a planet depends upon the size (mass and radius) of the planet and hence the value of acceleration due to gravity on its surface. It does not depend upon mass of the body. To throw an ant or an elephant out of the gravitational field, the required velocity of projection is same !

CHECK Point

- **Does the escape velocity of a body from the earth depend upon**
  (a) the mass of the body?
  (b) the location from where it is projected?
  (c) the direction of projection?
  (d) the height of the location from where the body is launched?
  Explain your answer.

Solution

(a) The escape velocity of a body does not depend on its mass.

(b) The escape velocity depends upon the location from where the body is projected.
A body of mass m on the surface of the earth has a gravitational potential energy –GMm/R.
The escape velocity v_0 gives the body so much KE so that the total energy of the body
i.e. $\left[-\dfrac{GMm}{R}+\dfrac{1}{2}mv_0^2\right]$ becomes zero, which is the total energy of any body at infinity. Since earth in not a perfect sphere, value of R will be different at different locations. It makes escape velocity dependent of location.

(c) The escape velocity of a body does not depend upon the direction of its projection.

(d) The escape velocity depends upon the height (altitude) from where the body is projected.

As shown in part (b), in this case, we have $-GMm/(R+h)+\dfrac{1}{2}mv_0^2=0$ for a place at height h. It makes escape velocity dependent of altitude.

ILLUSTRATION : 10

Determine the escape velocity of a body from the Moon. Take the Moon to be a uniform sphere of radius 1.74×10^6 m and mass 7.39×10^{22} kg. Does your answer throw light on why the Moon has no atmosphere?

SOLUTION :

Escape velocity on the surface of Moon is given by

$$v_e=\sqrt{\frac{2GM}{R}}=\sqrt{\frac{2\times6.67\times10^{-11}\times7.36\times10^{22}}{1.74\times10^6}}=2.38\times10^3\ \text{ms}^{-1}$$

All constituents of the atmosphere like oxygen, nitrogen, carbon dioxide and water vapours have root mean square velocities at (0º C) of their molecules slightly greater than escape velocity on the surface of Moon (2.38 kms^{-1}). Hence, there is practically no atmosphere on Moon.

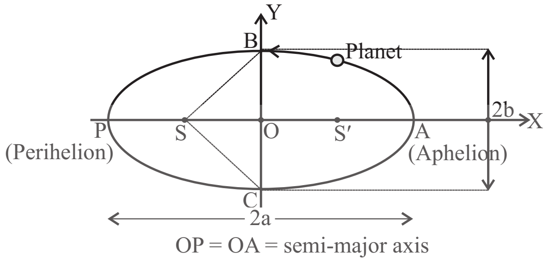
ILLUSTRATION : 11

Mass of moon is 1/81 times that of earth and its radius is 1/4 the earth's radius. If escape velocity at surface of earth is 11.2 km/sec, then what is its value at the surface of the moon ?

SOLUTION :

Escape velocity at the surface of the earth $v_e=11.2=\sqrt{\dfrac{2GM}{R}}$, $v_m=\sqrt{\dfrac{2GM_m}{R_m}}$, $v_m=\sqrt{\dfrac{G\times M\times4}{81\times R}}=\dfrac{11.2\times2}{9}=2.5\,km/\text{sec}$

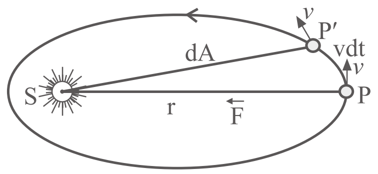
KEPLER'S LAWS OF PLANETARY MOTION



Kepler worked out three laws, which govern the motion of planets and are known as kepler's laws of planetary motion.

1. **Law of orbits (first law) :** *All planets revolve in elliptical orbits around the sun and the sun is situated at one of the two foci of the elliptical path.*

2. **Law of areas (second law) :** *The areal velocity* $\left(\frac{dA}{dt}\right)$ *of a planet remains constant, i.e. the line joining the sun to planet covers equal areas in same intervals of time.*

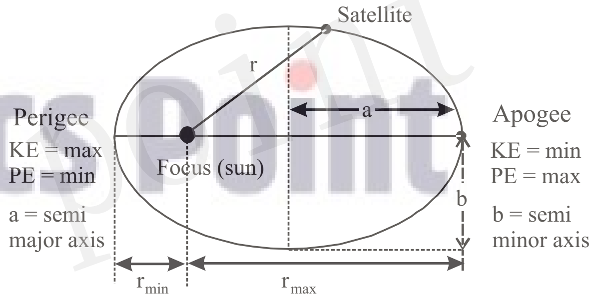


i.e., $\frac{dA}{dt}$ = constant

$$= \frac{1}{2}\frac{r(vdt)}{dt} = \frac{1}{2}rv = \frac{L}{2m} \quad \text{[since L = mvr]}$$

3. **Law of periods (third law) :** *The square of the period of revolution of the planet is directly proportional to the cube of semi major axis of its orbit.*

i.e., $T^2 \propto a^3$ or, $\frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3}$



The position of a planet nearest to the sun is known as **perigee**. In this position the speed of planet is maximum.

The position of a planet at the maximum distance from sun is known as **apogee.** In this position the speed of the planet is minimum.

ILLUSTRATION : 12

The planet Neptune travels around the Sun with a period of 165 years. Show that the radius of its orbit is approximately thirty times that of Earth's orbit, both being considered as circular.

SOLUTION :

$T_1 = T_{Earth} = 1$ year, $T_2 = T_{Neptune} = 165$ year

Let R_1 and R_2 be the radii of the circular orbits of Earth and Neptune respectively

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} \qquad \therefore \quad R_2^3 = \frac{R_1^3 T_2^2}{T_1^2} \quad \text{or} \quad R_2^3 = \frac{R_1^3 \times 165^2}{1^2}$$

$$\Rightarrow \quad R_2^3 = 165^2 R_1^3 \quad \text{or} \quad R_2 \approx 30\, R_1$$

ILLUSTRATION : 13

Suppose Earth's orbital motion around the Sun is suddenly stopped. What time will the Earth take to fall into the Sun?

SOLUTION :

When the Earth's motion is suddenly stopped, it would fall into the Sun and (suppose) it comes back. If the effect of temperature of Sun is ignored, we can say that the Earth would continue to move along a strongly extended flat ellipse whose extreme points are located at the Earth's orbit and at the centre of the Sun.

The semi major axis of such ellipse is $R/2$.

Now $\frac{T'^2}{T^2} = \left[\frac{R}{2}\right]^3 \left[\frac{1}{R^3}\right]$

Where T is the time period of normal orbit of earth.

or $T'^2 = \frac{T^2}{8}$ or $T' = \frac{T}{2\sqrt{2}}$

Now, time required to fall into the Sun, $t = \frac{T'}{2} = \frac{T}{4\sqrt{2}} = \frac{365}{4\sqrt{2}} \approx 65$ day

So, the Earth would take slightly more than 2 months to fall into the Sun.

SATELLITES

Just as the planets revolve around the sun, in the same way few celestial bodies revolve around these planets. These bodies are called 'Satellites'. For example moon is the natural satellite of Earth. Artificial satellites are launched from the Earth. Such satellites are used for telecommunication, weather forecast etc. The path of these satellites are elliptical with the centre of Earth at a focus. However, the difference in major and minor axes is so small that they can be treated as nearly circular for not too sophisticated calculations. Let us derive certain characteristics of the motion of satellites by assuming the orbit to be perfectly circular, and mass of the satellite is much smaller than the Earth's mass.

(a) Orbital velocity (v_0) : Let a satellite of mass m revolves around the Earth in circular orbit of radius r with speed v_0. The gravitational pull between satellite and earth provides the necessary centripetal force.

Centripetal force required for the motion $= \frac{mv_0^2}{r}$

Gravitational force $= \frac{GMm}{r^2}$

$\frac{mv_0^2}{r} = \frac{GMm}{r^2}$ or $v_0^2 = \frac{GM}{r}$

or $v_0 = \sqrt{\frac{GM}{r}} = \sqrt{gR}$ ... (1)

or $v_0 = R\sqrt{\frac{g}{R+h}}$ $\because$ $g = \frac{GM}{R^2}$ and $r = (R+h)$

Relation between escape speed (v_e) and orbital speed (v_0) : $v_e = \sqrt{2}v_0$

(i) Value of orbital velocity does not depend on the mass of satellite but it depends on the mass and radius of the planet around which the rotation is taking place.
(ii) The orbital velocity for a satellite near the surface of earth is 7.92 km/sec.
(iii) Greater is the height of the satellite, smaller is the orbital velocity.
(iv) The direction of orbital velocity is along the tangent to the path.
(v) The work done by the satellite in a complete orbit is zero.

(b) Angular momentum (L) : For satellite motion angular momentum will be given by

$$L = mvr = mr\sqrt{\frac{GM}{r}} \quad \text{i.e.,} \quad \boldsymbol{L} = [m^2GMr]^{1/2}$$

Angular momentum of a satellite depends on both, the mass of orbiting and central body. It also depends on the radius of the orbit.

(c) Period of revolution (T) :

Period of revolution $T = \frac{\text{circumference of the orbit}}{\text{orbital velocity}}$ or $T = \frac{2\pi r}{v_0} = \frac{2\pi}{\text{ù}}$ [as $v_0 = r\omega$]

$\Rightarrow T = \frac{2\pi r}{v_0} = \frac{2\pi(R+h)}{v_0}$ or $T = \frac{2\pi r}{\sqrt{GM/r}}$

$$T = 2\pi\sqrt{\frac{r^3}{GM}} \quad ...(2)$$

On squaring eq^n. (2), $T^2 = \frac{4\pi^2}{GM}r^3$ $T^2 \propto r^3$ or $r^3 = \frac{GMT^2}{4\pi^2}$

or $r = \left(\frac{GM}{4\pi^2}T^2\right)^{\frac{1}{3}}$ ...(3)

or $r = \left(\frac{GM}{4\pi^2 R^2} \times R^2 T^2\right)^{\frac{1}{3}}$

or $r = \left(\frac{gR^2T^2}{4\pi^2}\right)^{\frac{1}{3}}$ ...(4)

and $R + h = \left(\frac{gR^2T^2}{4\pi^2}\right)^{\frac{1}{3}}$

$\therefore$ Height of satellite $h = \left(\frac{gR^2T^2}{4\pi^2}\right)^{\frac{1}{3}} - R$

(i) Time period is independent of the mass of orbiting body and depends on the mass of central body and radius of the orbit.

As $T^2 = \frac{4\pi^2}{GM} r^3$ i.e. $M = \frac{4\pi^2 r^3}{GT^2}$

If the radius of the orbit and time period, are known then we can calculate the mass of central body M. This is how we compute the mass of sun.

(ii) If the satellite is revolving close to the surface of the Earth $h \to 0$, i.e., $r = R$.

$T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{6.4\times10^6}{10}} = 2\pi \times 800\text{s} = 84.6 \text{ minute} \simeq 1.4 \text{ hours.}$

(d) Energy of satellite : A satellite revolving around a planet has both kinetic and potential energy.

(i) Kinetic energy : The kinetic energy of the satellite is due to motion of the satellite.

$\frac{mv_0^2}{r} = \frac{GMm}{r^2}$ or $mv_0^2 = \frac{GMm}{r}$ or $\frac{1}{2}mv_0^2 = \frac{GMm}{2r}$

Kinetic energy of orbiting satellite, $K = \frac{GMm}{2r}$

(ii) Potential energy : As for external point a spherical mass behaves as whole of its mass is concentrated at its centre; potential energy of the satellite. $U = -\frac{GMm}{r}$

The negative sign is because of zero potential energy at infinity.

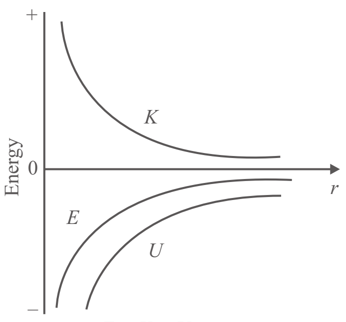
(iii) Total energy : Total energy of orbiting satellite,

$E = K + U = \frac{GMm}{2r} + \left[-\frac{GMm}{r}\right] = \frac{GMm}{2r} - \frac{GMm}{r}$

or $E = -\frac{GMm}{2r}$

The total energy of a satellite is negative.

$\frac{K}{E} = -1$ i.e., $K = -E$ and $\frac{U}{E} = 2$, i.e., $U = 2E$



$E = K + U$

The variation of kinetic energy K, potential energy U, and total energy E with radius r for a satellite in a circular orbit.

For any value of r, the values of U and E are negative, the value of K is positive, and $E = -K$.

As $r \to \infty$, all three energy curves approach a value of zero.

Kinetic, potential or total energy of a satellite depends on the mass of the orbiting satellite and the central body and also on the radius of the orbit.

(e) Binding energy : Total energy of satellite in its orbit is negative. Negative energy means that the satellite is bound to the central body by an attractive force and energy must be supplied to remove it from the orbit to infinity.

The energy required to remove the satellite from its orbit to infinity is called binding energy of the system.

Binding energy of satellite $= -E = \frac{GMm}{2r}$

(i) When the satellite is orbiting in its orbit, then no energy is required to keep it in its orbit.

(ii) When the energy of the satellite is negative then it moves in either a circular or an elliptical orbit.

(iii) When the energy of the satellite is zero then it escapes away from its orbit and its path becomes parabolic.
(iv) When the energy of the satellite is positive then it escapes from the orbit following a hyperbolic path.
(v) When the velocity of the satellite is increased then its energy increases and it starts moving in a circular path of smaller radius.
(vi) When the height of the satellite is increased then its potential energy increases and kinetic energy decreases.
(vii) The potential energy of a satellite orbiting in circular orbit is always more than its kinetic energy.

Geo-stationary Satellite

A satelite which appears to be stationary for a person on the surface of the Earth is called geostationary satellite.
It is also known as Communication Satellite or Synchronous Satellite.

(i) The orbit of the satellite must be circular and in the equatorial plane of the Earth.
(ii) The angular velocity of the satellite must be in the same direction as the angular velocity of rotation of the earth i.e., from west to east.
(iii) The period of revolution of the satellite must be equal to the period of rotation of earth about its axis. i.e. 24 hours = 24 × 60 × 60 = 86400 sec.
(iv) Height from the surface of the earth is nearly 35600 km.

$$T^2 = \frac{4\pi^2}{GM} r^3 \quad \text{or} \quad r = \left(\frac{GMT^2}{4\pi^2}\right)^{\frac{1}{3}} = \left(\frac{GM}{R^2} \times \frac{R^2T^2}{4\pi^2}\right)^{\frac{1}{3}} = \left[9.8 \times (6.38 \times 10^6)^2 \times \frac{(86400)^2}{4\pi^2}\right]^{\frac{1}{3}}$$

$= 42237 \times 10^3$ m = 42,237 km. ≈ 42000 km. approximately.

$h = r - R = 42000 - 6400 = 35600$ km.

(v) The orbital velocity of this satellite is nearly 3.08 km/sec.
(vi) The relative velocity of geostationary satellite with respect to earth is zero.
This type of satellite is used for communication purposes.
The orbit of a geostationary satellite is called '***Parking Orbit***'.

Polar Satellite

Polar Satellites go around the poles of the earth in north-south direction and the earth rotates around its axis in east-west direction. The altitude of polar satellite is around 500 to 800 km and its time period is around 100 minutes.

Different orbital shapes corresponding to different velocities of a satellite

[1] When $v < v_0$
(i) The path is spiral. The satellite finally falls on the Earth.
(ii) Kinetic energy is less then potential energy.
(iii) Total energy is negative.

[2] When $v = v_0$
(i) The path is circular.
(ii) Eccentricity is zero.
(iii) Kinetic energy is less than potential energy.
(iv) Total energy is negative.

[3] When $v_0 < v < v_e$
(i) The path is elliptical.
(ii) $e < 1$.
(iii) Kinetic energy is less than potential energy.
(iv) Total energy is negative.

[4] When $v = v_e$
(i) The path is a parabola.
(ii) $e = 1$.
(iii) Kinetic energy is equal to potential energy.
(iv) Total energy is zero.

[5] When $v > v_e$
(i) The path is a hyperbola.
(ii) $e > 1$.
(iii) Kinetic energy is greater than potential energy.
(iv) Total energy is positive.

If the orbit of a satellite is elliptical

(i) The energy $E = -\frac{GMm}{2a}$ = const. with a as semi-major axis;

(ii) KE will be maximum when the satellite is closest to the central body (at perigee) and minimum when it is farthest from the central body (at apogee) [as for a given orbit L = const., i.e., mvr = const., i.e., $v \propto 1/r$]

(iii) PE = (E – K) will be minimum when KE = max, i.e., the satellite is closest to the central body (at perigee) and maximum when KE = min, i.e., the satellite is farthest from the central body (at apogee).

FREE FALL

The motion of a body under the influence of gravity alone is called a free fall. When a body falls freely towards the earth, its velocity continuously increases. The acceleration developed in its motion is called acceleration due to gravity (g).

Calculation of value of 'g'

If the point mass m is situated on the earth's surface, then r = R, and the gravitational force on mass m is

$$F = \frac{GMm}{R^3}.R \quad \text{or} \quad F = \frac{GMm}{R^2}. \qquad (\text{for } r = R)$$

Suppose the mass m experiences acceleration g, called the acceleration due to gravity, then according to Newton's second law of mation,

$$F = mg$$

$$\therefore \quad mg = \frac{GMm}{R^2} \quad \text{or } g = \frac{GM}{R^2}$$

This gives the acceleration due to gravity on the surface of the earth.

Putting the value of $G = 6.67 \times 10^{-11}$ Nm^2/kg^2; M (mass of earth) = 6×10^{24}kg; R (radius of earth) = 6.4×10^6m

we get, $g = 9.8$ m/s^2

WEIGHTLESSNESS

The phenomenon of "weightlessness" occurs when there is no force of support on your body. When your body is effectively in "free fall", accelerating downward at the acceleration due gravity, then you are not being supported. The sensation of apparent weight comes from the support that you feel from the floor, from the seat, etc. Different sensations of apparent weight can occur on a roller-coaster or in an aircraft because they can accelerate either upward or downward.

If you travel in a curved path in a vertical plane, then when you go over the top on such a path, there is necessarily a downward acceleration. Taking the example of the roller-coaster which is constrained to follow a track, then the condition for weightlessness is met when the downward acceleration of your seat is equal to the acceleration of gravity. Considering the path of the roller-coaster to be a segment of a circle so that it can be related to the centripetal acceleration, the condition for weightlessness is

$$v_{\text{weightless}} = \sqrt{gR} \text{ from the centripetal acceleration relationship } \frac{v^2}{r} = g$$

The "weightlessness" you may feel in an aircraft occurs any time the aircraft is accelerating downward with acceleration 1g. It is possible to experience weightlessness for a considerable length of time by turning the nose of the craft upward and cutting power so that it travels in a ballistic trajectory. A ballistic trajectory is the common type of trajectory you get by throwing a rock or a baseball, neglecting air friction. At every point on the trajectory, the acceleration is equal to g downward since there is no support. A considerable amount of experimentation has been done with such ballistic trajectories to practice for orbital missions where you experience weightlessness all the time.

The satellite is moving in a circular orbit, it has a radial acceleration

$$a = \frac{v_0^2}{r} = \frac{GM}{r^2} \qquad \left[as\ v_0 = \sqrt{\left(\frac{GM}{r}\right)}\right]$$

i.e., it is falling towards earth's centre with acceleration a, so apparent weight of the body in it $W_{ap} = m(g' - a)$

where g' is the acceleration due to gravity of earth at the position (height) of satellite, i.e. $g' = (GM/r^2)$, so that

$$W_{ap} = m\left[\frac{GM}{r^2} - \frac{GM}{r^2}\right] = 0$$

i.e., the apparent weight of a body in a satellite is zero and independent of the radius of the orbit.

CHECK Point

- **The astronauts in the satellite orbiting the earth feel weightlessness. Does the weightlessness rely upon the distance of the satellite from the earth. If so, how? Explain your answer.**

Solution

Weightlessness does not rely upon distance. Weightlessness occurs because force of gravity of earth is utilised by the centripetal force giving orbital motion to the satellite. It is true whatever may be the distance of the satellite from the earth.

ILLUSTRATION : 14

Two satellites have their masses in the ratio of 3 : 1. The radii of their circular orbits are in the ratio of 1 : 4. What is the ratio of total mechanical energy of *A* and *B*?

SOLUTION :

As we know,

$$E = -\frac{GMm}{2r}$$

$$\therefore \quad \frac{E_1}{E_2} = \frac{m_1}{r_1} \times \frac{r_2}{m_2} = \left[\frac{m_1}{m_2}\right]\left[\frac{r_2}{r_1}\right] = \frac{3}{1} \times \frac{4}{1} = \frac{12}{1}$$

ILLUSTRATION : 15

Consider an earth satellite so positioned that it appears stationary to an observer on earth and serves the purpose of a fixed relay station for intercontinental transmission of TV and other communications. What would be the height at which the satellite should be positioned and what would be the direction of its motion? Given that the radius of the earth is 6400 km and acceleration due to gravity on the surface of the earth is 9.8 m/s^2.

SOLUTION :

For a satellite to remain above a given point on the earth's surface, it must rotate with the same angular velocity as the point on earth's surface. Therefore, the satellite must rotate in the equatorial plane from west to east with a time period of 24 hours.

Now as for a satellite $v_0 = \sqrt{GM/r}$

$$T = \frac{2\pi r}{v_0} = 2\pi r\sqrt{\frac{r}{GM}} = 2\pi r\sqrt{\frac{r}{gR^2}} \qquad \left[\because g = \frac{GM}{R^2}\right]$$

or $\quad r = \left[gR^2 \frac{T^2}{4\pi^2}\right]^{1/3} = 4.23 \times 10^7 \text{ m} = 42300 \text{ km}$

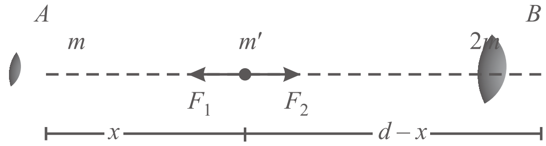
So the height of the satellite above the surface of earth, $h = r - R = 42300 - 6400 \approx 36000$ km

[The speed of a geostationary satellite $v_0 = R\sqrt{g/r} = r\omega = 3.1$ km/sec

MISCELLANEOUS SOLVED EXAMPLES

1. Two bodies A and B of masses *m* and 2*m* respectively are kept *a* distance *d* apart. Where should a small particle be placed, so that the net gravitational force on it due to the bodies A and B is zero ?

Sol. It is the clear that the particle must be placed on the line AB between the bodies A and B. Suppose it is at a distance x from A. Let its mass be m′.



The force on m′ due to A is

$$F_1 = \frac{Gmm'}{x^2} \text{ towards A}$$

and that due to *B* is

$$F_2 = \frac{G(2m)m'}{(d-x)^2} \text{ towards B.}$$

The net force will be zero if $F_1 = F_2$.

Thus, $\frac{Gmm'}{x^2} = \frac{G(2m)m'}{(d-x)^2}$ or $(d-x)^2 = 2x^2$

or $d - x = \pm\sqrt{2}$ or $d = (1 \pm \sqrt{2})\,x$.

Thus, $x = \frac{d}{1+\sqrt{2}}$ or $\frac{d}{1-\sqrt{2}}$.

As *x* cannot be negative,

$$x = \frac{d}{1+\sqrt{2}}.$$

2. A body has a weight *W* on the surface of earth. What is its weight on a planet which has mass 10 times that of the earth and a radius of 4 times that of the earth ?

Sol. Let the mass of the body be m and *g* be the acceleration due to gravity on the earth.

Therefore, $W = mg$ ...(i)

The ratio of acceleration due to gravity on planet and on the earth is

$$\frac{g_p}{g} = \frac{\frac{GM_p}{R_p^2}}{\frac{GM}{R^2}} = \frac{GM_p}{R_p} \times \frac{R^2}{GM} = \frac{M_pR^2}{R_p^2 \times M}$$

Given that $M_p = 10\text{ M}$ and $R_p = 4\text{ R}$

Therefore, $\frac{g_p}{g} = \frac{10M \times R^2}{(4R)^2 \times M} = \frac{10}{16} = \frac{5}{8}$ or $g_p = \frac{5}{8}g$

Therefore, weight of the body on that planet

$$W_p = mg_p = \frac{M \times 5}{8}g = \frac{5}{8}mg$$

or $W_p = \frac{5}{8}W$ [from eq. (i)]

3. Calculate the value of acceleration due to gravity on the surface of the moon.
(Given: Mass of the moon = 7.4 × 10^{22} kg, radius of moon = 1740 m, G = 6.7 × 10^{-11} Nm^2/kg^2).

Sol. As we know, $g = G \times \frac{M}{R^2}$

Radius of the moon,

$R = 1740$ km $= 1740 \times 1000$ m $= 1.74 \times 10^6$ m

Putting values of G, M and R in the above formula, we get

$$g = \frac{6.7\times10^{-11}\times7.4\times10^{22}}{(1.74\times10^{6})^2} \quad \text{or} \quad g = 1.63 \text{ m/s}^2$$

Thus, the acceleration due to gravity, g on the surface of the moon is 1.63 m/s^2.

SOLVED EXAMPLES BASED ON CONNECTING TOPICS

4. A particle is taken to a height of $2R_e$ above the earth's surface, where R_e is the radius of the earth. If it is dropped from this height, what would be its acceleration?

Sol. The acceleration due to gravity at height H above the surface of the earth is

$$g' = g\left(\frac{R_e}{R_e + H}\right)^2 \quad \text{here } H = 2R_e.$$

$$\text{Thus, } g' = g\left(\frac{R_e}{3R}\right)^2 = \frac{g}{9} = \frac{9.8 \text{m/s}^2}{9} \approx 1.1 \text{m/s}^2.$$

5. Two satellites A and B of equal mass move in the equatorial plane of the earth, close to earth's surface. Satellite A moves in the same direction as that of the rotation of the earth, while satellite B moves in the opposite direction. Calculate the ratio of the kinetic energy of B to that of A in the reference frame fixed to the earth (Take $g = 9.8$ m s^{-2} and radius of the earth = 6.37×10^6 km)

Sol. Let ω_A and ω_B be the absolute angular speeds of A and B. Since they are in the same orbit, their time periods must be the same, i.e., $\omega_A = \omega_B$. Considering the dynamics of circular motion in the cases,

$$m\omega_A^2 R = \frac{GMm}{R^2} \Rightarrow \omega_A = \sqrt{\frac{g}{R}} \quad (\because GM = gR^2) \text{ Similarly } \omega_B = \sqrt{\frac{g}{R}}$$

$$\text{And } \omega_A = \omega_B = \sqrt{\frac{9.8}{6.37\times10^6}} = 124 \times 10^{-5} \text{ rad s}^{-1}$$

Now $\omega_{AE} = \omega_A - \omega_E = 124 \times 10^{-5} - 7.3 \times 10^{-5} = 116.7 \times 10^{-5}$ rad s^{-1}

ω_{BE} = (velocity of B relative to E) $= \omega_B - (-\omega_E) = \omega_B + \omega_B = 131.1 \times 10^{-5}$ rad s^{-1}

$$\text{Therefore, } \frac{K_B}{K_A} = \frac{\frac{1}{2}m\omega_{BE}^2 r^2}{\frac{1}{2}m\omega_{AE}^2 r^2} = \frac{131.3^2}{116.7^2} = 1.27$$

6. Determine the escape velocity of a body from the moon. Take the moon to be a uniform sphere of radius 1.76×10^6 m, and mass 7.36×10^{22} kg. Given $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}.

Sol. Here $R = 1.76 \times 10^6$ m, $M = 7.36 \times 10^{22}$kg

$$\text{Escape velocity, } v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2\times6.67\times10^{-11}\times7.36\times10^{22}}{1.76\times10^6}}$$

$= 2375$ ms^{-1} $= 2.375$ km s^{-1}.

7. Calculate the escape velocity for an atmospheric particle 1600 km above the earth's surface, given that the radius of the earth is 6400 km and acceleration due to gravity on the surface of earth is 9.8 ms^{-2}.

Sol. At a height h above the earth's surface, we have

$$v_e = \sqrt{2g_h(R+h)}, \quad g_h = \frac{gR^2}{(R+h)^2}$$

$$\therefore \quad v_e = \sqrt{\frac{2\times gR^2}{(R+h)^2}\times(R+h)} = \sqrt{\frac{2gR^2}{R+h}}$$

But $g = 9.8$ms^{-2}, $R = 6.4 \times 10^6$m, $h = 1600$ km $= 1.6 \times 10^6 = 8 \times 10^6$m

$$\therefore \quad v_e = \sqrt{\frac{2\times9.8\times(6.4\times10^6)^2}{8\times10^6}} = 10.02 \times 10^3 \text{ ms}^{-1} = 10.02 \text{kms}^{-1}.$$

1 EXERCISE

Fill in the Blanks :

DIRECTIONS : *Complete the following statements with an appropriate word / term to be filled in the blank space(s).*

1. The force of .... is the centripetal force on the moon.
2. The force of gravity between two objects is inversely proportional to the square of the ....
3. The acceleration due to gravity at the surface of a planet depends on the .... and the .... of the planet.
4. g_e and g_p denotes the acceleration due to gravity on the surface of the earth and another planet whose mass and radius are twice that of the earth. The relation that holds is ....
5. The constant of gravitation G is related to g by ....
6. If the density of planet is increased, then the acceleration due to gravity at its surface will ....
7. Every object inside the satellite is ....
8. The value of g will become 10% of its value at the earth's surface at a height .... above the surface of earth. Take radius of earth as 6400 km.
9. Acceleration due to gravity is ....proportional to the density of the planet.
10. Dimensional formula of universal gravitational constant is ....
11. ....velocity of the planet is constant when it revolves in an elliptical orbit around the sun.
12. Weight of an object is ....at equator than at poles.
13. Acceleration due to gravity ....with depth below the surface of the earth.
14. If the earth rotates faster, the acceleration due to gravity at equator will ....
15. The weight of an object at ....is zero.
16. The orbit of a geostationary orbit is called ....

True/False :

DIRECTIONS : *Read the following statements and write your answer as true or false.*

1. For a spherically symmetric earth, the acceleration due to gravity should be same at the eqator and at the poles.
2. Copernicus discovered that the earth moves around the sun.
3. If earth suddenly stops rotating about its axis, then the value of g will be same at all the places.
4. Weightlessness experienced while orbiting the earth in a spaceship is the result of zero gravity.
5. Force of gravity is least at the equator.
6. The gravitation force between a spherical shell and a point mass inside it is negative and finite.
7. An astronaut cannot use a straw to sip a drink on the surface of the moon.
8. It is possible to put on artificial satellite in to orbit in such a way that it will remain directly over New Delhi.
9. Acceleration due gravity at poles is greater than that at equator.
10. Weight of body at centre of the earth is zero.
11. If the earth stops rotating, acceleration due to gravity at poles increases.
12. Work is to be done on the system of two particles in increasing the distance between them.
13. An object in a satellite experiences weightlessness.
14. The orbital speed of a satellite is inversely proportional to radius of its orbit.

Match the Columns :

DIRECTIONS : *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column I have to be matched with statements (p, q, r, s) in Column II.*

1. Match the columns

	Column I		Column II
(A)	A Newton's law of gravitation	(p)	zero
(B)	At height $h = R$ value of g	(q)	inverse square law
(C)	At height $h = R/2$ value of g	(r)	decreases by a factor 1/4
(D)	At depth $h = R$ value of g	(s)	decreases by a factor 1/2

Very Short Answer Questions :

DIRECTIONS : *Give answer in one word or one sentence.*

1. The mass of two bodies are doubled and the distance is halved, how will the gravitational force change?
2. If two masses are equal and made of same material, how will the force of attraction vary with separation?

3. Earth is continuously pulling moon towards its centre. Why does not moon fall on to earth?
4. Which has a longer time period, a satellite revolving close or away from the surface of the earth?
5. Will 1 kg sugar weigh more at poles or at the equator?
6. Which is greater, the attraction of the earth for 1 kg of iron or attraction of 1 kg iron for the earth? Why?
7. Why does weight of a body change from place to place whereas mass remains constant?
8. At what place on earth, the value of *g* does not change due to its rotational motion?

Short Answer Questions :

DIRECTIONS : *Give answer in 2-3 sentences.*

1. A planet reduces its radius by 1% keeping its mass unchanged. How does its acceleration due to gravity change?
2. If a planet existed where mass and radius both were half of those of the earth, what would be the value of the acceleration due to gravity on its surface as compared to what it is on the earth's surface?
3. What is the importance of the universal law of gravitation in nature?
4. How did Galileo prove that different bodies fall towards the earth at the same rate?
5. How do 'g' and 'G' differ from each other ?
6. If the force of gravity somehow vanished today, why would we be sent flying in space ?

Long Answer Questions :

DIRECTIONS : *Give answer in four to five sentences.*

1. Define acceleration due to gravity. Deduce an expression for it in terms of mass of the earth (M) and universal gravitational constant (G).
2. Show that weight of an object on the moon is 1/6th its weight on the earth. Given that the mass of the earth is 100 times the mass of the moon and its radius is 4 times that of the moon.
3. State Newton's universal law of gravitation. Hence define universal gravitational constant. Give the value of G.

2 EXERCISE

Text-Book Questions :

1. State the universal law of gravitation.
2. Write the formula to find the magnitude of gravitational force between the earth and an object on the surface of the earth.
3. What do you mean by free fall?
4. What do you mean by acceleration due to gravity?
5. What are the differences between the mass of an object and its weight ?
6. Why is the weight of an object on the moon $\frac{1}{6}$ th its weight on the earth?

Text-Book Exercise :

1. How does the force of gravitation between two objects change when the distance between them is reduced to half ?
2. Gravitational force acts on all objects in proportion to their masses. Why, then, a heavy object does not fall faster than a light object ?
3. What is the magnitude of the gravitational force between the earth and a 1 kg object on its surface ? (Mass of the earth = 6×10^{24} kg and radius of the earth = 6.4×10^{6} m.)
4. The earth and the moon are attracted to each other by gravitational force. Does the earth attract the moon with a force that is greater or smaller or the same as the force with which the moon attracts the earth ? Why?
5. If the moon attracts the earth, why does the earth not move towards the moon?
6. What happens to the force between two objects, if
   (i) the mass of one object is doubled?
   (ii) the distance between the object is doubled and tripled?
   (iii) the masses of both objects are doubled ?
7. What is the importance of universal law of gravitation?
8. What is the acceleration of free fall?
9. What do we call the gravitational force between the earth and an object ?
10. Amit buys few grams of gold at the poles as per the instruction of one of his friends. He hands over the same when he meets him at the equator. Will the friend agree with the weight of gold bought? If not, why? [Hint: The value of g is greater at the poles than at the equator.]
11. Why does a sheet of paper fall slower than one that is crumpled into a ball?
12. Gravitational force on the surface of the moon is only $\frac{1}{6}$ as strong as gravitational force on the earth. What is the weight in newtons of a 10 kg object on the moon and on the earth?
13. A ball is thrown vertically upwards with a velocity of 49 m/s. Calculate
    (i) the maximum height to which it rises,
    (ii) the total time it takes to return to the surface of the earth.
14. A stone is released from the top of a tower of height 19.6 m. Calculate its final velocity just before touching the ground.
15. A stone is thrown vertically upward with an initial velocity of 40 m/s. Taking g = 10 m/s^2, find the maximum height reached by the stone. What is the net displacement and the total distance covered by the stone ?
16. Calculate the force of gravitation between the earth and the sun, given that the mass of the earth = 6×10^{24} kg and of the sun = 2×10^{30} kg . The average distance between the two is 1.5×10^{11} m.
17. A stone is allowed to fall from the top of a tower 100 m high and at the same time another stone is projected vertically upwards from the ground with a velocity of 25 m/s. Calculate when and where the two stones will meet?
18. A ball thrown up vertically returns to the thrower after 6 s. Find
    (a) the velocity with which it was thrown up,
    (b) the maximum height it reaches, and
    (c) its position after 4 s.

Exemplar Questions :

1. What is the source of centripetal force that a planet requires to revolve around the Sun? On what factors does that force depend?
2. Suppose gravity of earth suddenly becomes zero, then in which direction will the moon begin to move if no other celestial body affects it?
3. Identical packets are dropped from two aeroplanes, one above the equator and the other above the north pole, both at height h. Assuming all conditions are identical, will those packets take same time to reach the surface of earth. Justify your answer.

4. The weight of any person on the moon is about 1/6 times that on the earth. He can lift a mass of 15 kg on the earth. What will be the maximum mass, which can be lifted by the same force applied by the person on the moon?

5. How does the weight of an object vary with respect to mass and radius of the earth. If the diameter of the earth becomes half of its present value and its mass becomes four times of its present value, then how would the weight of any object on the surface of the earth be affected?

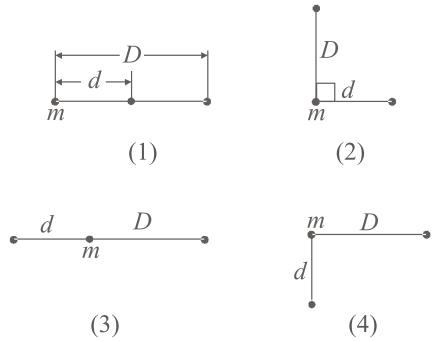
Hots Questions :

DIRECTIONS: *Answer the following questions.*

1. Gravitational force acts on all bodies in proportion to their masses. Why, then, doesn't a heavy body fall faster than a light body?

2. If the earth were somehow expanded to a larger radius, with no change in mass, how would your weight be affected? How would it be affected if the earth were shrunk instead ?

3. The figure shows four arrangements of three particles of equal masses. (a) Rank the arrangements according to the magnitude of the net gravitational force on the particle labeled m, greatest first. (b) In arrangement 2, is the direction of the net force, closer to the line of length d or to the line of length D?



4. If there is an attractive force between all objects, why do we not feel ourselves gravitating toward massive buildings in our vicinity?

5. In what sense is drifting in space far away from all celestial bodies like stepping off a chair?

3 EXERCISE

Single Option Correct :

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. The value of the universal gravitational constant
(a) changes with change of place
(b) does not change from place to place
(c) becomes more at night
(d) becomes more during day

2. Two iron and wooden balls identical in size are released from the same height in vacuum. The time taken by them to reach the ground are
(a) zero (b) not equal
(c) roughly equal (d) exactly equal

3. Two stars of masses m and 5m are 3000 parsec apart. If the force on the large one is F, the force on the small star is
(a) F/25 (b) F/5
(c) F (d) 5F

4. What is the acceleration of a meteor passing by the earth at an altitude of 55000 km ?
(a) 0.11 m/s^2 (b) 11 m/s^2
(c) 1.11 m/s^2 (d) 5.18 m/s^2

5. Determine the gravitational attraction between two asteroids separated by 22000 meters if their masses are 450000 kg and 700000 kg, respectively
(a) 1.2×10^{-6} N (b) 1.2×10^{-8} N
(c) 4.3×10^{-8} N (d) 2.1×10^{-6} N

6. The radius of the earth is 6400 km. What is its mass ?
(a) 6.0×10^{24} kg (b) 5.0×10^{24} kg
(c) 1.0×10^{22} kg (d) 6.5×10^{14} kg

7. How much is the gravitational force that keeps an artificial satellite of mass 3500 kg in orbit around the earth at an altitude of 4200 km
(a) 12000 N (b) 12500 N
(c) 13000 N (d) 10000 N

8. The radius of earth is about 6400 km. and that of Mars is about 3200 km. The mass of earth is about 10 times the mass of Mars. An object weighs 200 N on earth's surface. Then its weight on the surface of Mars will be
(a) 8 N (b) 20 N
(c) 40 N (d) 80 N

9. The value of g is maximum
(a) at poles of earth (b) at equator of earth
(c) in a mine (d) at a high hill

10. Let us say F_1 is the magnitude of the force exerted on sun by earth and F_2 is the magnitude of force exerted on earth by sun, then
(a) $F_1 > F_2$ (b) $F_1 = F_2$
(c) $F_1 < F_2$ (d) None of above

11. The gravitational force of attraction between two bodies is F Newtons. If the mass of each body and the distance between them are doubled, then the gravitational force between them in Newton is
(a) $16F$ (b) $F/16$
(c) $F/4$ (d) F

12. If the earth stops rotating about its axis, the acceleration due to gravity will remain unchanged at
(a) equator (b) latitude 45°
(c) latitutde 60° (d) poles

13. If a man weighs 60 kg on the surface of the earth, the height above the surface of the earth where his weight is 30kg is
(a) 0.41 R (b) $\sqrt{2}$R
(c) $R/\sqrt{2}$ (d) R/2

14. An object weighs 10N when measured on the surface of earth. What should be its weight on moon?
(a) 1.67 N (b) 4.18 N
(c) 2.20 N (d) 16.7 N

15. A hypothetical planet has a mass of half that of the Earth and a radius of twice that of the Earth. What is the acceleration due to gravity on the planet in terms of g
(a) g (b) $g/2$
(c) $g/4$ (d) $g/8$

16. The force of gravitation between objects of ordinary size kept at a distance of 1 metre from each other is of the order of ?
(a) 10 N (b) 100 N
(c) 10^{-8} N (d) 10^{-11} N

17. A ball is thrown vertically upwards and attains a maximum height of 100 m. It was thrown with a speed
(a) 9.8 m/s (b) 44.3 m/s
(c) 19.6 m/s (d) 98 m/s

18. A stone is thrown vertically upwards and caught at the point of projection after 10 seconds. The time taken by the stone to reach the highest point is
(a) 5 sec. (b) 10 sec.
(c) 9.8 sec. (d) 4.9 sec.

19. A stone is dropped from the roof of a house and is found to reach the ground in one second. The height of the house is
(a) 4.9 m (b) 9.8 m
(c) 12 m (d) 19.6 m

20. A ball of relative density 0.8 fall into water from a height of 2m. The depth to which the ball will sink is
(a) 8 m (b) 2 m
(c) 6 m (d) 4 m

21. Two point masses each equal to 1 kg attract one another with a force of 10^{-10} N. The distance between the two point masses is ($G = 6.6 \times 10^{-11}$ MKS units)
(a) 8 cm (b) 0.8 cm
(c) 80 cm (d) 0.08 cm

22. According to Newton's Universal Laws of Gravitation
(a) $F = \frac{GM}{r^2}$ (b) $F = G\frac{Mm}{R^2}$
(c) $F = \frac{GM}{R}$ (d) $F = \frac{GMm}{R^3}$

More Than One option Correct :

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

1. When an object is falling from rest position its velocity after t seconds would be
(a) $\frac{1}{2}gt^2$ (b) gt
(c) $2gh$ (d) $\sqrt{2gh}$

2. What is the weight of man weighing 600 newton on earth?
(a) 10 m/s^2 (b) 60 kg
(c) 600/g kg (d) 1.66 kg

3. The edges of blades and knives are made sharp to
(a) increase effective surface area
(b) decrease effective surface area
(c) increase effect of pressure
(d) decrease the total thrust

4. The mass of an object is
(a) measured by its inertia
(b) constant at all places
(c) scalar quantity
(d) vector quantity

5. During free fall of an object
(a) direction of motion is same
(b) velocity of object remains constant
(c) mass of the object changes
(d) velocity is uniformly accelerated

6. Choose the correct statements from the following :
(a) Gravitational constant (G) is scalar but acceleration due to gravity (g) is a vector
(b) G and g are constant everywhere
(c) The gravitational forces between two particles are an action and reaction pair.
(d) The values of G and g are to be determined experimentally.

Multiple Matching Questions :

DIRECTIONS: *Following question has four statements (A, B, C, D....) given in Column I and five statements (p, q, r, s, t) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in Column I with entries in Column II.*

1. Match the items of column I against the items of column II.

	Column-I		Column-II
(A)	Acceleration due to gravity (g)	(p)	Vector quantity
(B)	Gravitational constant (G)	(q)	$F = \frac{GMm}{r^2}$
(C)	Force of gravitation	(r)	Scalar quantity
(D)	Weight	(s)	9.8 m/s^2
		(t)	6.67×10^{11} Nm^2\kg^2

	A	B	C	D
(a)	p,s	r, t	q, p	p
(b)	p, q, r, s	q	p, q, r, s	p, q, r, s
(c)	p, s	q	r, s, t	r
(d)	p, t	t, q	r	s

Passage Based Questions :

DIRECTIONS : *Study the given paragraph(s) and answer the following questions.*

PASSAGE

The quantity of matter contained in an object is called mass. It remains constant whether the object is on earth, moon or even in the outer space. Weight on the other hand is the force of attraction of earth with which an object is attracted towards the earth. Now, suppose a man weighs 600 N on earth, his weights on moon would be 100 N.

1. The mass of man on earth, if g is 10 m/s^2 is
(a) 60 kg (b) 10 kg
(c) 6000 kg (d) 1000 kg

2. The mass of man on moon is
(a) 60 kg (b) 10 kg
(c) 6000 kg (d) 1000 kg

3. Acceleration due to gravity on moon is
(a) 10 m/s^2 (b) 9.8 m/s^2
(c) 1.66 m/s^2 (d) 1 m/s^2

Assertion & Reason :

DIRECTIONS : *Each of these questions contains an Assertion followed by Reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.

(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.
(c) If **Assertion** is **correct** but **Reason** is **incorrect**.
(d) If **Assertion** is **incorrect** but **Reason** is **correct**.

1. **Assertion :** The value of acceleration due to gravity does not depend upon mass of the body.
Reason : Acceleration due to gravity is a constant quantity.

2. **Assertion :** When distance between two bodies is doubled and also mass of each body is also doubled, gravitational force between them remains the same.
Reason : According to Newton's law of gravitation, force is directly proportional to mass of bodies and inversely proportional to distance between them.

3. **Assertion :** Newton solved the apple-Earth problem using Shell theorem.
Reason : Newton's law of gravitation applies strictly to a particles.

4. **Assertion :** According to Newton's law of motion, if earth attracts a stone towards itself, then the stone should also attract the earth.
Reason : Earth attracts towards stone but with a very small acceleration.

5. **Assertion :** Whether the object is falling towards or moving away from earth, the direction of acceleration is always towards earth.
Reason : The value of '*g*' is 9.8 m/s^2.

6. **Assertion :** The value of '*g*' is greater at the equator than at the poles.
Reason : Radius is more at the equator than at the poles.

Integer/Numeric type Questions :

DIRECTIONS : *Following are integer based/Numeric based questions. Each question, when worked out will result in one integer or numeric value.*

1. A particle weighs 120 N on the surface of the earth. At what height above the earth's surface will its weight be 30N ?
Radius of the earth = 6400 km.

2. How much less will a mass of 1.000 kg weigh at a ceiling of height h = 3.00 m compared to its weight on the floor ? Assume that the local gravitational strength at floor level has standard value of 9.80665 N/kg. (Rearth = 6400 km)

3. At what height above the surface, the value of the gravity would be half of what it is on the surface of the earth. Take radius of the earth as Re = 6400 km.

4. Imagine that you are visiting planet Mars. You want to record your weight in a notebook on this planet. If your weight on the earth is 450 N, what would you record as your weight on the Mars? Take mass of Mars = 6 × 1023 kg and g = 10 m/s^2.

5. If your weight is 60 kg on earth, how far must you go from the centre of the earth so that you weigh 30 kg?

6. A body weighs 63 N on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the earth?

7. Calculate the force of gravitation between the earth and the sun, given that the mass of the earth = 6 × 1024 kg and of the sun = 2 × 1030. The average distance between the two is 1.5 × 1011 m.

4 ADVANCED EXERCISE
BASED ON CONNECTING TOPICS

DIRECTIONS (Qs. 1 - 11): *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. An astronaut in a spacecraft goes into circular orbit above a strange planet that has a radius of 2100 km. The craft is going at 1200 meters per second at an altitude of 6500 km. The astronaut's life-support pack has a mass of 60 kg. What will the life-support pack weigh on the surface of the planet?
(a) 150 N (b) 170 N
(c) 90 N (d) 190 N

2. When a satellite is in the synchronous orbit above the equator, it stays in one place with reference to the earth by making each revolution in just the same time as it takes the earth to rotate once. What is the altitude of the synchronous orbit?
(a) 20000 km (b) 30000 km
(c) 32500 km (d) 36000 km

3. If G is gravitational constant and R is the radius of the earth, then acceleration due to gravity 'g' and the mean density of earth D are related by the equation

(a) $D = \frac{g}{G}\left(\frac{4\pi R^3}{3}\right)$ (b) $D = \frac{g/G}{\left(\frac{4}{3}\pi R\right)}$

(c) $D = \left(\frac{g}{G}\right)\left(\frac{4}{3}\pi R^2\right)$ (d) $D = \frac{g/G}{\frac{4}{3}\pi R^3}$

4. A balloon filled with CO_2 released on earth would (neglect viscosity of air)
(a) climb with an acceleration 9.8 m/s^2
(b) fall with an acceleration 9.8 m/s^2
(c) fall with a constant acceleration 3.4 m/s^2
(d) fall with acceleration and then would attain a constant velocity

5. If the density of the earth is doubled keeping its radius constant then acceleration due to gravity (present value 9.8 m/s^2) will be
(a) 2.45 m/s^2 (b) 4.9 m/s^2
(c) 9.8 m/s^2 (d) 19.6 m/s^2

6. The mass of the moon is 1/81 of earth's mass and its radius 1/4 that of the earth. If the escape velocity from the earth's surface is 11.2 km/sec, its value for the moon will be
(a) 0.14 kms^{-1} (b) 0.5 kms^{-1}
(c) 2.5 kms^{-1} (d) 5.0 kms^{-1}

7. Two bodies of masses 10 kg and 100 kg are separated by a distance of 2m (G = 6.67 × 10^{-11} Nm^2 kg^{-2}). The gravitational potential at the mid-point on the line joining the two is
(a) 7.3 × 10^{-7} J/kg (b) 7.3 × 10^{-9} J/kg
(c) –7.3 × 10^{-9} J/kg (d) 7.3 × 10^{-6} J/kg

8. The ratio of the radii of the planets R_1 and R_2 is k. The ratio of the acceleration due to gravity is r. The ratio of the escape velocities from them will be
(a) kr (b) $\sqrt{kr}$
(c) $\sqrt{(k/r)}$ (d) $\sqrt{(r/k)}$

9. The escape velocity of a body depends upon mass as
(a) m^0 (b) m^1
(c) m^2 (d) m^3.

10. The time period of an earth satellite in circular orbit is independent of
(a) both the mass and radius of the orbit
(b) radius of its orbit
(c) the mass of the satellite
(d) neither the mass of the satellite nor the radius of its orbit.

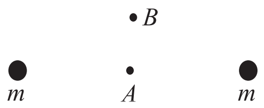
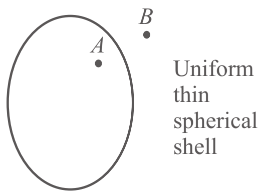
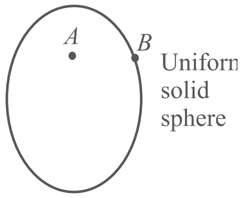
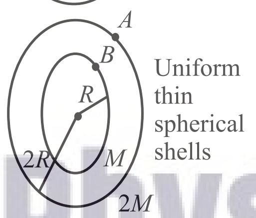
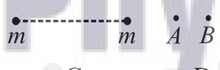
11. For a satellite moving in an orbit around the earth, the ratio of kinetic energy to potential energy is
(a) $\frac{1}{\sqrt{2}}$ (b) 2
(c) $\frac{1}{2}$ (d) $\sqrt{2}$

DIRECTIONS (Qs. 12) : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

12. If two satellites of different masses are revolving in the same orbit, they have the same
(a) angular momentum (b) speed
(c) energy (d) time period

DIRECTIONS (Qs. 13) : *Following question has four statements (A, B, C, D....) given in Column I and five statements (p, q, r, s, t) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in Column I with entries in Column II.*

13. If U_A and U_B represent magnitude of gravitational potential at points A and B respectively and I_A and I_B represent magnitude of gravitational fields at those points in different configuration of column II, then match the following.

Column-I	Column-II
(A) $U_A > U_B$	(p) 
(B) $U_A < U_B$	(q) 
(C) $I_A > I_B$	(r) 
(D) $I_A < I_B$	(s) 
	(t) 

	A	B	C	D
(a)	p, q, r, t	s	s, t	p, q, r
(b)	t, q	p, r	s, t	p, q
(c)	p, s	q	r, s, t	r
(d)	p,	q, r	r	s

DIRECTIONS (Qs. 14 -15) : *Study the given paragraph(s) and answer the following questions.*

PASSAGE

Two satellites S_1 and S_2 revolve round a planet in coplanar circular orbits in the same sense. Their periods of revolution are 1 h and 8 h respectively. The radius of the orbit of S_1 is 10^4 km. When S_2 is closest to S_1 find :

14. The speed of S_2 relative to S_1

(a) 2×10^4 km/h (b) $\pi \times 10^4$ km /h.

(c) $2\pi \times 10^4$ km/h (d) 10^4 km/h

15. The angular speed of S_2 as observed by an astronaut in S_1 is

(a) π rad /h (b) $2\pi/3$ rad/h

(c) $\pi/4$ rad/h (d) $\frac{\pi}{3}$ rad / h.

DIRECTIONS (Qs. 16-17) : *Each of these questions contains an Assertion followed by Reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.

(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.

(c) If **Assertion** is **correct** but **Reason** is **incorrect**.

(d) If **Assertion** is **incorrect** but **Reason** is **correct**.

16. **Assertion :** The binding energy of a satellite does not depend upon the mass of the satellite.

Reason : Binding energy is the negative value of total energy of satellite.

17. **Assertion :** The time period of geostationary satellite is 24 hours.

Reason : Geostationary satellite must have the same time period as the time taken by the earth to complete one revolution about its axis.

DIRECTIONS (Qs. No. 18 - 20) : *Following are integer based/ Numeric based questions. Each question, when worked out will result in one integer or numeric value.*

18. What is the true weight of an object in a geostationary satellite that weighed exactly 10.0 N at the north pole?

19. The gravitational field in a region is given by $\vec{E} = (10\hat{i} + 10\hat{j})$ N/kg. Find the work done by an external agent to slowly shift a particle of mass 2 kg from the point (0,0) to a point (5 m, 4 m).

20. A planet of mass M moves along a circle around the sun with a velocity v = 34.9 km/s (relative to the heliocentric reference frame). Find the period of revolution around the sun.

SOLUTIONS

Brief Explanations of Selected Questions

1 EXERCISE

FILL IN THE BLANKS :

1. gravity
2. distance between them
3. mass, radius
4. $g_p = \frac{1}{2} g_e$
5. $g = \frac{GM_e}{r^2}$
6. increase
7. weightless
8. 57,600
9. directly
10. $[M^{-1} L^3 T^{-2}]$
11. Areal
12. less
13. decreases
14. decreases
15. centre of the earth
16. parking orbit

TRUE/FALSE :

1. False
2. True
3. True
4. True
5. False
6. False
7. True
8. False
9. True
10. True
11. False
12. True
13. True
14. False

MATCH THE COLUMNS :

1. (A) → (q); (B) → (r); (C) → (s); (D) → (p)

VERY SHORT ANSWER QUESTIONS :

1. $F = \frac{GM_1M_2}{R^2}$

New force $= F' = \frac{G(2M_1)(2M_2)}{(R/2)^2}$

$= 16\frac{GM_1M_2}{R^2} = 16F$

∴ Force will become 16 times.

2. $F = \frac{GM^2}{r^2} = \frac{G\left(\frac{4}{3}\pi r^3 \rho\right)^2}{r^2} \propto r^4$

∴ Force of attraction is directly proportional to fourth power of separation.

3. Because the gravitational pull of the earth on the moon provides the necessary centripetal force for its orbital motion.

4. ∵ $T^2 \propto R^3$

∴ The satellite away from the surface of the earth will have a longer time period.

5. The value of 'g' is larger at poles than at the equator.

∴ 1 kg sugar will weigh more at poles when weighed by a spring balance calibrated at the equator.

6. In accordance with Newton's law of gravitation, the force is equal in the two cases. It is because, when the two bodies interact due to their masses (gravitational interaction), they exert equal forces on each other but in opposite directions.

7. Mass (m) is the amount of matter contained in a body, which is same everywhere. Weight (W = mg) is force of gravity which changes according to the value of g.

8. At the poles.

SHORT ANSWER QUESTIONS :

1. When mass is same, $g \propto \frac{1}{R^2}$

$\therefore \frac{\Delta g}{g} = 2\frac{\Delta R}{R}$

$\Rightarrow \frac{\Delta g}{g} \times 100 = 2\frac{\Delta R}{R} \times 100 = 2 \times 1\% = 2\%.$

2. $g = \frac{GM}{R^2}$ and $g' = \frac{GM'}{R'^2}$ where $M' = \frac{1}{2}M$ and R'

$= \frac{1}{2}R \;\therefore\; \frac{g'}{g} = \frac{1}{2} \times 2^2 = 2 \;\Rightarrow g' = 2g.$

3. The universal law of gravitation successfully explained many phenomena occurring in nature. Some of these phenomena are as follows :
   1. The force that binds us to the earth.
   2. The motion of the moon around the earth.
   3. The motion of planets around the sun.
   4. The tides due to the moon and the sun.

4. Galileo climbed to the top of the leaning tower of Pisa in the presence of a large gathering. He dropped spheres of different masses and materials simultaneously from the top. To the great surprise of the gathering, all the spheres reached the earth's surface at the same time. So, he concluded that the acceleration of an object falling freely towards the earth does not depend on the mass of the object.

Galileo further explained that the difference in the rate of fall of heavy and light bodies is due to difference in the resistance or friction of air experienced by them. He predicted that if there were no air, the two objects of different masses would reach the ground simultaneously when dropped from the same height.

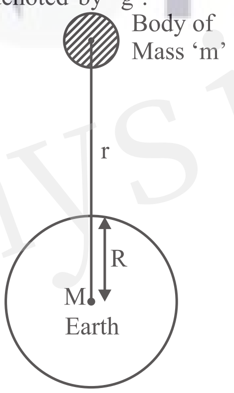
5.

Acceleration due to gravity (g)	Universal gravitational constant (G)
1. It is the acceleration aquired by a body due to earth's gravitational pull on it.	It is equal to the force of attraction between two masses of 1 kg each separate by a distance of 1 m.
2. The value of g is different at different places on the surface of the earth. Its value varies from one celestial (heavenly) body to another.	'G' is a universal constant i.e., its value is the same everywhere in the universe. $(G = 6.67 \times 10^{11}\ Nm^2\ kg^{-2}$
3. It is a vector quantity	It is a scalar quantity

6. It is the force of gravity which binds any object on the surface of the earth. If this force vanishes then there is no other force which will binds the object on earth and objects would be seen flying in space.

LONG ANSWER QUESTION :

1. The acceleration produced in the motion of a body falling under the force of gravity is called acceleration due to gravity. It is denoted by 'g'.



Relation between g and G : Suppose the earth is a sphere of mass M and radius R, as shown in figure given below. Consider a body of mass m situated at distance r from the centre of the earth. According to Newton's law of gravitation, the force of attraction between the earth and the body is given by

$$F = \frac{GMm}{r^2} \quad \text{... (i)}$$

This force of gravity produces an acceleration 'g', called acceleration due to gravity in the body of mass m.

Hence, from Newton's second law,

F = Mass × Acceleration = mg ... (ii)

From equations (i) and (ii), we get

$$mg = \frac{GMm}{r^2}$$

or $$g = \frac{GM}{r^2} \quad \text{... (iii)}$$

This equation gives acceleration due to gravity at points far away from the earth.

If body is located on the surface of the earth, then r = R, the radius of the earth.

The above equation becomes

$$g_{surface} = \frac{GM}{R^2} \quad \text{... (iv)}$$

As the radius of the earth does not change much over its entire surface, the value of 'g' is almost constant near or on the earth.

The value of 'g' on the earth's surface = 9.8 ms–2

2. Let mass of the object = m

mass of the earth = M_e

mass of the moon = M_m

radius of the earth = R_e

radius of the moon = R_m

then, $M_e = 100\ M_m$ and $R_e = 4\ R_m$

Weight of the object on the earth is

We = The force with which the earth attracts the object

$$= G.\frac{M_e.m}{R_e^2}$$

Weight of the object on the moon is

W_m = The force with which the moon attracts the object.

Weight of the object on the moon is

W_m = The force with which the moon attracts the object

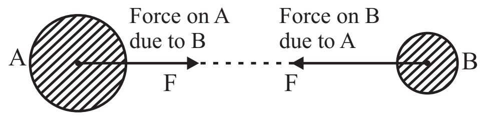
$$= G\frac{M_m.m}{R_m^2}$$

$$= \frac{W_m}{W_e} = \frac{GM_m.m}{R_m^2} \times \frac{R_e^2}{GM_e.m}$$

or $$\frac{W_m}{W_e} = \frac{M_m.m}{M_e} \times \left(\frac{R_e}{R_m}\right)$$

$$= \frac{M_m}{100M_m} \times \left(\frac{4R_m}{R_m}\right)^2 = \frac{16}{100} = \frac{1}{6}$$

3. This law states that every body in this universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. The direction of the force is along the line joining the centres of two bodies.



Law of gravitation

Value of G is 6.67 × 1011 Nm^2kg^{-2}

2 EXERCISE

TEXT-BOOK QUESTIONS :

1. According to the universal law of gravitation, every body in the universe attracts every other body with a force which is directly proportional to product of their masses and inversely proportional to the square of the distance between them. This force acts along the line joining the centres of the two bodies.
2. Suppose the earth is a sphere of mass M and radius R. If an object of mass m is placed on the surface of the earth, then the magnitude of the gravitational force between the body and the earth will be $F = G\dfrac{Mm}{R^2}$
3. When an object falls towards the earth under the influence of attraction force of earth (force of gravity only) this is called free fall of the object.
4. The acceleration produced in a body falling under the influence of gravitational attraction of the earth is called acceleration due to gravity.
5. Differences between mass and weight :

Mass	Weight
1. Mass is the quantity of matter contained in a body and is the measure of its inertia.	1. Weight of a body is the force with which a body is attracted towards the centre of the earth.
2. Its value remains constant at all places	2. Its value (W = mg) changes from place to place due to the change in the value of acceleration due to gravity 'g'.
3. Mass of a body is never zero.	3. Weight of a body is zero at the centre of the earth because there 'g' becomes zero.
4. It is measured by a pan balance	4. It is measured by a spring balance
5. It is scalar quantity	5. It is vector quantity
6. Its unit is kg.	6. Its unit is newton or kg-wt

6. The acceleration due to gravity on the surface of the moon is 1/6th of the acceleration due to gravity on the surface of the Earth. Therefore the weight (w = mg) of an object on the moon will be 1/6th the weight on the earth.

TEXT-BOOK EXERCISE :

1. According to the law of gravitation, the force of attraction between two bodies is

$$F = G\frac{m_1 m_2}{d^2}$$

When the distance is reduced to half,

$$d' = \frac{d}{2}, \text{ so}$$

$$F' = G\frac{m_1 m_2}{d'^2} = G\frac{m_1 m_2}{(d/2)^2}$$

$$= 4G\frac{m_1 m_2}{d^2} = 4F$$

Thus the gravitational force becomes four times the original force.

2. If F be the gravitational force on a body of mass m, then

$$F = G\frac{Mm}{r^2} = mg$$

or

$$g = \frac{GM}{r^2} \text{ (g is the acceleration due to gravity)}$$

Clearly, $F \propto m$ but g does not depend on mass of an object . Hence all objects fall with the same rapidness when there is no air resistance.

3. Here, Mass of the body (m) = 1 kg
Mass of the earth (M) = 6×10^{24} kg
Radius of the earth (R) = 6.4×10^6 m
Magnitude of the gravitational force between the earth and 1 kg body,

$$F = G\frac{Mm}{R^2} = \frac{6.67\times10^{-11}\times6\times10^{24}\times1}{(6.4\times10^6)^2}\text{N}$$

$$= \frac{6.67\times6\times10}{6.4\times6.4} = 9.77 \text{ N} \approx 9.8 \text{ N}.$$

4. The earth attracts the moon with the same force as the force with which the moon attracts the earth. According to Newton's third law, these two forces are equal and opposite.
5. According to Newton's third law, the moon also attracts earth with a force equal to that with which the earth attracts the moon. But the earth is much larger than the moon. So, the acceleration produced in the earth ($a \propto 1/m$) is not noticeable.
6. Force of gravitation, $F = G\dfrac{m_1 m_2}{r^2}$

(i) When mass of one body (m_1 or m_2) is doubled, the force gets doubled.

$$F' = G\frac{(2m_1)m_2}{r^2} = 2G\frac{m_1 m_2}{r^2} = 2F$$

(ii) When distance between the bodies is doubled,

$$F' = G\frac{m_1 m_2}{(2r)^2} = \frac{1}{4}G\frac{m_1 m_2}{r^2} = \frac{1}{4}F$$

i.e., the force becomes one fourth of the original force.
When the distance between the two bodies is tripled,

$$F' = G\frac{m_1 m_2}{(3r)^2} = \frac{1}{9}G\frac{m_1 m_2}{r^2} = \frac{1}{9}F$$

i.e., the force becomes one ninth of the original force.

(iii) When the masses of both bodies are doubled,

$$F' = G\frac{(2m_1)(2m_2)}{r^2} = 4G\frac{m_1 m_2}{r^2} = 4F$$

i.e., the force becomes four times the original force.

7. The universal law of gravitation successfully explained several phenomena which were believed to be unconnected:
(i) the force that binds us to the earth;
(ii) the motion of the moon around the earth;
(iii) the motion of planets around the Sun; and
(iv) the tides due to the moon and the Sun.

8. It is the acceleration produced when a body falls under the influence of the force of gravitation of the earth alone. Near the surface of the earth, its value is 9.8 ms^{-2}.

9. The gravitational force between the earth and an object is called weight of the object.

10. No. The friend will not agree with the weight of gold bought. The value of g at the equator is less than that at the poles. Hence, the few grams of gold at poles will measure less when taken to the equator.

11. A sheet of paper has larger surface area and while falling down it has to overcome the force exerted by air/wind current called as air resistance.
The crumpled paper has smaller surface area and it has to overcome very less amount of air current.
The air resistance offered to plain sheet is more than the resistance to crumpled sheet.
Hence the sheet of paper fall slower than the crumpled one.

12. Mass of the object on the moon = 10 kg
Mass of the object on the earth = 10 kg
Weight of the object on the earth
= mg = 10 × 9.8 = 98 N
Weight of the object on the moon

$$= \frac{1}{6} \times 98 = 16.3 \text{ N}$$

13. (i) In Cartesian sign convention, upward velocity is taken positive and acceleration due to gravity is taken negative.

$\therefore$ u = + 49 ms^{-1}
g = – 9.8 ms^{-2}

At the highest point, v = 0

As $v^2 - u^2$ = 2gs
$\therefore$ $0^2 - 49^2$ = 2(– 9.8) × s

Maximum height, $s = \frac{49 \times 49}{2 \times 9.8}$
= 122.5 m

(ii) Let t be the time taken by the stone to reach the highest point.

As $v = u + gt$
$\therefore$ $0 = 49 - 9.8 \times t$

$$t = \frac{49}{9.8} = 5\ s$$

$\because$ Time of ascent =Time of descent
$\therefore$ Time taken by the stone to return to the earth's surface = $2t$ = 2 × 5 = 10 s

14. Here, u = 0, g = + 9.8 ms^{-2}, s = + 19.6 m
$v^2 - u^2 = 2gs$
$v^2 - 0^2$ = 2 × (+ 9.8) × (+ 19.6)
$v^2 = (19.6)^2$ or v = + 19.6 ms^{-1}

15. Here, u = 40 ms^{-1} , g = – 10ms^{-2}
As the maximum height h, final velocity , v = 0
As $v^2 - u^2 = 2gs$
or $0^2 - 40^2$ = 2 × (–10)h
or $h = \frac{40 \times 40}{20} = 80\,\text{m}$
Total distance covered = $h + h$ = 80 + 80 =160 m
Net displacement is zero.

16. Here,
$M_e = 6 \times 10^{24}$ kg, $M_s = 2 \times 10^{30}$kg, $r = 1.5 \times 10^{11}$m

$$F = G\frac{M_e M_s}{r^2}$$

$$= \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 2 \times 10^{30}}{(1.5 \times 10^{11})^2}\ \text{N}$$

$$= \frac{6.67 \times 12 \times 10^{21}}{1.5 \times 1.5} = 3.56 \times 10^{22}\ \text{N}$$

17. Suppose the two stones meet at a height x from the ground, after time t from the start.
For the downward motion of stone A:
u = 0, g = + 10ms^{-2} , s = (100 – x)

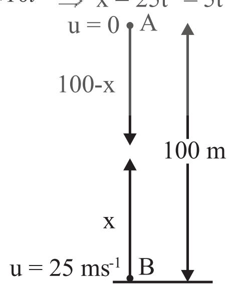
As s = $ut \quad - gt$

$\therefore$ $(100 - x) = 0 + \frac{1}{2} \times 10 \times t^2 = 5t^2$ ...(1)

For upward motion of stone B:
u = +25 ms^{-1} , g = – 10 ms^{-2}, s = x

As s = $ut + \frac{1}{2}gt^2$

$\therefore$ $x = 25t - \frac{1}{2} \times 10t^2 \Rightarrow$ x = 25t^2 – 5t^2 ...(2)



From equations (1) and (2)

or $100 = 25t$

$t = 4s$

From (2),

$$x = 25 \times 4 - \frac{1}{2} \times 10 \times (4)^2 = 100 - 80 = 20\,\text{m}$$

Hence, the two stones meet after 4s at a height of 20 m from the ground or 80 m from the top.

18. Time of ascent = time of descent = $\frac{6}{2} = 3\,\text{s}$

(a) For upward motion of the ball,

$v = 0, t = 3s, g = -9.8\ \text{ms}^{-2}$

As $v = u + gt$

$\therefore\ 0 = u - 9.8 \times 3$

or $u = 9.8 \times 3 = 29.4\ \text{ms}^{-1}$

(b) The height, $h = ut + \frac{1}{2}gt^2$

$$h = 29.4 \times 3 - \frac{1}{2} \times 9.8 \times 3^2$$

$h = 88.2 - 44.1 = 44.1$ m

(c) The position of the ball after 4 s is given by

$$h = ut + \frac{1}{2}gt^2$$

$$h = 29.4 \times 4 - \frac{1}{2} \times 9.8 \times 4^2$$

$h = 117.6 - 78.4 = 39.2$ m

The ball is at a height of 39.2 m from the ground or 4.9 m from the top.

Exemplar Questions :

1. Gravitational force and this force depends on the product of the masses of the planet and sun and the distance between them

2. The moon will begin to move in a straight line in the direction in which it was moving at that instant because the circular motion of moon is due to centripetal force provided by the gravitational force of earth.

3. The value of 'g' at the equator of the earth is less than that at poles. Therefore, the packet will remain in air for longer time interval, when it is dropped at the equator.

4. $g_e = g$ and $g_m = \frac{g}{6}$

Force applied to lift a mass of 15 kg at the earth. $F = mg_e = 15\ g_e$ N

Therefore, the mass lifted by the same force on the moon,

$$m = F/g_m = \frac{15g}{g/6} = 90\text{kg}$$

5. Weight of an object is directly proportional to the mass of the earth and inversely proportional to the square of the radius of the earth i.e.,

Weight of a body = $\frac{M}{R^2}$

When hypothetically M becomes 4 M and R becomes then weight becomes $\frac{R}{2}$

$$W_n = mG\frac{4M}{\left(\frac{R}{2}\right)^2} = (16mG)\frac{M}{R^2} = 16 \times W_o$$

The weight will become 16 times.

Hots Questions :

1. Heavy and light bricks fall with the same acceleration because both have the same ratio of weight to mass. Newton's second law ($a = F/m$) reminds us that greater force acting on greater mass does not result in greater acceleration.

2. Expanded radius corresponds to a lower force of gravity between the earth and the person. Thus, the weight will be reduced on expanding the radius of earth. If earth were shrunk, the force of gravity would increase and hence the weight would also increase.

3. (a) (i) → (1)

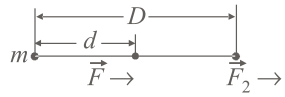
(ii) → (2) and (4) have the same magnitude

(iii) → (3).

Using the formula $F = G\frac{m_1 m_2}{r^2}$

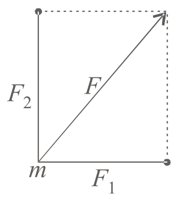
We find that the force is maximum for (1)

$$F = G\frac{m.m}{d^2} + G\frac{m.m}{D^2}$$

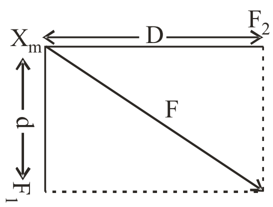


$$= Gm^2\left(\frac{1}{d^2} + \frac{1}{D^2}\right)$$

For (2) and (4) $F = \sqrt{F_1^2 + F_2^2}$



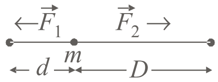
$$F_1 = \frac{G\,m.m}{d^2}$$



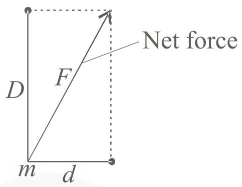
$$F_2 = \frac{Gm.m}{D^2}$$

For (3) $F = \frac{Gm.m}{d^2} - \frac{Gm.m}{D^2}$

$$= Gm^2\left[\frac{1}{d^2} - \frac{1}{D^2}\right]$$



(b) In case (2), the direction of the net force is closer to the line of length d.



4. Gravity pulls us to massive buildings and to everything else in the universe. Physicist Paul A.M. Dirac, winner of the 1933 Nobel prize for physics, put in this way. "Pick a flower on earth and you move the farthest star" How much we are influenced by buildings or how much interaction there is between flowers and stars is another story. The forces between us and buildings are relatively small because their masses are small compared with the mass of earth. The forces due to the stars are also small because of their great distances from us. These tiny forces escape our notice when they are overwhelmed by the overpowering attraction to earth.

5. In both cases you would experience weightlessness. When drifting in deep space, you would remain weightless because no force acts on you. When stepping from a chair, you would be only momentarily weightless because of a temporary lapse of support force.

3 EXERCISE

SINGLE OPTION CORRECT :

1. (b) **2.** (a) **3.** (c)

4. (a) The acceleration is due to gravity along, so $g \propto 1/r^2$ and gr^2 is a constant. We know g and r at the surface of the earth. For the meteor r is its altitude plus the radius of the earth : 55000 km + 6400 km = 61400 km.

Then, $g_1 r_1^2 = g_2 r_2^2$

or $g_2 = g_1\left(\frac{r_1}{r_2}\right)^2 = (9.8 \text{ m/s}^2)\left(\frac{6400 \text{ km}}{61400 \text{ km}}\right)^2$

$= 0.11 \text{ m/s}^2$

5. **(c)** $F = \frac{Gm_1 m_2}{r^2}$

$$= \frac{(6.67\times10^{-11}\text{ Nm}^2/\text{kg}^2)(4.5\times10^5\text{ kg})(7\times10^5\text{ kg})}{(22000\text{ m})^2}$$

$= 4.3 \times 10^{-8}$ N

6. **(a)** At the earth's surface, 6400 km from its center, a kilogram weighs 9.8 N. Then

$$m_1 = \frac{F_{grav} r^2}{Gm_2} = \frac{(9.8\text{N})(6.4\times10^6\text{ m})^2}{(6.67\times10^{-11}\text{ Nm}^2/\text{kg}^2)(1\text{ kg})}$$

$m_1 = 6.0 \times 10^{24}$ kg

7. **(b)** First find the acceleration due to gravity

$$g = (9.8\text{ m/s}^2)\left(\frac{6400\text{ km}}{6400\text{ km}+4200\text{ km}}\right)^2 = 3.57\text{ m/s}^2$$

Then $w = mg = (3500 \text{ kg})(3.57 \text{ m/s}^2) = 12500$ N

8. **(c)** The value of g is $g = \frac{GM}{R^2}$

Weight of object of mass m on earth,

$$W_e = mg_e = \frac{GM_e m}{R_e^2} \quad \text{.......... (1)}$$

Weight of object on Mars,

$$W_m = mg_m = \frac{GM_m m}{R_m^2}$$

$$\therefore \frac{W_m}{W_e} = \frac{(GM_m m/R_m^2)}{(GM_e m/R_e^2)} = \frac{M_m}{M_e}.\left(\frac{R_e}{R_m}\right)^2$$

or $W_m = \frac{M_m}{M_e}.\left(\frac{R_e}{R_m}\right)^2 .W_e$

Given $R_e = 6400$ km, $R_m = 3200$ km

$\frac{M_e}{M_m} = 10$, $W_e = 200$N

$$\therefore W_m = \frac{1}{10}\times\left(\frac{6400\text{ km}}{3200\text{ km}}\right)^2 \times 200N$$

$$= \frac{4}{10}\times 200\text{N} = 40\text{N}$$

9. (a) **10.** (b) **11.** (d) **12.** (d) **13.** (a)

14. (a) Weight on earth $W_e = 10$ N

Weight on moon $W_m = ?$

$W_e = mg_e$ .... (1)

$W_m = mg_m$ .... (2)

On dividing eq. (2) by eq. (1)

$$\frac{W_m}{W_e} = \frac{mg_m}{mg_e}$$

$$\frac{W_m}{W_e} = \frac{g_m}{g_e} \qquad \because \frac{g_m}{g_e} = \frac{1}{6}$$

$$\frac{W_m}{W_e} = \frac{1}{6}$$

$$W_m = \frac{W_e}{6} = \frac{10}{6} = 1.67\ N$$

15. (d) **16.** (c) **17.** (b) **18.** (a)

19. (a) **20.** (a)

21. (c) According to Newton's Gravitation Law

$$F_g = \frac{GM_1M_2}{r^2} \quad \text{here } F_g = 10^{-10} \text{ Newton,}$$

$m_1 = m_2 = 1$kg,

$G = 6.6 \times 10^{-11}$

$$\text{So } r^2 = \frac{GM_1M_2}{F_g} = \frac{6.6\times10^{-11}\times1\times1}{10^{-10}} = 0.66$$

or $r = 0.8125$ metre $= 81.25$ cm ≈ 80 cm

22. (a) According to Newton's universal law of gravitation,

$$F = \frac{GMm}{R^2}$$

More Than One Option Correc :

1. (b, d) **2.** (b, c) **3.** (b, c) **4.** (a, b, c)

5. (a, b) **6.** (a, c, d)

Multiple Matching Questions :

1. (a) (A) → (p, s); (B) → (r, t); (C) → (q, p); (D) → (p)

Passage Based Questions :

1. (a) $W = m \times g$

$600 = m \times 10$

$m = 60$ kg

2. (a) Mass remains same everywhere. So mass on moon $= 60$ kg

3. (c) $W = m \times g$

$100 = 60 \times g$

$g = 100/60 = 1.66$ m/s^2

Assertion & Reason :

1. (c) **2.** (a) **3.** (b) **4.** (a) **5.** (b) **6.** (d)

Integer/Numeric Type Questions :

1. 6400 km. **2.** – 9.24 × 10–6 N

3. h = 0.41 × 6400 km = 2524 km

4. 97.4 N **5.** 9.05 × 106 m

6. 28 N **7.** 3.56 × 1022 N

4 ADVANCED EXERCISE
BASED ON CONNECTING TOPICS

1. (b) The craft is going in a circle of radius 6500 + 2100 = 8600 km.

Its centripetal acceleration is thus

$$a_c = \frac{v^2}{r} = \frac{(1200\text{ m/s})^2}{8.6\times10^6\,m} = 0.167\text{m/s}^2$$

This is the value of g at altitude. At the surface, it will be

$$g_0 = g\left(\frac{r}{r_0}\right)^2 = (0.167\text{m/s}^2)\left(\frac{8600\ km}{2100\ km}\right)^2 = 2.80\text{ m/s}^2$$

Then the weight of the pack at the surface is

w = mg = (60 kg) (2.80 N/kg) = 170 N

2. (d) The period of revolution of the satellite must be exactly one day, or 86400s. The centripetal acceleration of the satellite must be $4\pi^2 r/T^2$, the gravitational field must be $g = g_0\,(r_0/r)^2$. In free fall, $a = g$, so

$$\frac{4\pi^2 r}{T^2} = g_0\left(\frac{r_0}{r}\right)^2 \text{ so, } \frac{4\pi^2 r}{T^2} = g_0\left(\frac{r_0}{r}\right)^2$$

$$r = \sqrt[3]{\frac{(9.8\text{m/s}^2)\,(6.4\times10^6\,m)^2\,(86400s)^2}{4\pi^2}} = 4.23\times10^7\text{ m}$$

To get the altitude, subtract the radius of the earth. The satellite must be at an altitude of 36000 km.

3. (b) $$g = \frac{GM}{R^2} \Rightarrow g = \frac{G\left(\frac{4}{3}\pi R^3 D\right)}{R^2}$$

$$\Rightarrow D = \frac{g}{G\left(\frac{4}{3}\pi R\right)}$$

4. (c) If B is upthrust of air on balloon, and a is downward acceleration, then

$Mg - B = Ma$

$$\Rightarrow a = \frac{Mg - B}{M} = g - \frac{V\rho_{air}g}{V\rho_{CO_2}}$$

$$= \left(1 - \frac{V\rho_{air}}{V\rho_{CO_2}}\right)g = \left(1 - \frac{28.8}{44}\right)\times 9.8\text{ m/s}^2$$

$= 3.4$ m/s^2

5. (d)

6. (c) $$v_e = \sqrt{\frac{2GM_e}{R_e}};\quad v_m = \sqrt{\frac{2G\frac{M_e}{81}}{\frac{R_e}{4}}} = \frac{2}{9}v_e$$

$= 2/9 \times 11.2$ kms^{-1} $= 2.5$ kms^{-1}

7. (c) $V_g = \dfrac{-6.67\times10^{-11}\times10}{1} - \dfrac{6.67\times10^{-11}\times100}{1}$

$= -6.67 \times 10^{-10} - 6.67 \times 10^{-9}$

$= -6.67 \times 10^{-10} \times 11 = -7.3 \times 10^{-9}$ J/kg

8. **(b)** We know that, $v_e = \sqrt{(2gR)}$

$\therefore \dfrac{(v_e)_{P_1}}{(v_e)_{P_2}} = \dfrac{\sqrt{(2g_1R_1)}}{\sqrt{(2g_2R_2)}} = \sqrt{\left(\dfrac{g_1}{g_2}\right)} \times \sqrt{\left(\dfrac{R_1}{R_2}\right)}$

$= \sqrt{kr}$

9. **(a)** $v_{esc} = \sqrt{2gR}$, where R is radius of the planet.

Hence escape velocity is independent of m.

10. (c) $\dfrac{mv^2}{R+x} = \dfrac{GmM}{(R+x)^2}$

x = height of satellite from earth surface

m = mass of satellite

$\Rightarrow v^2 = \dfrac{GM}{(R+x)}$ or $v = \sqrt{\dfrac{GM}{R+x}}$

$T = \dfrac{2\pi(R+x)}{v} = \dfrac{2\pi(R+x)}{\sqrt{\dfrac{GM}{R+x}}}$

which is independent of mass of satellite.

11. **(c)** Reqd. ratio $= \dfrac{(GMm/2r)}{(GMm/r)} = \dfrac{1}{2}$

12. **(b, d)**

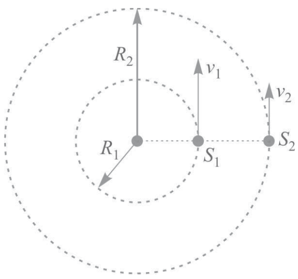
Orbital speed and time period are independent of mass of satellite.

13. (a) (A) $\to$ (p, q, r, t); (B) $\to$ (s); (C) $\to$ (s, t); (D) $\to$ (p, q, r)

14. (b) We have $T^2 \propto r^3$

$\therefore \left(\dfrac{T_1}{T_2}\right)^2 = \left(\dfrac{R_1}{R_2}\right)^3$

or $\left(\dfrac{1}{8}\right)^2 = \left(\dfrac{R_1}{R_2}\right)^3$



$\Rightarrow R_2 = 4R_1 = 4 \times 10^4$ km.

Let v_1 and v_2 be the linear speeds of S_1 and S_2 with respect to the planet. Then

$v_1 = \dfrac{2\pi R_1}{T_1} = 2\pi\times10^4\, km/h$

and $v_2 = \dfrac{2\pi R_2}{T_2} = \pi\times10^4$ km/h.

At the closest separation, they are moving in the same direction. Therefore the speed of S_2 with respect to S_1 is $|v_2 - v_1| = \pi \times 10^4$ km /h.

15. **(d)** As seen from S_1 the satellite S_2 is at a distance $r = R_2 - R_1 = 3 \times 10^4$ km. At the closest separation

$\omega = \dfrac{|v_2 - v_1|}{R_2 - R_1} = \dfrac{\pi\times10^4}{3\times10^4}$

$= \dfrac{\pi}{3}$ rad / h.

16. **(d)** B.E. = $\dfrac{GMm}{2r}$ depends on mass of the satellite.

17. **(a)**

18. The value of g at a height of geostationary satellite

$g' = \dfrac{g}{\left(1+\dfrac{h}{R}\right)^2} = \dfrac{g}{\left(1+\dfrac{36000}{6400}\right)^2}$

$= 0.023\, g.$

Thus weight of 10 N object at a height of geostationary satellite

$W = 0.023 \times 10 = 0.23$ N

19. Work done is given by $W = \vec{\mathbf{F}}\cdot\vec{\mathbf{s}} = \mathbf{m}\vec{\mathbf{E}}\cdot(\vec{\mathbf{s}}_2 - \vec{\mathbf{s}}_1)$

$= 2(10\hat{\mathbf{i}} + 10\hat{\mathbf{j}})\cdot(5\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 0)$

$= 180$ J

$\therefore$ Work done by external agent $= -180$ J

20. We know that $T = 2\pi\sqrt{\dfrac{r^3}{GM}}$ ...(i)

and $v = \sqrt{\dfrac{GM}{r}}$ ...(ii)

From above equations, we get

$T = \dfrac{2\pi GM}{v^3} = 225$ days

Chapter 4

WORK AND ENERGY

INTRODUCTION

In common sense, all physical activities such as reading, writing, carrying a bag, pushing a wall etc. are considered to be work. Let, for example, you employ a worker to push and move a high wall of a building. He tries his best whole day but he doesn't become able to move the wall a little bit. But he asks for his labouring charge. Would you pay him the charge or not? Definitely, your answer would be 'yes' because he has performed his task or work. But in language of physics he hasn't done any work.

The meaning of work in physics is different from its meaning in common language. Actually, in physics work has a meaning only when a displacement is caused on a body by the applied force on it. If there is no displacement in a body by an applied force, no work is said to be done on the body by the force.

Suppose Pari gets tired after four hours of study but Dolly doesn't tire even after eight hours of study. Who does more work-Pari or Dolly? We can't answer it. But we can surely say that Dolly has more capacity of doing work than Pari. The capacity of doing work is defined as the 'energy'.

This chapter is all about work and energy.

WORK

In physics, if force applied on object displaces the object in the direction of force then the work is said to be done. Here all three terms force, displacement and direction of force are important. If force is zero, work is zero; if force is non-zero but displacement is zero (like pushing the wall) work is zero and if force is non-zero, displacement is non-zero but no part of force is in the direction of displacement, work is zero. Hence, we define the work as *the product of the force and displacement in the direction of applied force or product of displacement and force in the direction of displacement*.

W = Force × displacement in the direction of force = F.S = FScos θ

where θ is the angle between F and S.

Note that the force in above formula is the component of force in the direction of displacement.

Unit of Work

The **SI unit** of force is newton and the unit of length is metre (m). So, the **SI unit** of work is newton-metre which is written as Nm. This unit (Nm) is also called **joule (J)**, i.e. 1 joule = 1 newton . 1 metre

Abbreviated as 1 J = 1 Nm

When a force of 1 newton moves a body through a distance of 1 metre in its own direction the work done is 1 joule.

Other units of work

In **c.g.s. system** of measurement force is measured in dyne, displacement in cm, and work is measured in **erg**.

From $W = Fs$

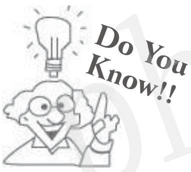
If $F = 1$ dyne, $s = 1$ cm, then $W = 1$ erg.

Thus, if a force of 1 dyne, displaces the point of application, by 1 cm, in the direction of force, then work done by the force is said to be 1 dyne.

Relation between 1 joule and erg is as follows.

1 joule = 1 N × 1m = 10^5 dyne × 10^2 cm = 10^7 erg

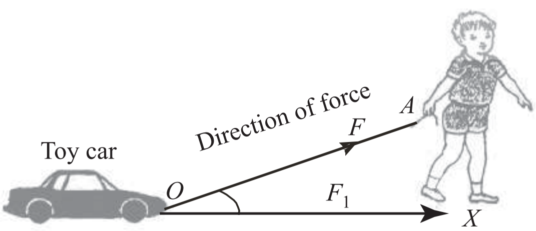
Higher units of work are kJ = 10^3 joule and 1 MJ = 10^6 joule

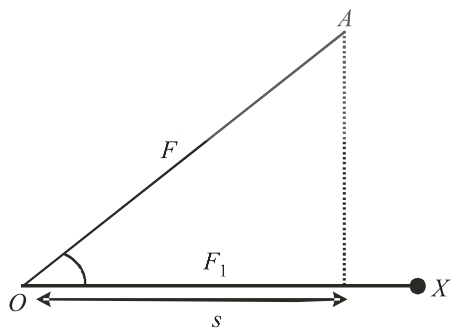


If nothing is actually moving, no work is done–no matter how great the force involved!

WORK DONE BY A FORCE APPLIED AT AN ANGLE

Let Rishabh pulls a toy car through a string, as shown in figure. The force applied by him is along the string (direction OA whereas the toy car moves horizontally (direction OX). The pulling force (F) makes an angle θ with the displacement of car. In this case only a part of the force, say F_1, which acts along the horizontal direction is being actually used for the motion of the car. In such a case the amount of work done by the force on the car is defined as the product of its component along the motion and the magnitude of displacement.

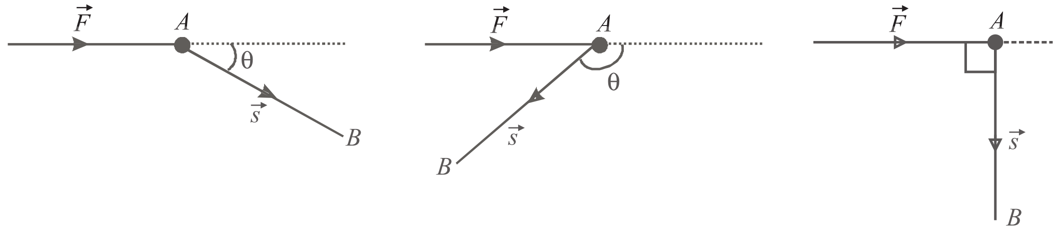




W = component of force in the direction of displacement × magnitude of displacement = F cos θ s

∴ Work done by a force can be positive, negative or zero as the value of cos θ is positive, negative or zero.

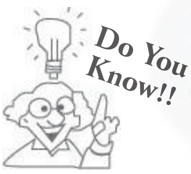
(∵ *F* and *s*, being magnitudes, are always positive)



W = +ve for θ = acute angle W = –ve for θ = obtuse angle W = 0 for θ = 90°

Thus, if the displacement (of the point of application) has a component along the direction of applied force, then work done is positive. On the other hand, if the displacement has a component opposite to the direction of applied force, then work done will be negative. And if the applied force and particle's displacement be mutually perpendicular, then work done by the force on the particle is zero. (cos 90° being zero).

It is important to understand that work done by the force does not depend on the time taken in the displacement of point of action. For example, one porter takes 5 minutes to put a box on the roof of a bus while other put the same box on the roof in 10 minutes, work done by both the porter is same.



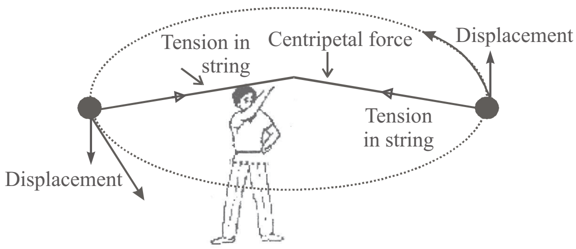
Work is a scalar quantity but you can have positive and negative work. Positive work is where the force pulls in the same direction as the movement. Negative work is where the force is in the opposite direction.

Example of zero work or no work, positive and negative work :

Zero work :

(i) A coolie with a luggage on his head, moving on a horizontal platform, does no work, since the direction of force is vertically up and displacement horizontal (even though he might feel physically tired).

(ii) A body attached to a string revolves in a horizontal circle (figure). The tension *T* in the string does no work on the body, because it has no component in the direction of displacement. In general, for a body moving with uniform speed the centripetal force is always perpendicular to displacement, hence no work will be done by this force.



(iii) If the wall doesn't move, the person does no work.

(iv) If a boy tries to push a heavy boulder, by applying a force, but unable to displace it, then work done by the boy is zero.

Positive work:

(v) When a horse pulls a cart, the applied force and the displacement are in the same direction. So, work done by the horse is positive.

Negative work:

(vi) When brakes are applied to a moving vehicle, the work done by the braking force is negative. This is because the braking force and the displacement act in opposite directions.

(vii) When a spring is compressed then the force applied by the spring and the displacement will be in opposite to each other, so work done by the spring will be negative.

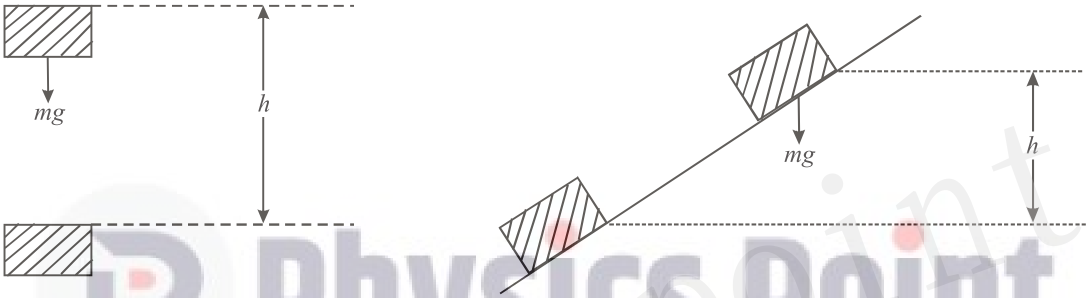
Work Done Against Gravity

Consider a body of mass m which is raised a vertical distance h.

The work done by the weight is – mgh. mgh is called the work done against gravity.

If an agent, such as crane, is responsible for lifting the body, then mgh is referred to as the work done by the crane against gravity.

Similarly if a vehicle of mass m climbs a hill, and in doing so raises itself a vertical distance h, then mgh is called the work done by the vehicle against gravity.

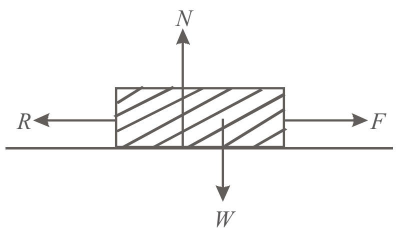


Work Done by a Moving Vehicle

The diagram shows the forces that commonly act on a moving vehicle. R is the resistance to motion (this is always in the direction opposite to the direction of motion) and F is the driving force of the engine.

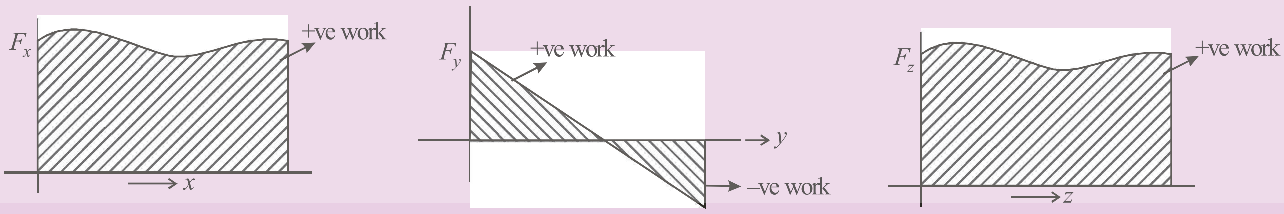
The work done by F is referred to as the work done by the vehicle.

If the vehicle is not accelerating, the forces acting on it are in equilibrium.



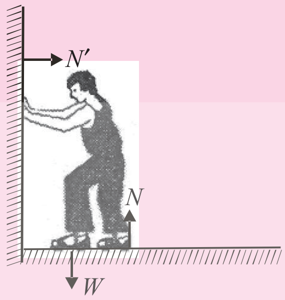
Knowledge ENHANCER

(i) Area under force-displacement graph gives work done.



The work done can be positive or negative as per the area above the x-axis (positve) or below the x-axis (negative) respectively.

(ii) Internal work : Suppose that a man sets himself in motion backward by pushing against a wall. The forces acting on the man are his weight '*W*', the upward force *N* exerted by the ground and the horizontal force *N*' exerted by the wall. The works of '*W*' and of *N* are zero because they are perpendicular to the motion. The force *N*' is the unbalanced horizontal force that imparts to the system a horizontal acceleration. The work of *N*', however, is zero because there is no motion of its point of application. We are, therefore, confronted with a curious situation in which a force is responsible for acceleration, but its work, being zero, is not equal to the increase in kinetic energy of the system.



The new feature in this situation is that the man is a composite system with several parts that can move in relation to each other and thus can do work on each other, even in the absence of any interaction with externally applied forces. Such work is called internal work. Although internal forces play no role in acceleration of the composite system, their points of application can move so that work is done; thus the man's kinetic energy can change even though the external forces do no work.

CHECK Point

Figure shows four situations in which a force acts on a box while the box slides rightward a distance d across a frictionless floor. The magnitudes of the forces are identical, their orientations are as shown. Rank the situations according to the work done on the box during the displacement, from most positive to most negative.

Solution

We know that the positivity or negativity of work depends upon the angle between the force and the displacement caused by the force. The work done becomes less positive as θ increases from 0º to 90º and becomes negative afterwards upto 270°. Hence, the correct order is D, C, B, A

ILLUSTRATION : 1

How much work is done by a force of 250 N in moving an object through a distance of 100m in the direction of force ?

SOLUTION :

The work done is calculated by the formula

$$W = F.s = 250 \times 100$$
$$= 25 \times 10^3 \text{ joule}$$

ILLUSTRATION : 2

A body of mass 2 kg is raised to a height of 1m. Find the work done by the force of gravity (Take g = 9.8 m/s^2).

SOLUTION :

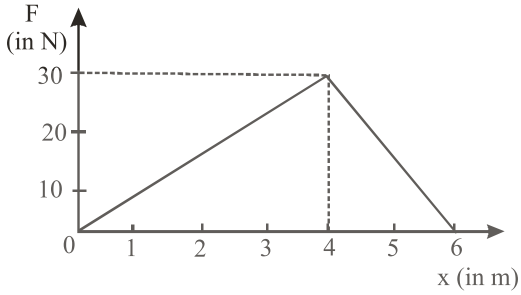
The force of gravity on the body, is the force exerted by earth on it and is *mg* (weight) acting vertically down. However, the displacement *s* = 1m vertically up.

Now from $W = Fs\cos\theta = mg\cos\theta$

$$W = 2 \times 9.8 \times 1 \times \cos 180°$$
$$= -19.6 \text{ joule} \quad [\because \cos 180° = -1]$$

ILLUSTRATION : 3

From the graph of force versus displacement of a particle, find the total work done by the force.



SOLUTION :

The total work done = total area under the $(F-x)$ curve

$$=\frac{1}{2}\times 4\times 30+\frac{1}{2}\times 2\times 30$$

$$=\frac{1}{2}\times 6\times 30=90 \text{ joule}$$

ILLUSTRATION : 4

A force of 10N displaces a body by 5m, the angle between force and displacement is 60°, then find the work done.

SOLUTION :

Force, $F = 10\text{N}$, displacement, $d = 5\text{m}$

Angle between force and displacement, $\theta = 60°$

Work done, $W = Fd\cos\theta = 10 \times 5 \times \cos 60°$

or, $W = 10 \times 5 \times \frac{1}{2} = 25$ joule $(\because \cos 60° = 1/2)$

ENERGY

Some people have a lot of energy when they get up in the morning. Carbohydrates are high-energy foods. Oil is the main source of energy that keeps industry and cars going.

The word energy has a different meaning in physics, the word energy has a very precise meaning, although it is a little difficult to define because energy takes many different forms. We can approach a definition by noting the relationship between energy and work. *Energy is defined as the capacity to do work.*

In other words, anything which has the capacity to do work is said to possess energy. This implies that work can be done only at the expense (cost) of energy i.e., to do work, we need to spend energy, whatsoever be its form.

Its **S.I unit** is same as that of work i.e., joule (J).

idea box

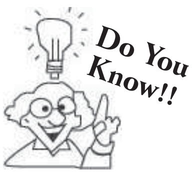
Energy and work are one and same, but used in different context e.g. it will be wrong to give a statement as "I have done 50 joule of energy". The correct statement is "I have done 50 joule of work". Similarly it would be incorrect to say that "The body possesses 100 joule of work". The correct statement would be to say that "the body possesses 100 joule of energy".

Let us understand energy-work equivalence with some examples

(i) When a fast moving cricket ball hits a stationary stump, the stump is thrown away. Here the work is done on the stump by the ball and the ball has the capacity to do this work because of its motion (kinetic energy).

(ii) A body can acquire the ability to do work when it is deformed temporarily. For example, a compressed watch spring is able to drive the wheels of a watch.

(iii) If a boy (mass = m) climbs upstairs to a height (h) then work done by him would be mgh and consequently he would have lost mgh joule of energy.



Energy is a promise of work to be done in future. It is the stored ability to do work.

MECHANICAL ENERGY

The energy in a body may be by virtue of its motion (kinetic energy) or by virtue of its position (potential energy). Energy in a body due to these conditions is called mechanical energy. For example, energy of water in a water tank on the roof, energy of moving bullet, energy of small spring in ball-pen, energy of moving air etc. are the forms of mechanical energy.

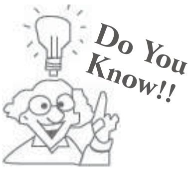
Kinetic Energy

Energy possessed by a body by virtue of its state of motion is called kinetic energy. Kinetic energy is always positive and is a scalar. The fact, that moving bodies carry energy with them is proved by some of the several happenings in day to day life.

Examples :

(i) A stone thrown with some velocity, breaks the window pane.

(ii) A moving vehicle, when accidently happens to collide with another vehicle at rest or motion, leads to destruction.

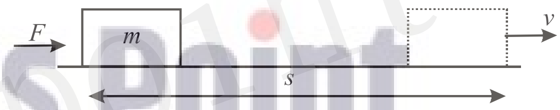


The Kinetic energy of a body is always positive.

Expression of kinetic energy

Suppose that a constant force F is applied on a body of mass m. Its velocity becomes v in a displacement s, then according to Newton's 3rd equation of motion $v^2 = u^2 + 2$ as

$$v^2 = 0 + 2\left(\frac{F}{m}\right)s \quad \text{or} \quad (F) = m\left(\frac{v^2}{2s}\right)$$



Work done by force F in displacing the body by a distance s in the direction of force

$$W = F.s = m\left(\frac{v^2}{2s}\right)s \quad \text{or} \quad W = \frac{1}{2}mv^2$$

This work done by the force which makes a stationary body to move with a velocity v, is measured as its kinetic energy

i.e. *Kinetic energy* $K = \frac{1}{2}mv^2$

From this expression it is clear that the kinetic energy possessed by a moving body is directly proportional to its mass and to the square of its velocity, if velocity is doubled KE becomes 4 times.

Knowledge ENHANCER

Relation between kinetic energy and momentum :

A body of mass m is moving with velocity v, then the momentum of the body, $p = mv$

The kinetic energy of the body, $K = \frac{1}{2}mv^2$

or $K = \frac{1}{2}mv^2 \times \frac{m}{m}$ or $K = \frac{1}{2}\frac{m^2v^2}{m}$ (here $m^2v^2 = p^2$)

$\therefore$ $K = \frac{p^2}{2m}$ or $p = \sqrt{2mK}$

(i) For same momentum : K-energy varies inversely as the mass $K \propto \frac{1}{m}$

(ii) For same K-energy : Momentum varies directly as the square root of mass of the body $p \propto \sqrt{m}$

CHECK Point

(i) If a golf ball and a Ping-Pong ball both move with the same kinetic energy, can you say which has the greater speed? Explain in terms of K.E. Similarly, in a gaseous mixture of massive molecules and light molecules with the same average KE, can you say which has the greater speed?

(ii) How is it possible that a flock of birds in flight can have a momentum of zero but not have zero kinetic energy?

Solution

(i) The kinetic energy of an object depends on the mass and square of the velocity. As the kinetic energy of a golf ball and a ping-pong ball is the same therefore a ping-pong ball has greater speed because its mass is less than a golf-ball. Similarly, in a gaseous mixture of massive molecules and light molecules with the same average K.E, the light molecules have the greater speed.

(ii) Momentum is a vector quantity having both magnitude and direction whereas kinetic energy is a scalar quantity having only magnitude. Momentum that is directional is capable of being cancelled entirely.
The vector sum of the momenta of a flock of birds in flight can be zero because of birds flying in different directions in the flock.
Each flying bird has some kinetic energy and the algebraic addition of the kinetic energies of all birds in the flock cannot be zero.

ILLUSTRATION : 5

If a stone of mass 3 kg be thrown with a kinetic energy of 37.5 joule, find its velocity.

SOLUTION :

Here, mass m = 3kg; K.E. = 37.5J; velocity of stone, v = ?

From K.E. $=\frac{1}{2}mv^2$

$\Rightarrow \quad 37.5 = \frac{1}{2} \times 3v^2 \quad \Rightarrow v^2 = \frac{75}{3} = 25 \quad \Rightarrow v = 5 \text{ m/s}$

ILLUSTRATION : 6

A bullet is fired from a gun. What will be the ratio of kinetic energy of bullet and gun ?

SOLUTION :

When a bullet is fired from a gun, the gun has same momentum backward, which the bullet has a forward momentum

$\because \quad K \propto \frac{1}{m}$

$$\frac{\text{K.E. of bullet}}{\text{K.E. of gun}} = \frac{\text{mass of gun}}{\text{mass of bullet}} = \frac{M}{m}$$

So bullet has more Kinetic energy than the gun.

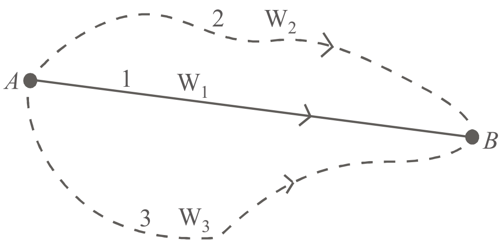
CONNECTING TOPIC

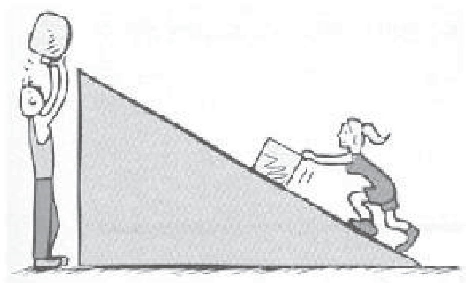
CONSERVATIVE AND NON-CONSERVATIVE FORCES

Nature has been gifted by several types of forces, coming into operation, now and then, as and when possible. To name a few, the wind force, the force of friction, the viscous drag, the gravitational force, the electric force, etc.
The forces which always tend to oppose the motion e.g., frictional forces, viscous forces, etc. always tend to dissipate energy when a body moves relative to another, in their presence i.e., work has always to be done against them, to move a body from one position to another, which cannot be recovered, after regaining the original position. These are hence known as non-conservative

forces. At the same time, there are several forces which are conservative in nature and in which energy can be recovered, provided, the body is restored to its original configuration. Commonly encountered such forces are gravitational, elastic forces.
Hence we can define conservative and non-conservative forces as follows.
1st definition : *If the work done by a force in moving a body from an initial location to a final location is independent of the path taken between the two points, then the force is conservative, otherwise it is non-conservative.*

2nd definition : If a body moves under the action of a force that does no net work during any round trip, then the force is conservative, otherwise it is non–conservative.





$W_1 = W_2 = W_3$ for a conservative force.
Both blocks acquire the same gravitational potential energy, *mgh*. The same work is done on each block.

Conservative Force and Work done

As we have already discussed in the previous chapter that work done by a conservative force doesn't depend upon the path taken i.e. it only depends upon the initial and final positions of the body. There is another important very useful relation between work done by conservative force and potential energy. The work done by a conservative force is equal to the negative of change in the potential energy.

$$W_c = -(U_f - U_i) = -\Delta U$$

U_f = final potential energy; U_i = initial potential energy

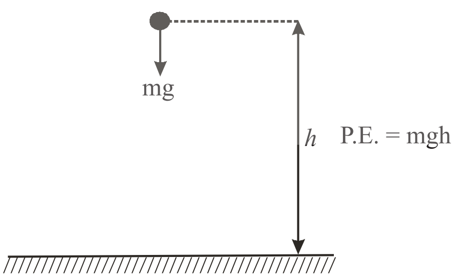
or, $F_c \Delta x = -\Delta U \Rightarrow F_c = -\dfrac{\Delta U}{\Delta x}$

or $F_c = -\dfrac{dU}{dx}$ or, $U_f - U_i = -\int_i^f \vec{F}.\vec{dr}$

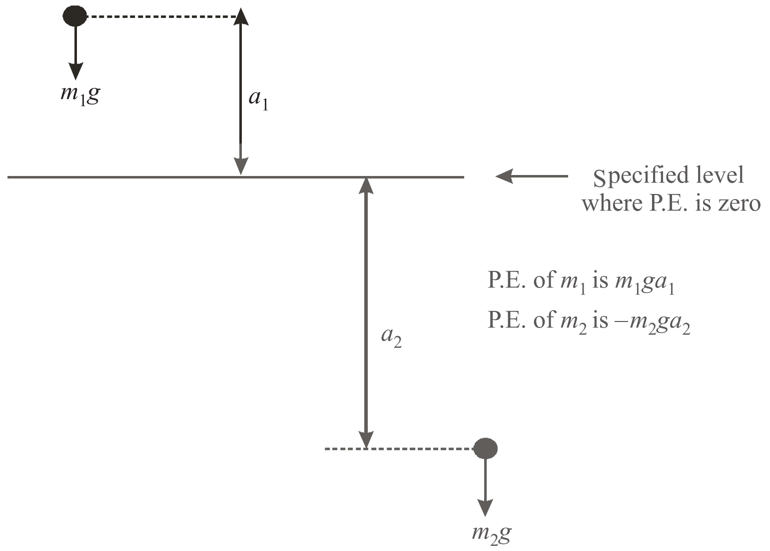
POTENTIAL ENERGY

Potential energy is energy due to position. If a body is in a position such that if it were released it would begin to move, it has potential energy. There are two common forms of potential energy, gravitational and elastic.

(i) Gravitational Potential Energy



When an object is allowed to fall from higher level to a lower level it gains speed due to gravitational pull, i.e. it gains kinetic energy. Therefore, in possessing height, a body has the ability to convert its height into kinetic energy, i.e. it possesses potential energy.
The magnitude of its gravitational potential energy is equivalent to the amount of work done by the weight of the body in causing the descent.
If a mass *m* is at a height *h* above a lower level, the P.E. possessed by the mass is (*mg*) (*h*).
Since *h* is the height of an object above a specified level, an object below the specified level has negative potential energy.



NOTE: The chosen level from which height is measured has no absolute position. It is, therefore, important to indicate clearly the zero P.E. level in any problem in which P.E. is to be calculated.

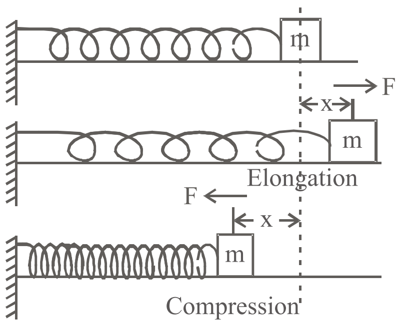
IDEA BOX — idea box

The potential energy of a body may be positive or negative.

Do You Know!!

It does not matter, what path is taken; work done to move a body between any two points in a vertical plane is equal to work done against gravity through the vertical distance between the points!

(ii) Elastic Potential Energy



This is a kind of potential energy which is due to a change in the shape of a body. The change in shape of a body can be brought about by stretching, compressing, bending and twisting the body. Some work has to be done to change the shape of a body. This work gets stored in the deformed body in the form of elastic potential energy.

For example, the energy stored in a stretched rubber band or a spring is elastic potential energy and is equal to the work done in stretching the rubber band or spring. When this deformed body is released, it attains its original shape and the potential energy is converted into some other form, usually in kinetic energy. Elastic potential energy is never negative whether due to extension or to compression.

Consider a massless spring whose length is ℓ on its natural state. Now it is elongated by 'x' then the work done by external force will be $\frac{1}{2}kx^2$.

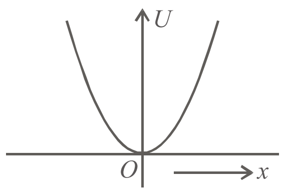
Similarly, if the spring is compressed by 'x' then also the work will be $\frac{1}{2}kx^2$ only.

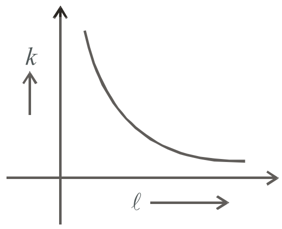
So, the energy associated with the state of elongation or compression of a spring is called elastic potential energy of spring or simply potential energy (U).

$$U = \frac{1}{2}kx^2$$

Where 'k' is force constant of spring. It is also called spring factor, its unit is N/m or dyne/cm. Remember spring contant (k) is inversely proportional to the lengths of spring (ℓ)

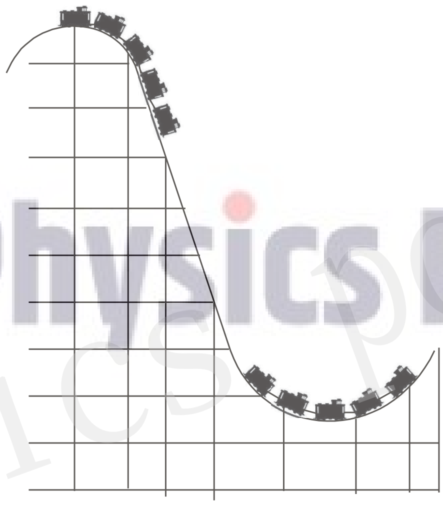
i.e., $k \propto \frac{1}{\ell}$ or $k\ell$ = constant





CHECK Point

In which car will you be moving the fastest at the very bottom of the incline?



Solution

Of course, at any moment all cars have the same speed but the gravitational potential energy of the system of cars is least when the middle car is at the bottom—when the centre of mass of the system is lowest. Lowest potential energy means highest kinetic energy. So sit in the middle car for the fastest ride at the very bottom.

LAW OF CONSERVATION OF ENERGY

According to this law, *energy can only be converted from one form to another, it can neither be created nor destroyed*. The total energy before and after the transformation always remains the same.

Conservation of Mechanical Energy

Kinetic and potential energy are both forms of mechanical energy. The total mechanical energy of a body or system of bodies will be changed in value if :

(a) an external force other than weight causes work to be done (work done by weight is potential energy and is therefore already included in the total mechanical energy),

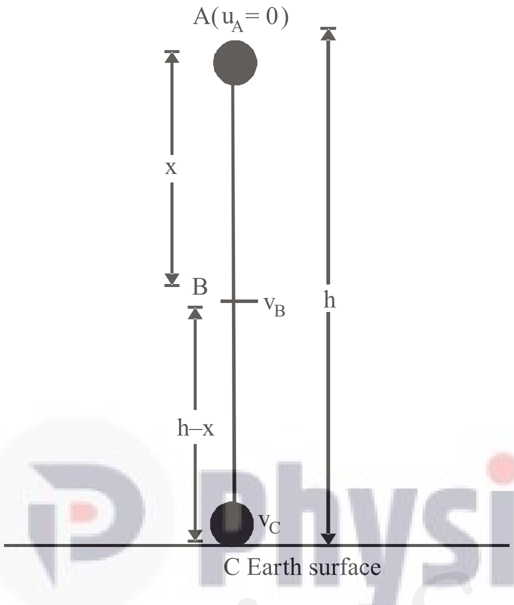
(b) some mechanical energy is converted into another form of energy (e.g. sound, heat, light etc.). Such a conversion of energy usually takes place when a sudden change in the motion of the system occurs. For instance, when two moving objects collide some mechanical energy is converted into sound energy which is heard as bang at impact. Another common example is the conversion of mechanical energy into heat energy when two rough objects rub against each other.

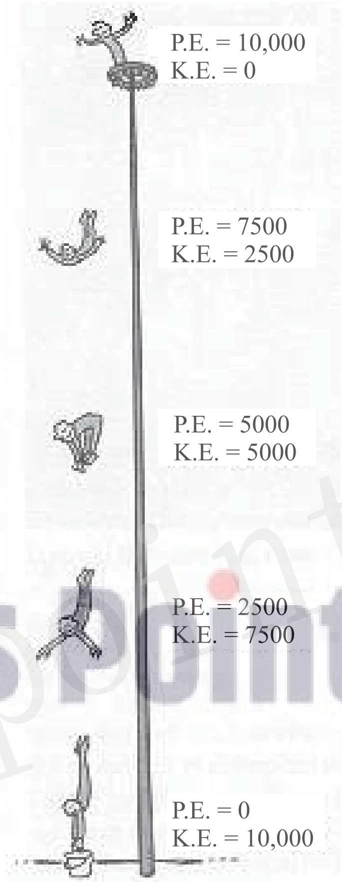
If neither (a) nor (b) occurs then the total mechanical energy of a system remains constant. This is the principle of conservation of Mechanical Energy can be expressed in the form :

The total mechanical energy (K.E. + P.E.) of a system remains constant provided that no external work is done and no mechanical energy is converted into another form of energy.

Examples :

(1) Body falling freely : Considering earth as a body of zero mechanical energy, if a body of mass *m* is situated at point *A* whose height from earth surface is *h*, then the total mechanical energy of the body/system will be potential energy only. i.e. $E_A = mgh$





Now, as body falls freely from point *A*, its velocity increases due to acceleration due to gravity. At the point B, body possesses both K.E. and P.E.

So, potential energy at *B* : $U_B = mg\,(h - x)$

Kinetic energy at *B* : $K_B = \frac{1}{2}mv_B^2$

Total mechanical energy at *B* : $E_B = U_B + K_B$

or $E_B = mg\,(h - x) + \frac{1}{2}mv_B^2$

or $E_B = mg\,(h - x) + \frac{1}{2}m.2gx$ or $E_B = mgh$

When the body reaches at point *C* on earth's surface, *h* becomes zero. Therefore, the potential energy (U_c) at *C* is zero. Total mechanical energy at *C* is K.E. only which equals the work done by gravitational force from *A* to *C*. Therefore,

$$E_C = 0 + \frac{1}{2}mv_C^2$$

or $E_C = 0 + \frac{1}{2}m.2gh$ or $E_C = mgh$

Thus, the total mechanical energy remains conserved at every point on the path of a freely falling body.

(2) When the pendulum is pulled to position C, it gains height.
At position C, it has maximum potential energy and zero kinetic energy, as the pendulum is held by hand in position C.
When the pendulum is released from position C, it moves towards position A. In doing so, its velocity increases, due to the increase in velocity, its kinetic energy increases, at the expense of potential energy. At position A, it has maximum kinetic energy and zero potential energy, as it is at its lowest position. (we can call it as reference level)
When the pendulum swings from A to B, it again gains height and hence, its potential energy increases. However, due to gain in height, its velocity decreases and hence, the kinetic energy decreases. At position B, it has maximum potential energy and zero kinetic energy, as pendulum comes to rest at B for a moment, before swinging back to position A.
Hence in the system of pendulum and earth, the energy is conserved. It is the potential energy, which changes to the kinetic energy and vice versa.

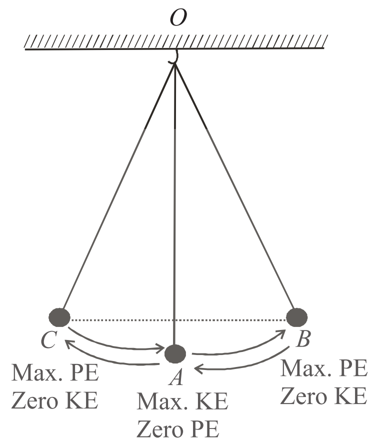


ILLUSTRATION : 7

A mass of 10 kg is released from a height of 40m above the surface of the earth. Calculate its mechanical energy when it is at 20m above the surface of the earth. (Take g = 10 m/s^2)

SOLUTION :

Here, initially the body is at rest $v = u = 0$; $m = 10$ kg, $h = 40$m and $g = 10$ m/s²

The initial energy of the body $(U + K) = mgh + \frac{1}{2}mv^2$

$\therefore$ initial energy $= 10 \times 10 \times 40 + \frac{1}{2} m \times 0 = 4000$ J

When the body is at a height of $h_1 = 20$m above the ground, it has fallen by a distance of $s_1 = (40 - 20) = 20$m from its initial position, at this position it will have both the potential and kinetic energy.

At this position, kinetic energy $K = \frac{1}{2}mv_1^2$

From $v_1^2 = u^2 + 2gs_1$; $v_1^2 = 2gs_1$

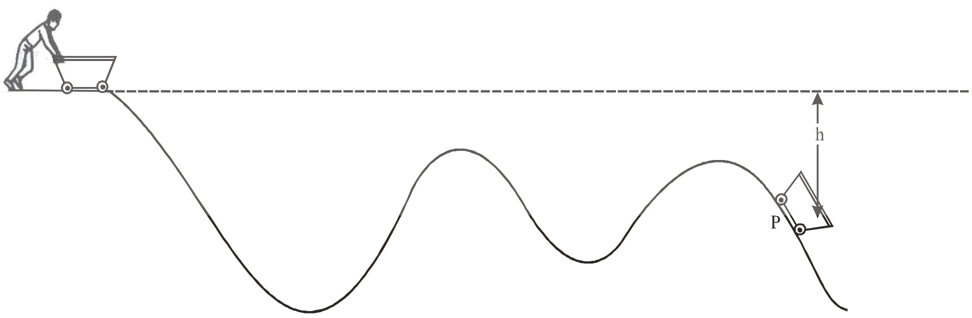
$$K = \frac{1}{2}m \times 2 \times g \times s_1 = \frac{1}{2} \times 10 \times 2 \times 2 \times 10 \times 20$$

or $K = \frac{1}{2} \times 10 \times 400 = 2000$ J

Here the body is at height h_1 above the surface of earth hence the (gravitational) potential energy
$U = mgh_1 = 10 \times 10 \times 20 = 2000$ J
$\therefore$ Total energy $= K + U = 2000 + 2000 = 4000$ J

ILLUSTRATION : 8

Starting at rest, the cart of figure slides frictionlessly to point P, which is 4.5 meters below the top of the hill. How fast is it going at P ?



SOLUTION :

Considering its gravitational energy to have dropped to zero at P, its gravitational energy when it is at the top of the hill is $(mgh)_{\text{start}}$. Its kinetic energy starts at zero and increases to $\left(\frac{1}{2}mv^2\right)_{\text{end}}$. Since its total energy does not change,

$$(mgh)_{\text{start}} = \left(\frac{1}{2}mv^2\right)_{\text{end}}$$

The m's drop out, and the equation becomes

$$v = \sqrt{2gh} = \sqrt{2(9.8)\,(4.5)} = 9.4 \text{ m/s}$$

WORK-ENERGY THEOREM

According to the work-energy theorem, *total work done on a system by forces (external, internal, conservative or non-conservative) equals the change in kinetic energy.*

$$W_c + W_{nc} + W_{ext} = K_f - K_i$$

Where, W_c = Work done by conservative forces

W_{nc} = Work done by non-conservative forces

W_{ext} = Work done by external forces

K_f = Final kinetic energy

K_i = Initial kinetic energy.

But work done by conservative forces equals the negative of change in potential energy,

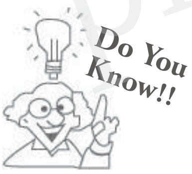
$$W_c = -(U_f - U_i)$$

So, $W_{nc} + W_{ext} = (K_f + U_f) - (K_i + U_i)$

$W_{nc} + W_{ext} = ME_f - ME_i$ (ME – Mechanical energy)

If $W_{nc} = 0$; then $W_{ext} = ME_f - ME_i$

So, if non-conservative forces do not act then work done by external forces equals the change in mechanical energy and if there is no external force acting then mechanical energy of the system remains conserved.



Work-energy theorem is particularly useful in calculation of minimum stopping force or minimum stopping distance. If a body is brought to a halt, the work done to do so is equal to the kinetic energy lost.

ILLUSTRATION : 9

A bullet leaving the muzzle of a rifle barrel with a velocity v penetrates a plank and loses one fifth of its velocity. It then strikes second plank, which it just penetrates through. Find the ratio of the thickness of the planks supposing average resistance to the penetration is same in both the cases.

SOLUTION :

Let R = resistance force offered by the planks, t_1 = thickness of first plank, t_2 = thickness of second plank.

For first plank :

Loss in KE = Work against resistance

$$\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{4}{5}v\right)^2 = Rt_1 \Rightarrow \frac{1}{2}mv^2\left(\frac{9}{25}\right) = Rt_1 \qquad ......(1)$$

For second plank :

$$\frac{1}{2}m\left(\frac{4}{5}v\right)^2 - 0 = Rt_2 \Rightarrow \frac{1}{2}mv^2\left(\frac{16}{25}\right) = Rt_2 \quad ......(2)$$

Dividing eq. (1) and (2) $\frac{t_1}{t_2} = \frac{9}{16}$

ILLUSTRATION : 10

A particle of mass 0.5 kg travels in a straight line with velocity $v = ax^{3/2}$ where a = 5 $m^{-1/2}\ s^{-1}$. What is the work done by the net force during its displacement from $x = 0$ to $x = 2$m ?

SOLUTION :

$m = 0.5$ kg, $v = ax^{3/2}$, $a = 5\ m^{-1/2}\ s^{-1}$, $W = ?$

Initial velocity at $x = 0$, $v_0 = a \times 0 = 0$

Final velocity at $x = 2$, $v_2 = a \times 2^{3/2} = 5 \times 2^{3/2}$

Work done = increase in kinetic energy $= \frac{1}{2}m(v_2^2 - v_0^2) = \frac{1}{2} \times 0.5[(5 \times 2^{3/2})^2 - 0] = 50$ J

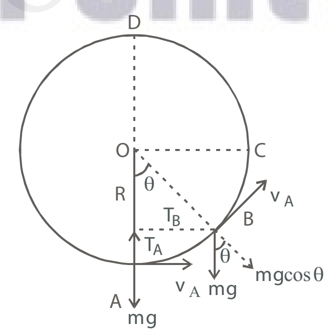
CONNECTING TOPIC

MOTION IN A VERTICAL CIRCLE

Consider a particle of mass 'm' tied to an inelastic, massless string of length R moving in a vertical circle as shown in figure.

At A : $T_A - mg = \frac{mv_A^2}{R}$ (Newton's second law $\sum f = ma$, here $a = \frac{v^2}{R}$)

$$\Rightarrow T_A = mg + \frac{mv_A^2}{R} \quad ...... (i)$$



At B : $T_B - mg\cos\theta = \frac{mv_B^2}{R}$

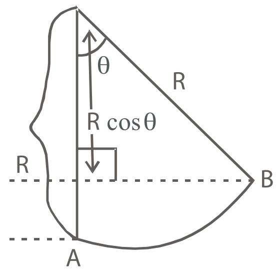
$$\Rightarrow T_B = mg\cos\theta + \frac{mv_B^2}{R} \quad .... (ii)$$

Applying conservation of mechanical energy between A and B

$$\frac{1}{2}mv_A^2 = \frac{1}{2}mv_B^2 + mgR(1-\cos\theta)$$

$$v_A^2 = v_B^2 + 2gR(1-\cos\theta) \quad \{\text{Taking '}A\text{' as reference line of zero } PE\}$$

$$\Rightarrow V_B^2 = V_A^2 - 2gR\,(1 - \cos\theta)$$



Substituting value of V_B^2 in eq. (ii), we get

$$T_B = \frac{mv_A^2}{R} - 2mg + 3mg\cos\theta \quad .... (iii)$$

Using equation (iii), tension at any point can be found by simply substituting the value of angle 'θ'.

At C : $\theta = 90°$, $T_c = \frac{mv_A^2}{R} - 2mg \quad (v_c^2 = v_A^2 - 2gR)$

$$\Rightarrow T_c = \frac{mv_c^2}{R}$$

At D : $\theta = 180°$, $T_D = \frac{mv_A^2}{R} - 5mg \quad (v_c^2 = v_A^2 - 4gR)$

$$\Rightarrow T_D = \frac{mv_D^2}{R} - mg$$

Condition for just completing vertical circle :

Using $T_D \geq 0$; we get $v_D = \sqrt{Rg}$; $v_A = \sqrt{5Rg}$ and $v_c = \sqrt{3Rg}$

Similarly, $T_A = 6mg$; $T_c = 3mg$

Condition for leaving circle :

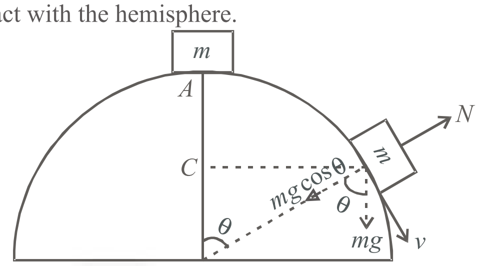
$\sqrt{2Rg} < v_A < \sqrt{5Rg}$ when $90° < \theta < 180°$

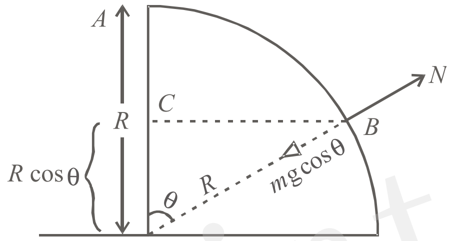
At this point tension will be zero but velocity will not be zero.

Condition for oscillation :

When $0 < v_A < \sqrt{2Rg}$ i.e. $0 < \theta < 90°$, the particle will oscillate.

Motion of a body on a smooth hemisphere : Consider a small body of mass '*m*' initially at rest kept over a smooth hemisphere of radius '*R*'. It is given a little push and it starts sliding over the surface of hemisphere and at a point '*B*' it leaves off the contact with the hemisphere.





Using Newton's 2nd law at *B*

$$mg\cos\theta - N = \frac{mv^2}{R} \quad \text{..... (i)}$$

Applying conservation of energy between *A* and *B*

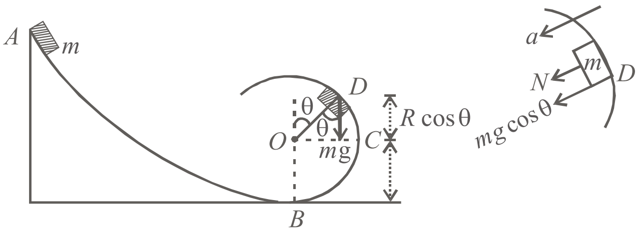
$$mgR = mgR\cos\theta + \frac{1}{2}mv^2 \quad \text{..... (ii)}$$

When body leaves the contact, $N = 0$

$$mg\cos\theta = \frac{mv^2}{R}, \text{ using eq. (ii)}$$

$$\cos\theta = \frac{2}{3} \text{ or, } \theta = 48.2° \text{ and } v = \sqrt{\frac{2gR}{3}} \text{ and AC} = \frac{R}{3}$$

Looping the loop : Consider a smooth inclined plane of vertical height *H* which is connected to a smooth vertical circular track of radius '*R*'. A mass '*m*' slides down from the point *A* and moves along the circular track and at point '*CD*' leaves the contact from the track with speed *v*.



Using Newton's 2nd law at *D*, $N + mg\cos\theta = m\left(\frac{v^2}{R}\right)$

As it leaves the contact, $N = 0$

$$\text{So, } mg\cos\theta = \frac{mv^2}{R} \quad \text{.... (i)}$$

Applying conservation of mechanical energy at A and D,

$$mgH = mg(R + R\cos\theta) + \frac{1}{2}mv^2 \quad \text{.... (ii)}$$

Solving eqs. (i) and (ii),

$$v = \sqrt{\frac{2g(H-R)}{3}}$$

$$\text{Also, } \cos\theta = \frac{2(H-R)}{3R}$$

ILLUSTRATION : 11

A small sphere tied to the string of length 0.8m is describing a vertical circle so that the maximum and minimum tensions in the string are in the ratio 3 : 1. The fixed end of the string is at a height of 5.8 m above ground.

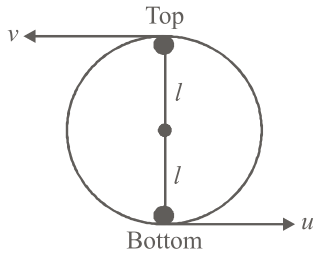
(a) Find the velocity of the sphere at the lowest position.

(b) If the string suddenly breaks at the lowest position, when and where will the sphere hit the ground? (Take $g = 10\ m/s^2$)

SOLUTION :

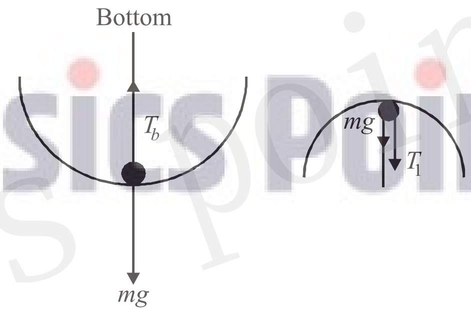
(a) Let u and v be the speeds of sphere at the bottom and the top positions and m be the mass.

Radius of circle = length of string = r = 0.8 m



T_b = tension at the bottom or the maximum tension

T_t = tension at the top or the minimum tension



$$T_b - mg = \frac{mu^2}{r}$$

$$T_t + mg = \frac{mv^2}{r}$$

$$\Rightarrow T_b = 3T_t$$

$$\left(\frac{mu^2}{r} + mg\right) = 3\left(\frac{mv^2}{r} - mg\right)$$

$$(3v^2 - u^2) = 4rg \qquad \text{...(i)}$$

Using conservation of energy, loss in KE from bottom to the top = gain in GPE

$$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = mg(2r)$$

$$\Rightarrow \quad v^2 = u^2 - 4rg \qquad \text{...(ii)}$$

Using eqs. (i) and (ii), we get

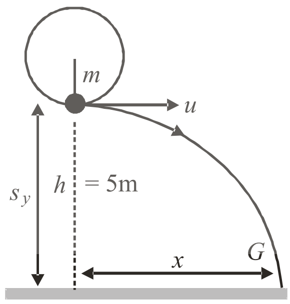
$$3(u^2 - 4gr) - u^2 = 4rg \quad \Rightarrow \quad 2u^2 = 16rg \quad \text{or} \Rightarrow \quad u = \sqrt{8rg} = \sqrt{8(0.8)10} = 8$$

(b) After breaking away from the string, the sphere moves along a parabolic path, and strikes the ground at G.

Vertical displacement of the sphere,

$s_y = 5.8 - 0.8 = 5$ m

Let t = time after which the sphere hits the ground.



$$\Rightarrow \quad s_y = 0t + \frac{1}{2}gt^2$$

$$\Rightarrow \quad t = \sqrt{\frac{2s_y}{g}} = 1\ s$$

The horizontal displacement, $x = ut = 8 \times 1 = 8$ m

Hence the sphere hits the ground 1s after breaking off the string and at point G.

COLLISIONS

Collision is an event in which two or more than two bodies interact with one another for a short time and exchange momentum and kinetic energy. Collisions are of two types :

(i) Elastic collision (ii) Inelastic collision

Do You Know!!

Linear momentum is always conserved, whereas kinetic energy is conserved only in elastic collision.

Elastic Collision

A collision in which there is no loss of kinetic energy is called elastic or perfectly elastic collision. The basic characteristics of perfectly elastic collision are

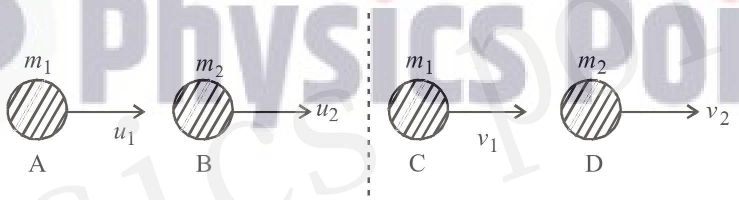
(i) linear momentum is conserved
(ii) kinetic energy is conserved
(iii) total energy is conserved
(iv) coefficient of restitution is unity ($e = 1$)

Coefficient of Restitution (e): It is defined as *the ratio of the velocity of separation to the velocity of approach.*

i.e., $e = \dfrac{\text{velocity of separation}}{\text{velocity of approach}} = \dfrac{v_2 - v_1}{u_1 - u_2}$

Elastic Collision in One Dimension

Let us discuss elastic collision in one dimension. Let two balls A and B of mass m_1 and m_2 move in the same direction along a straight line such that $u_1 > u_2$. After collision their velocities are v_1 and v_2 such that $v_1 < v_2$.



$u_1 > u_2$
Before collision

$v_2 > v_1$
After collision

Conserving linear momentum,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \text{.... (i)}$$

(**Note :** this is a vector equation so take care of sign)

Conserving kinetic energy,

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad \text{.... (ii)}$$

From (i) and (ii), we get

$$\frac{(m_1 - m_2)u_1 + 2m_2u_2}{(m_1 \quad m_2)} \; ; \quad v_2 = \frac{(m_2 - m_1)u_2 + 2m_1u_1}{(m_1 + m_2)}$$

Special cases

(a) When $m_1 = m_2$ then $v_1 = u_2$ and $v_2 = u_1$ i.e. velocities of two colliding bodies are interchanged.

(b) $m_1 >> m_2$ and $u_2 = 0$ (Ball A is very heavy and lighter ball B is at rest)
$v_1 = u_1$ and $v_2 = 2u_1$ i.e. heavier ball A keeps moving with the same speed and lighter ball B moves with double the speed of A.

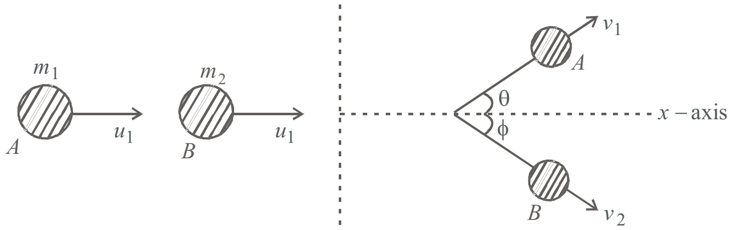
(c) $m_2 >> m_1$ and $u_2 = 0$ (Ball B is very heavy and it is at rest)
$v_1 = -u_1$, $v_2 = 0$ i.e. (lighter ball A rebounds with the same speed whereas heavy ball B remains at rest)

Do You Know!!

The collision in one dimension is also known as head-on collision.

Elastic Collision in Two Dimensions

Suppose two balls A and B moving with velocities u_1 and u_2 initially in a straight line collide and after collision move at angles θ and ϕ from the line of action (x-axis) with velocities v_1 and v_2 respectively :



Before collision After collision

Conserving kinetic energy,

$$\frac{1}{2}m_1u_1^2+\frac{1}{2}m_2u_2^2=\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2 \quad \text{.... (i)}$$

Conserving linear momentum along x-axis

$$m_1u_1+m_2u_2=m_1v_1\cos\theta+m_2v_2\cos\phi \quad \text{.... (ii)}$$

Conserving linear momentum along y-axis

$$0=m_1v_1\sin\theta-m_2v_2\sin\phi \quad \text{.... (iii)}$$

with the help of these three equations we can determine unknown quantity.

IDEA BOX idea box

When a body at rest is struck against by a moving body, the kinetic energy of the struck body is equal to the decrease in kinetic energy of the moving body.

Inelastic Collision

In an inelastic collision kinetic energy is lost during collision. The basic characteristic of an inelastic collision are :

(i) linear momentum is conserved

(ii) kinetic energy is not conserved

(iii) total energy is conserved

(iv) coefficient of restitution is $0 < e < 1$

For solving problems of inelastic collision we can use

$$m_1u_1+m_2u_2=m_1v_1+m_2v_2 \text{ and } e=\frac{v_2-v_1}{u_1-u_2}$$

And the loss in kinetic energy is given by

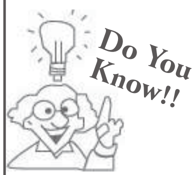
$$\Delta KE=\frac{1}{2}\frac{m_1m_2}{(m_1+m_2)}(u_1-u_2)^2(1-e)^2$$

*In **case of perfect inelastic collision*** the two bodies get stuck together and move with common velocity, that is why for perfectly inelastic collision;

Coeffiecient of restitution, $e = 0$ and common velocity, $v = \dfrac{m_1u_1 + m_2u_2}{m_1 + m_2}$

Loss in K.E. is completely inelastic collision $\Delta K.E. = \dfrac{1}{2}\dfrac{m_1m_2}{(m_1 + m_2)}.(u_1 - u_2)^2$

Also, the fraction of K.E. lost (when m_2 is at rest) is given by $= \dfrac{\text{loss of KE}}{\text{initial KE}} = \dfrac{m_2}{m_1 + m_2}$



A collision in which colliding bodies stick together, is always an inelastic collision. It is because, kinetic energy is never conserved in such collisions.

CHECK Point

The bob A of a pendulum released from 30° to the vertical hits another bob B of the same mass at rest on a table as shown in figure. How high does the bob A rise after the collision? Neglect the size of the bob and assume the collision to be elastic.

Solution

The positions of the bob of the pendulum and the target bob are shown in the figure.
The bob of the pendulum will not rise as explained below:
If the bob of the pendulum acquires a velocity, say v, on reaching the lower most position, then the target bob will start moving with the velocity v and the bob of the pendulum will comes to rest. It is because of the fact that in a perfectly elastic collision, when a moving object collides against a target object of the same mass, the two exchange their velocities.

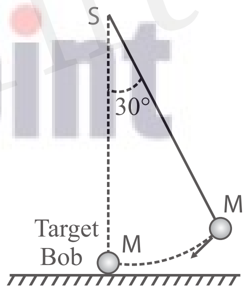


ILLUSTRATION : 12

A mass is released from a height H and after hitting the ground it rebounds to a height h, find the coefficient of restitution between the ground and mass.

SOLUTION :

Take the mass as body 1 and the earth as body 2.

So, $u_1 = \sqrt{2gH}$, $v_1 = -\sqrt{2gh}$ and $v_1 = v_2 = 0$

Using, $e = \dfrac{v_2 - v_1}{u_1 - u_2} \Rightarrow e = \sqrt{\dfrac{h}{H}}$

ILLUSTRATION : 13

A neutron having mass of 1.67×10^{-27} kg moving at 10^8 m s^{-1} collides with a deutron at rest and sticks to it. Given that the mass of neutron is 3.34×10^{-27} kg, calculate the speed of the combination. The composite particle is called triton.

SOLUTION :

Here, mass of the neutron, $M_1 = 1.67 \times 10^{-27}$ kg
Initial velocity of the neutron, $u_1 = 10^8$ m s^{-1};

Mass of the deutron, $M_2 = 3.34 \times 10^{-27}$ kgw
and initial velocity of the deutron, $u_2 = 0$ (at rest)
The mass of the composite particle i.e. triton, $M_1 + M_2 = 1.67 \times 10^{-27} + 3.34 \times 10^{-27} = 5.01 \times 10^{-27}$ kg
As the neutron sticks to the deutron after the collision, the collision is inelastic in nature. For an inelastic collision, only the law of conservation of linear momentum holds. If v is the velocity of the composite particle, then according to the law of conservation of linear momentum,

$$M_1 u_1 + M_2 u_2 = (M_1 + M_2) v$$

or $$v = \frac{M_1 u_1 + M_2 u_2}{M_1 + M_2} = \frac{1.67 \times 10^{-27} \times 10^8 + 3.34 \times 10^{-27} \times 0}{5.01 \times 10^{-27}} = 3.33 \times 10^7 \text{ms}^{-1}$$

POWER (RATE OF DOING WORK)

In several situations, it is not enough only to know that how much work is done but it is also required that how quickly it is done i.e, it is also important to know the rate of work done by the force.

The *time rate of doing work is defined as power* (P). If equal works are done in different times, power will be different. More quickly work is done, power will be more.

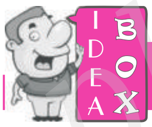
$$\text{Power (P)} = \frac{\text{work(w)}}{\text{time(t)}}$$

Unit of power : The unit of power is the joule per second and is called the **watt** (W). When large amounts of power are involved, a more convenient unit is the kilowatt (kW) where 1 kW = 1000 W.

1 Megawatt (MW) = 10^6 watt

Power was also measured earlier in a unit called **horse power**. Even these days, the unit of horse power is in common use.

1 horse power = 746 watt

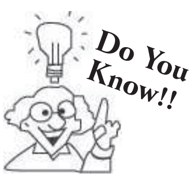


idea box

The term energy is different from power. Whereas energy refers to the *capacity* to perform the work, power determines the *rate* of performing the work. Thus, in determining power, time taken to perform the work is significant but it is of no importance for measuring energy of a body.

Power of a Moving Vehicle

The power of a vehicle is defined as the rate at which the driving force is working.
Consider a vehicle moving at a constant speed v meters per second. The driving force is F newton.
The distance moved in 1 second is v meters.
The work done by the driving force in 1 second is Fv joule.
Hence the power of the vehicle is Fv watts.
So, if P is the power, $\boldsymbol{P = Fv}$
i.e. the power of a vehicle is given by multiplying the driving force by the velocity.
When the velocity is not constant this relationship gives the power at the instant when the velocity is v.



Instantaneous power (P) of a body is defined as the dot product of force $(\vec{F})$ *and the instantaneous velocity* $(\vec{v})$ *of the body i.e.* $P = \vec{F}.\vec{v}$

Knowledge ENHANCER

Area under power-time curve gives the work done.

Commercial Unit of Energy

The SI unit of energy is the joule (J). But the commonly used unit for electrical-energy consumption is the kilowatt-hour (kWh). If energy is consumed at the rate of 1 kilojule/second, i.e., at the rate of 1 kilowatt, and this continues for 1 hour then the total energy consumed is called 1 kilowatt-hour.

Thus, $1\text{kWh} = 1\text{kW} \times 1 \text{ hour}$
$= (1000 \text{ W}) \times (3600 \text{ s})$
$= (1000 \text{ J/s}) \times (3600 \text{ s})$
$= (3600000 \text{ joules}) = 3.6 \times 10^6 \text{J}.$

For electrical-energy consumption in houses, factories, shops, etc., kilowatt-hour is simply called 'unit' (*Board of trade unit B.O.T.U.*). So, the cost of electrical energy is given in terms of ₹/unit. If electricity costs ₹ 5.00/ unit, it means that consumers pay ₹ 5.00 for every unit (kWh) of electrical energy consumed.

ILLUSTRATION : 14

What is the power of an engine which can lift 20 metric ton of coal per hour from a 20 metre deep mine ?

SOLUTION :

Mass, $m = 20$ metric ton $= 20 \times 1000$ kg, distance, $s = 20$m, time, $t = 1$ hour $= 3600$ s

$$\text{Power} = \frac{\text{work}}{\text{time}} = \frac{mg \times s}{t} = \frac{20 \times 1000 \times 9.8 \times 20}{3600} \text{watt} = 1.09 \times 10^3 \text{ W}$$

ILLUSTRATION : 15

One coolie takes one minute to raise a box through a height of 2 metre. Another one takes 30 second for the same job and does the same amount of work. Which one of the two has greater power and which one uses greater energy ?

SOLUTION :

$$\text{Power of first coolie} = \frac{\text{work}}{\text{time}} = \frac{M \times g \times s}{t} = \frac{M \times 9.8 \times 2}{60} \text{Js}^{-1}$$

$$\text{Power of second coolie} = \frac{M \times 9.8 \times 2}{30} = 2\left(\frac{M \times 9.8 \times 2}{60}\right) \text{Js}^{-1} = 2 \times \text{power of first coolie}$$

So, the power of the second coolie is double that of the first.
Both the coolies spend the same amount of energy.
Aliter, we know that $W = Pt$
For the same work, $W = P_1 t_1 = P_2 t_2$

$$\text{or} \quad \frac{P_2}{P_1} = \frac{t_1}{t_2} = \frac{1 \text{ minute}}{30 \text{ s}} = \quad \text{or} \quad P_2 = 2P_1$$

ILLUSTRATION : 16

A 100W bulb operates for 5 hours. How much electric energy will it consume ?

SOLUTION :

Work = power × time = 100 watt × 5 hour = 500 watt hour = 0.5 kilowatt hour

MISCELLANEOUS SOLVED EXAMPLES

1. A child pulls a toy car through a distance of 10 meters on a horizontal floor. The string held in child's hand makes an angle of 60° with the horizontal surface. If the force applied by the child be 10 N, calculate the work done by the child in pulling the toy car.

Sol. Here, the applied force and the displacement are not in same direction, we will calculate the work done by the formula

$W = Fs \cos\theta$

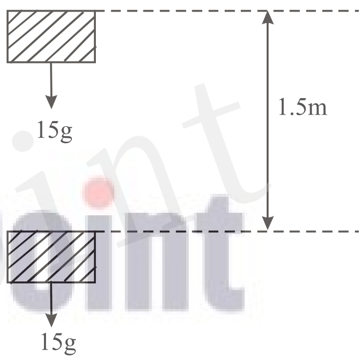
Here, force F = 10N; magnitude of the displacement s = 10m

Angle between force and displacement, $\theta = 60°$

Substituting these values in the above formula, we get

$W = (10) \times (10) \cos 60°$

$W = 10 \times 10 \times \frac{1}{2} = 50$ joule $\left(\because \cos 60° = \frac{1}{2}\right)$



2. A man lifts 20 boxes each of mass 15 kg to a height of 1.5m. Find the work done by the man against gravity (Take g = 9.8 ms^2).

Sol. The work done against gravity in lifting one box = mgh = 15g × 1.5 J = 22.5 g J

The work done against gravity in lifting 20 boxes = 20 × 22.5 gJ

= 450 g J = 4500 J ≃ 4.5 kJ

3. A scooter is moving with a velocity of 15 m/sec. Calculate its kinetic energy if its mass along with the rider is 150 kg.

Sol. Kinetic energy is given by

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2} \times 150 \times (15)^2 = 75 \times 225 = 16875 \text{ J}$$

4. A body of mass 10 kg is kept at a height 10m from the ground, when it is released, after sometime its kinetic energy becomes 450 joule. What will be the potential energy of the body at that instant?

Sol. At a height of 10m the mechanical energy of the body,

E = Kinetic energy + Potential energy

$E = \frac{1}{2}m(0)^2 + mgh$ ($\because$ initial velocity of the body is zero)

$E = 10 \times 10 \times 10 = 1000$ joule

After sometime the kinetic. energy is 450 joule, suppose at that instant potential energy is u, then by the law of conservation of mechanical energy

$E = 450 + u$

$1000 = 450 + u \Rightarrow u = 1000 - 450$

or $u = 550$ joule

5. A force acts on a body of mass 3 kg causing its speed to increase from 4m/s to 5 m/s. How much work has the force done ?

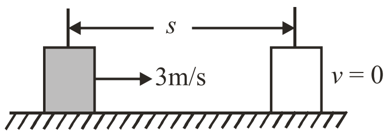
Sol. Initial K.E. $= \frac{1}{2}mv_1^2 = \frac{1}{2}(3)(4)^2 = 24$ J

Final K.E. $= \frac{1}{2}mv_2^2 = \frac{1}{2}(3)(5)^2 = 37.5$ J

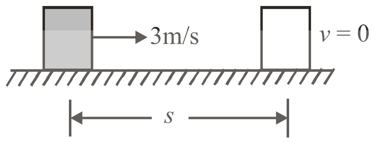
Work done = Change in energy

Hence, work done by force = (37.5 – 24) J = 13.5 J

6. A block of mass 2.0 kg is given an initital velocity of 3 m/s on a horizontal table where the coefficient of friction is 0.2. Find the distance covered by the block before it stops. Take $g = 10$ m/s^2.



Sol. Alternatively, you can apply work-energy methods in a very precise way as follows :
By using Work-Energy theorem
For the block, in the system under consideration, the statement of work-energy theorem can be written as
$W_{mg} = W_N = K_{fk} = K_2 - K_1$



$\Rightarrow \quad 0 + 0 + (-\mu_k mg \times s) = 0 - \frac{1}{2} mv_0^2$

$\Rightarrow \quad s = \frac{v_0^2}{2\mu_k g} = \frac{(3)^2}{2 \times 0.2 \times 10} = 2.25 \text{ m}$

Notice the difference between the "work done against friction" and the "work done by the frictional force". In this example, work done against friction is $\mu_k mgs$, and the work done by the frictional force is $(-\mu_k mgs)$.

7. A student pulls a bucket of water in 1 minute from a 30m deep well. If the mass of bucket with water is 25 kg, then calculate the power of this student.

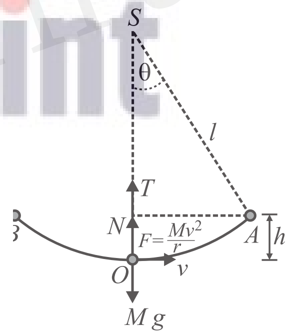
Sol. Work done by the student = mgh = 25 × 10 × 30 = 7500 J
Time duration of the work is = 1 min. = 60 sec.

Hence, the power of student $= \frac{\text{work}}{\text{time}} = \frac{7500\text{J}}{60\text{s}} = 125$ watt

SOLVED EXAMPLES BASED ON CONNECTING TOPICS

8. A pendulum consisting of a small heavy bob suspended from a rigid support oscillates in a vertical plane. When the bob passes through the position of equilibrium, the rod is subjected to a tension equal to twice the weight of the bob. Through what maximum angle from the vertical will the pendulum be deflected ? Disregard the weight of the rod and the resistance of the air.

Sol. Suppose that a small bob of weight M g is suspended from a rigid support S with a string of length l. In equilibrium position O, the bob is under the action of two forces : its weight M g (acting vertically downwards) and tension T in the string (acting vertically upwards) as shown in fig.



The resultant of these two forces provides the necessary centripetal force to the bob, so as to make it move along circular path of radius l

i.e. $T - Mg = \frac{M v^2}{l}$

Since $T = 2\,M\,g$, we have $2\,Mg - Mg = \frac{M v^2}{l}$ or $v = \sqrt{g\,l}$

As the bob gets deflected from its equilibrium position, its velocity decreases. It goes up to the point A, where its velocity becomes zero. Suppose that in this position, the string makes an angle θ with the vertical. If h is the height of the bob above its equilibrium position, then

$$h = SO - SN = l - l\cos\theta$$

From the principle of conservation of energy, we have kinetic energy of the bob at the point O = potential energy of the bob at the point A

or $\quad Mgh = \frac{1}{2} Mv^2$

or $\quad Mg(l - l\cos\theta) = \frac{1}{2} M(\sqrt{g\,l})^2$ or $\cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$

9. Two balls each of mass M moving in opposite directions with equal speeds v undergo a head-on collision. Calculate the velocity of the two balls after collision.

Sol. Here, $M_1 = M_2 = M$; $u_1 = v$ and $u_2 = -v$

Now, $\quad v_1 = \frac{(M_1 - M_2)\,u_1 + 2M_2 u_2}{M_1 + M_2} = \frac{(M - M)v + 2M(-v)}{M + M} = \frac{0 - 2M\,v}{2M}$

or $\quad v_1 = -v$

Also $\quad v_2 = \frac{(M_2 - M_1)\,u_2 + 2M_1\,u_1}{M_1 + M_2} = \frac{(M - M)\,(-v) + 2M\,v}{M + M} = \frac{0 + 2M\,v}{2M}$

or $\quad v_2 = v$

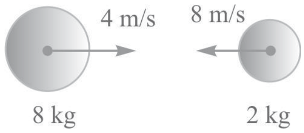
Thus, after the collision, the two balls bounce back with equal speeds v.

10. A 8 kg ball moving with velocity 4 m/s collides with a 2 kg ball moving with a velocity 8 m/s in opposite direction. If the collision be perfectly elastic, what are the velocities of balls after the collision.

Sol. By conservation of momentum

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$8 \times 4 - 2 \times 8 \quad = 8\, v_1 + 2v_2$ … (i)



As collision is elastic, so we have

$$\frac{1}{2}\, m_1u_1^2 + \frac{1}{2}\, m_2u_2^2 = \frac{1}{2}\, m_1v_1^2 + \frac{1}{2}\, m_2v_2^2$$

$$\frac{1}{2}\times 8\times 4^2 + \frac{1}{2}\times 2\times 8^2 = \frac{1}{2}\times 8\times v_1^2 + \frac{1}{2}\times 2\times v_2^2 \qquad \text{… (ii)}$$

Solving equations (i) and (ii), we get

$$v_1 = -\frac{4}{5}\text{ m/s}$$

and, $v_2 = \dfrac{56}{5}$ m/s.

1 EXERCISE

Fill in the Blanks :

DIRECTIONS: *Complete the following statements with an appropriate word/term to be filled in the blank space(s).*

1. The work done by the external force on a system equals the change in .... energy.
2. One horse-power = ... watt.
3. An electric motor exerts a force of 40N on a cable and pulls it through a distance of 30m in one minute. The power supplied by the motor in watts is ....
4. A one kilogram mass has a kinetic energy of one joule when its speed is .... meter/sec.
5. A truck and a car moving with the same kinetic energy are brought to rest by the application of brakes which provide equal retarding forces. The distance moved by the truck, in coming to rest, will then be .... the distance moved by car.
6. The work done in holding 15 kg suitcase while waiting for a bus for 15 minute is ....
7. Two bodies of masses m_1 and m_2 have equal moments. Their kinetic energies are in the ratio ....
8. The energy possessed by the body due to its .... is called kinetic energy.
9. Work done by a force is maximum when angle between force and displacement is ....
10. Energy is a .... quantity.
11. Energy stored in an elongated rubber is ....
12. Watt second is a unit of ....
13. .... energy can never be negative.
14. When angle between force and displacement is obtuse, work done by the force is ....

True/False :

DIRECTIONS: *Read the following statements and write your answer as true or false.*

1. Work is always done on a body when it experiences an increase in energy through a mechanical influence.
2. Work done by the resultant force is always equal to change in kinetic energy.
3. A light and a heavy body, having equal momenta, have equal kinetic energies.
4. Work done in the motion of a body over a closed loop is zero for every force in nature.
5. More work is done in compressing a litre of air than a litre of water from a pressure of one atmosphere to three atmospheres.
6. No work is done on a particle which remains at rest.
7. A man rowing a boat up stream is at rest with respect to the shore, is doing no work.
8. The total energy of a body in motion is equal to the work it can do in being brought to rest.
9. A body cannot have momentum when its energy is zero.
10. Kinetic energy of a body depends upon the direction of motion.
11. Work done by friction can-never be positive.
12. Work done by conservative force in round trip is zero.
13. Gravitational force is non-conservative.
14. Work done by conservative forces is equal to increase in potential energy.
15. A man carrying a bucket of water, walking on a level road with a uniform velocity does no work.

Match the Columns :

DIRECTIONS: *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column I have to be matched with statements (p, q, r, s) in Column II.*

1. A boy of mass 55 kg runs up a flight of 40 stairs, each measuring 0.15m in 5s. Column II given numerical value for quantities describe in Column I, match them correctly.

Column I	Column II
(A) Force acting on the boy	(p) 550
(B) Work done by the boy	(q) 3300
(C) Gain of potential energy by the boy	(r) 33000
(D) Power developed by the boy	(s) 220

2. Column I shows some devices and column II shows transformation of energy for which they are used. Then match the following.

Column I	Column II
(A) Electric motor	(p) Electrical energy to heat energy
(B) Engine on an automobile	(q) Electrical energy to mechanical energy.
(C) Electric heater	(r) Light energy to electrical energy
(D) Photocell	(s) Heat energy to mechanical energy.

Very Short Answer Questions :

DIRECTIONS: *Give answer in one word or one sentence.*

1. A man raises a mass '*m*' to a height '*h*' and then shifts it horizontally by a length '*x*'. What is the work done against the force of gravity?

2. Can a constant velocity be maintained in a body moving on a rough source without doing any work on it?

3. Does the work done in raising a box on to a platform depends upon how fast it is raised up? If not, why?

4. A man rowing a boat upstream is at rest with respect to the shore, is he doing work?

5. A light body and a heavy body have the same momentum. Which one will have greater kinetic energy?

6. A truck and a car are moving with the same kinetic energy on a straight road. Their engines are simultaneously switched off. Which one will stop at a lesser distance?

7. Is it possible that a force is acting on a body but still the work done is zero? Explain giving one example.
Or
What is the work done by a coolie walking on a level road with a load on his head ?

8. How does the energy of an object change when it performs work or work is done on it?

9. Mention the various energy transformations that take place when we throw a ball.

10. What is the commercial unit of energy? Establish the relationship between this unit and the SI unit of energy?
Or
Define one kilowatt hour. Express it in joules.

Short Answer Questions :

DIRECTIONS: *Give answer in 2-3 sentences.*

1. What do you understand by positive work and negative work? Give two examples of each.

2. A driver increases the speed of his car on approaching a hilly road. Why is it done ?

3. Can you say if the following objects have energy? If they do, identify whether the energy is kinetic, potential or combination of the two.
(i) A ceiling fan which has been switched off.
(ii) A man climbing a hill.
(iii) A flying bird.
(iv) Water in the reservoir of a dam.
(v) A spring expanded beyond its normal shape.
(vi) A rubber band lying on a table.
(vii) A stretched rubber band lying on the ground.

4. Water at the bottom of water-fall is warmer than at the top. Why?

5. In a thermal power station, coal is used for the generation of electricity. Mention how energy changes from one form to another before it is transformed into electrical energy?

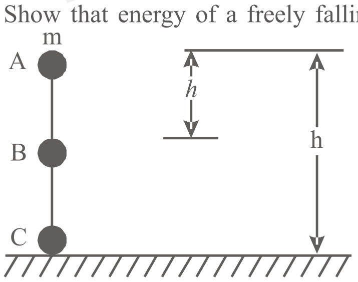
6. No work is done when we push immovable objects like a huge stone or a wall etc. However, we feel tired in doing so. Explain why ?

7. Identify energy transformation in the following :
(i) Hydro-electric power,
(ii) Battery,
(iii) Stretched bow with arrow,
(iv) Explosion of a cracker,
(v) Oscillating pendulum,
(vi) Hammering a nail.

Long Answer Questions :

DIRECTIONS : *Give answer in four to five sentences.*

1. Show that energy of a freely falling body is conserved.



2. Define work. Write its unit. How will you define 1 joule.

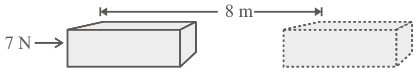
3. What do you mean by energy? Explain different kinds of the mechanical energy.

4. Define kinetic energy. Derive an expression for it.

2 EXERCISE

Text-Book Questions :

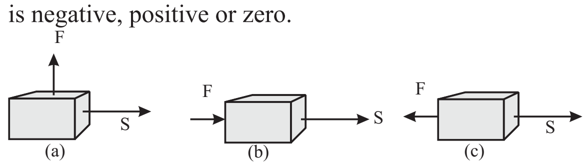
1. A force of 7 N acts on an object. The displacement is say 8m in the direction of the force. Let us take it that the force acts on the object through the displacement. What is the work done in this case?



2. When do we say that work is done?
3. Write an expression for the work done when a force is acting on an object in the direction of its displacement.
4. Define 1 J of work .
5. A pair of bullocks exerts a force of 140 N on a plough. The field being ploughed is 15 m long. How much work is done in ploughing the length of the field?
6. What is the kinetic energy of an object ?
7. Write an expression for the kinetic energy of an object.
8. The kinetic energy of an object of mass m moving with a velocity of 5 ms^{-1} is 25 J. What will be its kinetic energy when its velocity is doubled ? What will be its kinetic energy when its velocity is increased three times ?
9. What is power?
10. Define 1watt of power.
11. A lamp consumes 1000 J of electrical energy in 10s. What is its power?
12. Define average power.

Text-Book Exercise :

1. Look at the activities listed below. Reason out whether or not work is done in the light of your understanding of the term 'work'.
   (i) Suma is swimming in a pond.
   (ii) A donkey is carrying a load on its back.
   (iii) A wind-mill is lifting water from a well.
   (iv) A green plant is carrying out photosynthesis.
   (v) An engine is pulling a train.
   (vi) Food grains are getting dried in the sun.
   (vii) A sailboat is moving due to wind energy.
2. An object thrown at a certain angle to the ground moves in a curved path and falls back to the ground. The initial and the final points of the path of the object lie on the same horizontal line. What is the work done by the force of gravity on the object ?
3. A battery lights a bulb. Describe the energy changes involved in the process.
4. Certain force acting on a 20 kg mass changes its velocity from 5 ms^{-1} to 2 ms^{-1} . Calculate the work done by the force.
5. A mass of 10 kg is at a point A on a table. It is moved to a point B. If the line joining A and B is horizontal, what is the work done on the object by the gravitational force? Explain your answer.
6. The potential energy of a freely falling object decreases progressively. Does this violate the law of conservation of energy? Why ?
7. What are the various energy transformations that occur when you are riding a bicycle?
8. Does the transfer of energy take place when you push a huge rock with all your might and fail to move it ? Where is the energy you spend going?
9. A certain houshold has consumed 250 units of energy during a month. How much energy is this in joules ?
10. An object of mass 40 kg is raised to a height of 5 m above the ground. What is its potential energy? If the object is allowed to fall, find its kinetic energy when it is half-way down.
11. What is the work done by the force of gravity on a satellite moving round the earth ? Justify your answer.
12. Can there be displacement of an object in the absence of any force acting on it ? Think. Discuss this question with your friends and teacher.
13. A person holds a bundle of hay over his head for 30 minutes and gets tired. Has he done some work or not ? Justify your answer.
14. An electric heater is rated 1500 W. How much energy does it use in 10 hours ?
15. Illustrate the law of conservation of energy by discussing the energy changes which occur when we draw a pendulum bob to one side and allow it to oscillate. Why does the bob eventually come to rest ? What happens to its energy eventually ? Is it a violation of the law of conservation of energy ?
16. An object of mass, m is moving with a constant velocity, v. How much work should be done on the object in order to bring the object to rest ?
17. Calculate the work required to be done to stop a car of 1500 kg moving at a velocity of 60 km/h?
18. In each of the following a force F is acting on an object of mass, m. The direction of displacement is from west to east shown by the longer arrow. Observe the diagrams carefully and state whether the work done by the force F is negative, positive or zero.



19. Soni says that the acceleration in an object could be zero even when several forces are acting on it. Do you agree with her ? Why ?

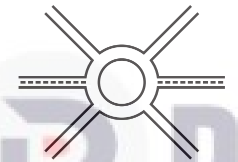
20. Find the energy in kWh consumed in 10 hours by four devices of power 500 W each.

21. A freely falling object eventually stops on reaching the ground. What happenes to its kinetic energy?

Exemplar Questions :

1. A rocket is moving up with a velocity *v*. If the velocity of this rocket is suddenly tripled, what will be the ratio of two kinetic energies?

2. A boy is moving on a straight road against a frictional force of 5N. After travelling a distance of 1.5 km he forgot the correct path at a round about of radius 100 m. However, he moves on the circular path for one and half cycle and then he moves forward upto 2.0 km. Calculate the work done by him.



3. The weight of a person on a planet A is about half that on the earth. He can jump upto 0.4m height on the surface of the earth. How high he can jump on the planet A?

4. The velocity of a body moving in a straight line is increased by applyng a constant force F, for some distance in the direction of the motion. Prove that the increase in the kinetic energy of the body is equal to the work done by the force on the body.

5. A ball is dropped from a height of 10m. If the energy of the ball reduces by 40% after striking the ground, how much high can the ball bounce back? ($g = 10ms^{-2}$)

Hots Questions :

1. It is generally much more difficult to stop a heavy truck than a light car when they move at the same speed. State a case in which the moving car could require more stopping force. (Consider relative times.)

2. Why is a punch more forceful with a bare fist than with a boxing glove?

3. A boxer can punch a heavy bag for more than an hour without tiring, but will tire quickly when boxing with an opponent for a few minutes. Why?

4. How is it possible that a flock of birds in flight can have a momentum of zero but not have zero kinetic energy?

5. If the K.E. of a body is increased by 300%, by what percentage will its momentum increase ?

6. A particle of mass m_1 is moving with a velocity v_1 and another particle of mass m_2 is moving with a velocity v_2. Both of them have the same momentum but their different kinetic energies are E_1 and E_2 respectively. If $m_1 > m_2$ then how are E_1 and E_2 related ?

7. A body of mass m accelerates uniformly from rest to v_1 in time t_1. As a function of time t, find the instantaneous power delivered to the body.

8. A pendulum, swinging back and forth, rises at the end of its swing to a position 15 cm. higher than its lowest point. How fast is it going at the lowest point ?

9. Does the work done in raising a box onto a platform depend on how fast is raised ?

10. If 20000 joules of work is done in pumping water up to a height of 12 meters, how much water is pumped ?

11. A body is thrown vertically upwards. Its velocity keeps on decreasing. What happens to its kinetic energy as its velocity becomes zero?

12. A man runs a distance ‘s’ on a level road. The same man ascends up a hill with the same velocity through the same distance. When does he do more work ?

3 EXERCISE

Single Option Correct :

DIRECTIONS : *This section contains multiple choice questions. Each questions has 4 choices (a), (b), (c) and (d), out of which only one is correct.*

1. How much work is done by a force of 10 N in moving an object through a distance of 5 m in the direction of the force ?
(a) 2 joule (b) 25 joule
(c) 50 joule (d) 100 joule

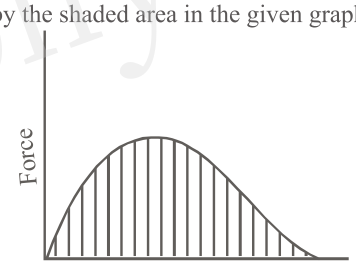
2. Which is not a unit of energy
(a) Watt second (b) kilo watt hour
(c) watt (d) joule

3. kilowatt hour is the unit of
(a) time (b) power
(c) energy (d) force

4. 1 kilowatt hour is equal to
(a) 1 joule (b) 100 joule
(c) 36 joule (d) 3.6×10^3 kilo joule

5. A stone of mass 1 kg is raised through 1m height
(a) The loss of gravitational potential energy by the stone is 1 joule
(b) The gain of gravitational potential energy by the stone is 1 joule
(c) The loss of gravitational potential energy is 9.8 joule
(d) The gain of gravitational potential energy is 9.8 joule

6. Which one of the following physical quantities is represented by the shaded area in the given graph?



(a) Torque (b) Impulse
(c) Power (d) Work done

7. The energy of 4900 J was expanded in lifting a 50 kg mass. The mass was raised to a height of
(a) 10m (b) 98m
(c) 960m (d) 245000 m

8. A body of mass 1 kg has kinetic energy 1J when its speed is
(a) 0.45 m/s (b) 1 m/s
(c) 1.4 m/s (d) 4.4 m/s

9. The kinetic energy of a body will become eight times if
(a) its mass is made four times
(b) its velocity is made four times
(c) both the mass and velocity are doubled
(d) both the mass and velocity are made four times

10. For a body falling freely under gravity from a height
(a) only the potential energy goes on increasing
(b) only the kinetic energy goes on increasing
(c) both kinetic energy as well as potential energy go on increasing
(d) the kinetic energy goes on increasing while potential energy goes on decreasing

11. A ball is dropped from a height of 10m. If the energy of the ball reduces by 40% after striking the ground, the ball will rebound to
(a) 4m (b) 6m
(c) 10m (d) 9.8 m

12. A 2 kg mass kept on a horizontal table is acted upon by a force of 5N at an angle of 60° from horizontal and moved through a distance of 3m on the table. The work done on the body is
(a) 5J (b) 7.5 J
(c) 10 J (d) 15 J

13. The kinetic energy acquired by a body of mass '*m*' after travelling a fixed distance from rest under the action of a constant force is
(a) directly proportional to mass *m*
(b) inversely proportional to mass *m*
(c) inversely proportional to mass $m^{1/2}$
(d) independent of mass *m*

14. If a force *F* is applied on a body and it moves with velocity *v*, the power will be
(a) Fv (b) F/v
(c) Fv^2 (d) F/v^2

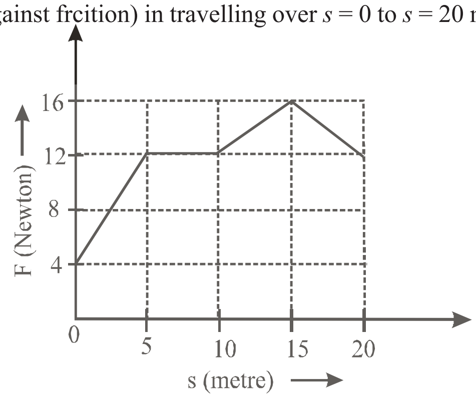
15. The kinetic energy of a body becomes twice its initial value. The new momentum of the body will be
(a) 2 times (b) $\sqrt{2}$ times
(c) 4 times (d) unchanged

16. A car weighing 500 kg. working against a resistance of 500N, accelerates from rest to 20 m/s in 100m. ($g = 10 m/s^2$). The work done by the engine of car is
(a) 1.0×10^5 J
(b) 1.5×10^5 J
(c) 1.05×10^5 J
(d) The information given is insufficient

17. A uniform force of 4N acts on a body of mass 8 kg for a distance of 2.0m. The K.E. acquired by the body is
(a) 8J (b) 64 J
(c) 4J (d) 16 J

18. A cord is used to lower vertically a block of mass *M* a distance *d* at a constant downward acceleration of *g*/4. Then the work done by the cord on the block is
(a) Mgd/4 (b) 3 Mgd/4
(c) –3Mgd/4 (d) Mgd

19. Figure shows the frictional force versus displacement for a particle in motion. The loss of kinetic energy (work done against frcition) in travelling over $s = 0$ to $s = 20$ m will be



(a) 80 J (b) 160 J
(c) 240 J (d) 24 J

20. A man A of mass 80 kg runs up a staircase in 12 seconds. Another man B of mass 60 kg runs up the same staircase in 11 seconds. The ratio of powers of A and B is
(a) 11 : 12 (b) 11 : 9
(c) 12 : 11 (d) 9 : 11

21. A man of weight 60 kg wt. takes a body of mass 15 kg at a height 10m on a building in 3 minutes. The efficiency of mass is
(a) 10% (b) 20%
(c) 30% (d) 40%

22. Kinetic energy of a body moving with speed 10 m/s is 30J. If its speed becomes 30 m/s, its kinetic energy will be
(a) 10 J (b) 90 J
(c) 180 J (d) 270 J

23. A weight-lifter lifts 200 kg from the ground to a height of 2 metre in 9 second. The average power generated by the man is
(a) 15680 W (b) 3920 W
(c) 1960 W (d) 980 W

24. Two masses m and $9m$ are moving with equal kinetic energies. The ratio of the magnitudes of their momenta is
(a) 1 : 1 (b) 1 : 3
(c) 3 : 1 (d) 1 : 9

25. If a stone of mass m falls a vertical distance d, the decrease in gravitational potential energy is
(a) mg/d (b) $md^2/2$
(c) mgd (d) md/g

26. No work is done when
(a) a nail is plugged into a wooden board
(b) a box is pushed along a horizontal floor
(c) there is no component of force parallel to the direction of motion
(d) there is no component of force normal to the direction of force

27. Potential energy of your body is minimum when you
(a) are standing
(b) are sitting on a chair
(c) are sitting on the ground
(d) lie down on the ground

28. A block of weight W is pulled a distance l along a horizontal table. The work done by the weight is
(a) Wl (b) 0
(c) Wgl (d) Wl/g

29. A child builds a tower from three blocks. The blocks are uniform cubes of side 2 cm. The blocks are initially all lying on the same horizontal surface and each block has a mass of 0.1 kg. The work done by the child is
(a) 4J (b) 0.04 J
(c) 6J (d) 0.06 J

30. A car is moving with a constant speed of 20 m/s against a resistance of 100N. The power exerted by the car is
(a) 2 kW (b) 5 W
(c) 200 W (d) 1 kW

31. A particle of mass m moves from rest under the action of a constant force F which acts for two seconds. The maximum power attained is
(a) $2Fm$ (b) F^2/m
(c) $2\,F/m$ (d) $2F^2/m$

32. Two bodies A and B having masses in the ratio of 3 : 1 possess the same kinetic energy. The ratio of linear momentum of B to A is
(a) 1 : 3 (b) 3 : 1
(c) $1:\sqrt{3}$ (d) $\sqrt{3}:1$

33. If the linear momentum is increased by 5%, the kinetic energy will increase by
(a) 50% (b) 100%
(c) 125% (d) 10%

More than One Option Correct :

DIRECTIONS: *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which One or More may be correct.*

1. The total energy of a swinging pendulum at any instant of time
(a) remains zero (b) remains conserved
(c) is same (d) is lost

2. The power of a body doing 20 Joules of work in 5 seconds is
(a) 400 J/s (b) 4 kW
(c) 4 J/s (d) 4 W

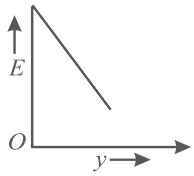
3. 1 unit of electricity consumed is same as
(a) 1 kwh (b) 3.6×10^6 J/s
(c) 3.6×10^6 J (d) 1 m/s^2

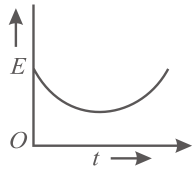
4. Which of the following is/are a scalar quantity?
(a) Acceleration (b) Velocity
(c) Work done (d) Energy

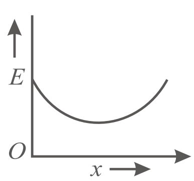
5. No work is done by a force on an object if:
(a) the object moves in such a way that the point of application of the force remains fixed
(b) the object is stationary but the point of application of the force moves on the object
(c) the force is always perpendicular to its acceleration
(d) the force is always perpendicular to its velocity

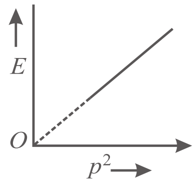
6. When a body is moving up with constant velocity:
(a) work done by force of gravity is positive
(b) work done by lifting force is positive
(c) work done by lifting force is negative
(d) work done by force of gravity is negative

7. A particle is projected from a point at an angle with the horizontal at $t = 0$. At an instant 't', if p is linear momentum, x is horizontal displacement, y is vertical displacement and E is kinetic energy of the particle, then which of the following graphs are correct?

(a)


(b)


(c)


(d)


8. In which of the following are no work done by the force?
   (a) A man walking upon a staircase
   (b) A man carrying a bucket of water, walking on a level road with a uniform velocity.
   (c) A drop of rain falling vertically with a constant velocity
   (d) A man whirling a stone tied to a string in circle with a constant speed.

Multiple Matching Questions :

DIRECTIONS : *Following question has four statements (A, B, C and D) given in Column I and five statements (p, q, r, s, t) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in Column I with entries in Column II.*

A body A of mass 3.0 kg and a body B of mass 10kg are dropped simultaneously from a height of 14.9m. Column II given numerical value for quantities describe in column I. Match them correctly.

1.

Column I	**Column II**
(A) The momentum of body A	(p) 294 J
(B) The potential energy of body B at a height 10 m above the ground	(q) 480 J
(C) The potential energy of body A at a height 10 m above the ground	(r) 294 N.M
(D) The K.E. of body B at a height 10 m above the ground	(s) 980J
	(t) 29.4 kg m/s

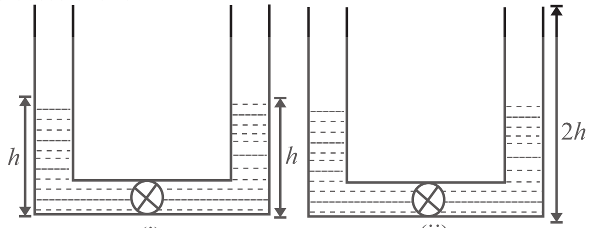
	A	B	C	D
(a)	p,q	q	r, s	q, r
(b)	t	s	p, r	q
(c)	p, s	q	r, s, t	r
(d)	p, s	r, s	q	p, r

Passage Based Questions

DIRECTIONS : *Study the given paragraph(s) and answer the following questions.*

PASSAGE

A liquid of density d is pumped by a pump p from situation (i) to situation (ii) as shown in figure. If the cross section of each of the vessels is a.



(i) (ii)

1. the work done in pumping (neglecting friction effect) is:
   (a) 2 dgh (b) dgha
   (c) 2 dgh^2a (d) dgh^2 a
2. p if the density of liquid is 0.2 kgm^{-3}, area of cross. Section is $2m^2$, height attainded by the liquid is 40 m and the pump takes 20s in this process. The power consumed (in JS^{-1}) in this process is: ($g = 9.8ms^{-2}$)
   (a) 78.4 (b) 0.785
   (c) 7.84 (d) 784

Assertion & Reason :

DIRECTIONS: *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.
(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.
(c) If **Assertion** is **correct** but **Reason** is **incorrect**.
(d) If **Assertion** is **incorrect** but **Reason** is **correct**.

1. **Assertion :** A moving hammer drives a nail into wood.
   Reason : A moving hammer has potential energy.
2. **Assertion :** A man gets completely exhausted in trying to push a stationary wall.
   Reason : Work done by the man on the wall is zero.
3. **Assertion :** The driver increases the speed of his car on approaching a hilly road.
   Reason : To give more kinetic energy to the car so that it may go up against gravity.
4. **Assertion :** Winding up the spring of a toy car gives it energy for moving.
   Reason : Work done in winding the spring get stored up as kinetic energy.
5. **Assertion :** The work done during a round trip is always zero.
   Reason : Displacement of body in round trip is zero.
6. **Assertion :** Work done in moving a body over a closed loop is zero for every force in nature.
   Reason : Work done depends on nature of force.
7. **Assertion :** A person walking on horizontal road with a load on his head does no work.
   Reason : No work is said to be done, if the directions of force and displacement of load are perpendicular to each other.

Integer/Numeric type Questions :

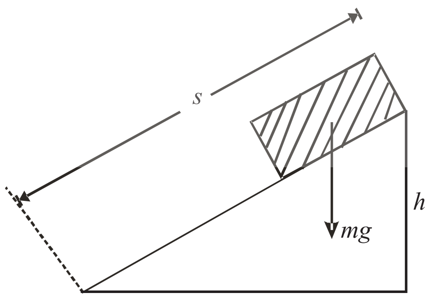
DRECTIONS : *Following are integer based/Numeric based questions. Each question, when worked out will result in one integer or numeric value.*

1. A particle describe a horizontal circle of radius 0.5 m with uniform speed. The centripetal force acting is 10 N. Find the work done in describing a semicircle.
2. A boy pushes a toy box 2.0 m along the floor by means of a force of 10 N directed downward at an angle of 60º to the horizontal. Determine the work done by the boy.
3. A particle moving in the xy plane undergoes a displacement of $\vec{s} = (2\hat{i} + 3\hat{j})$ while a constant force $\vec{F} = (5\hat{i} + 2\hat{j})$ N acts on the particle. Find the work done by the force F.
4. A motor of 100 H.P. moves a load with a uniform speed of 72 km/hr. What is the forward thrust applied by the engine on the car ?
5. Calculate the K.E and P.E. of the ball half way up, when a ball of mass 0.1 kg is thrown vertically upwards with an initial speed of 20 ms^{-1}.
6. If the linear momentum is increased by 5%, then find the increase in kinetic energy.

4 ADVANCED EXERCISE
BASED ON CONNECTING TOPICS

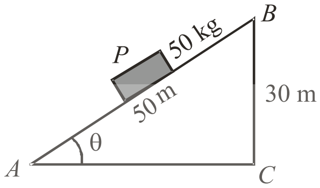
DIRECTIONS (Qs. 1–13) : *This section contains multiple choice questions. Each questions has 4 choices (a), (b), (c) and (d), out of which only one is correct.*

1. The total work done on a particle is equal to the change in its kinetic energy
   (a) always
   (b) only if the force acting on the body are conservative
   (c) only in the inertial frame
   (d) only if no external force is acting
2. With what speed must a ball be thrown down for it to bounce 10m higher than its original level ? Neglect any loss in energy against air resistance and in collision with the ground
   (a) 5 m/s
   (b) 14 m/s
   (c) 20 m/s
   (d) The information given is incomplete
3. When you compress a coil spring you do work on it. The elastic potential energy
   (a) increases (b) decreases
   (c) disappears (d) remains the same
4. The work done against gravity in moving the block a distance s up the slope is



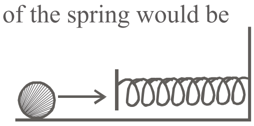
   (a) mh (b) mgs
   (c) ms (d) mgh
5. A small body is projected in a direction inclined at 45º to the horizontal with kinetic energy K. At the top of its flight, its kinetic energy will be
   (a) zero (b) $K/2$
   (c) $K/4$ (d) $K/\sqrt{2}$
6. A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1 mm. Then the elastic energy stored in the wire is
   (a) 10 J (b) 20 J
   (c) 0.1 J (d) 0.2 J
7. Consider the following two statement:
   I. Linear momentum of a system of particles is zero.
   II. Kinetic energy of a system of particles is zero.
   Then
   (a) I implies II but II does not imply I.
   (b) I does not imply II but II implies I.
   (c) I implies II and II implies I.
   (d) I does not imply II and II does not imply I.
8. A bullet of mass 'a' and velocity 'b' is fired into a large block of wood of mass 'c'. The bullet gets embedded into the block of wood. The final velocity of the system is
   (a) $\frac{b}{a+b} \times c$ (b) $\frac{a+b}{c} \times a$
   (c) $\frac{a}{a+c} \times b$ (d) $\frac{a+c}{a} \times b$
9. A ball is dropped from a height h. If the coefficient of restitution be e, then to what height will it rise after jumping twice from the ground?
   (a) $e\,h/2$ (b) $2\,e\,h$
   (c) $e\,h$ (d) $e^4\,h$
10. A body of mass 5 kg initially at rest explodes into 3 fragments with mass ratio 3 : 1 : 1. Two of fragments each of mass 'm' are found to move with a speed 60 m/s in mutually perpendicular directions. The velocity of third fragment is
   (a) $60\sqrt{2}$ (b) $20\sqrt{3}$
   (c) $10\sqrt{2}$ (d) $20\sqrt{2}$

11. In figure, a carriage P is pulled up from A to B. The relevant coefficient of friction is 0.40. The work done will be



(a) 10 kJ (b) 23 kJ
(c) 25 kJ (d) 28 kJ

12. A mass of 0.5 kg moving with a speed of 1.5 m/s on a horizontal smooth surface, collides with a nearly weightless spring of force constant $k = 50$ N/m. The maximum compression of the spring would be



(a) 0.5 m (b) 0.15 m
(c) 0.12 m (d) 1.5 m

13. A body of mass m moving with velocity v makes a head on elastic collision with another body of mass 2m which in initially at rest. The loss of kinetic energy of the colliding body (mass m) is

(a) $\frac{1}{2}$ of its initial kinetic energy

(b) $\frac{1}{9}$ of its initial kinetic energy

(c) $\frac{8}{9}$ of its initial kinetic energy

(d) $\frac{1}{4}$ of its initial kinetic energy

DIRECTIONS (Qs. 14–17) : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which One or More may be correct.*

14. When two blocks connected by a spring move towards each other under mutual interaction:
(a) their momentum are equal and opposite
(b) their velocities are equal and opposite
(c) their accelerations are equal and opposite
(d) the force acting on them are equal and opposite.

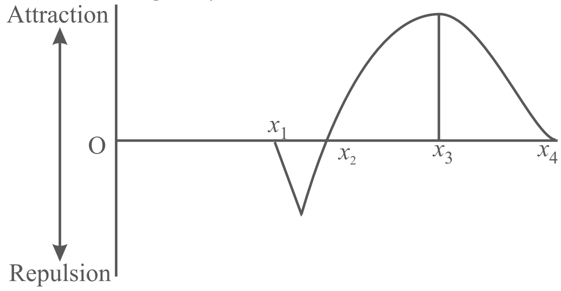
15. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows that:
(a) the acceleration of the particle is constant
(b) the velocity of the particle is constant
(c) the particle moves in a circular path
(d) the kinetic energy of the particle is constant

16. A body of mass m is moving in a straight line at a constant speed v. Its kinetic energy is K and the magnitude of its momentum is p. Which of the following relations are correct?

(a) $v = \frac{2K}{p}$ (b) $2K = pv$

(c) $p = \sqrt{2mK}$ (d) $p = \sqrt{\frac{2K}{m}}$

17. The figure given below shows how the net interaction force between two particles A and B is related to the distance between them. When the distance between them varies from x_1 to x_4, then:



(a) kinetic energy increases from x_1 to x_2 and decreases form x_2 to x_3
(b) potential energy of the system increases from x_1 to x_2
(c) potential energy of the system increases from x_3 to x_4
(d) potential energy of the system increases form x_2 to x_3

DIRECTIONS (Qs. 18) : *Following question has four statements (A, B, C and D) given in Column I and five statements (p, q, r, s, t......) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in Column I with entries in Column II.*

18. A particle of mass m is projected from ground with speed v making an angle θ with the vertical. Match the following for the motion of the particle.

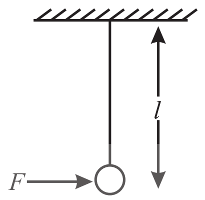
	Column I		**Column II**
(A)	Power of gravity at the highest point	(p)	zero
(B)	Power of gravity at the point of projection	(q)	$\frac{1}{2}m v^2 \cos^2 \theta$
(C)	Work done by gravity during ascend	(r)	$mgu \cos \theta$
(D)	Work done by gravity during descend	(s)	negative
		(t)	positive

	A	B	C	D
(a)	p	r, s	q, s	q, t
(b)	p, q, r, s	q	p, q, r, s	p, q, r, s
(c)	r, t	p, q, s	q, r	r
(d)	p,	q, r	r	s

DIRECTIONS (Qs. 19–24) : *Study the given paragraph(s) and answer the following questions.*

PASSAGE-1

A pendulum bob of mass m is hanging from a fixed point can oscillate in a vertical plane. A constant horizontal force F is applied on the bob as shown.



19. The maximum angle made by the string with the vertical is

(a) $\tan^{-1}\left(\frac{F}{mg}\right)$ (b) $2\tan^{-1}\left(\frac{F}{mg}\right)$

(c) $2\tan^{-1}\left(\frac{2F}{mg}\right)$ (d) $\tan^{-1}\left(\frac{mg}{F}\right)$

20. The maximum velocity of the *bob* will be

(a) $\left[\frac{2l(\sqrt{F^2+m^2g^2-mg})}{m}\right]^{1/2}$

(b) $\left[2l\frac{(\sqrt{F^2+m^2g^2}-F)}{m}\right]^{1/2}$

(c) $\left[\frac{l(\sqrt{F^2+m^2g^2}-mg)}{m}\right]^{1/2}$

(d) $\left[\frac{l(\sqrt{F^2+m^2g^2}-F)}{m}\right]^{1/2}$

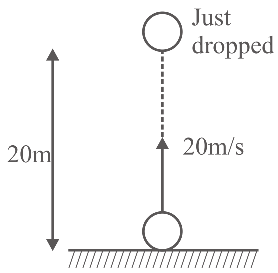
21. The tension in the bob in the position of problem 1 will be

(a) $\sqrt{F^2+(mg)^2}$ (b) $\frac{2F\,mg}{F+mg}$

(c) $\frac{F+mg}{2}$ (d) mg

PASSAGE-2

Two identical masses are as shown in figure. One is thrown upwards with velocity 20 m/s and another is just dropped simultaneously.



22. The masses collide in air and stick together. After how much time the combined mass will fall to the ground (calculate the time from the starting when the motion was started) –

(a) $(1+\sqrt{2})$ s (b) $2\sqrt{2}$ s

(c) $(2+\sqrt{2})$ s (d) None of these

23. In the above problem, to what maximum height (from ground) will the combined mass rise

(a) 25m (b) 18m (c) 15m (d) 20m

24. If collision between them is elastic, find the time interval between their striking with ground

(a) zero (b) 2s (c) 1s (d) 3s

DIRECTIONS (Qs. 25–26): *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.
(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.
(c) If **Assertion** is **correct** but **Reason** is **incorrect**.
(d) If **Assertion** is **incorrect** but **Reason** is **correct**.

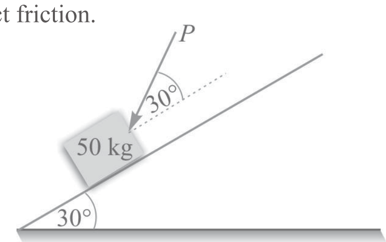
25. **Assertion:** No external force acts on system of two spheres which undergo a perfectly elastic head on collision. The minimum kinetic energy of this system is zero if the net momentum of this system is zero.
Reason: In any two body system undergoing perfectly elastic head on collision, at the instant of maximum deformation, the complete kinetic energy of the system is converted to deformation potential energy of the system.

26. **Assertion:** A sphere of mass m moving with speed u undergoes a perfectly elastic head on collision with another sphere of heavier mass M at rest (M > m), then direction of velocity of sphere of mass m is reversed due to collision [no external force acts on system of two spheres]
Reason: During a collision of spheres of unequal masses, the heavier exerts more force on lighter mass in comparison to the force which lighter mass exerts on heavier mass.

DIRECTIONS (Qs. 27–30) : *Following are integer based/ Numeric based questions. Each question, when worked out will result in one integer or numeric value.*

27. A block of mass 1.2 kg moving at a velocity of 20 cm/sec collides elastically with similar block at rest e = 3/5. Find loss in kinetic energy.

28. In fig. what constant force P is required to bring the 50 kg body, which starts from rest, to a velocity of 10 m/s in 7 m? Neglect friction.



29. A metal ball of mass 2 kg moving with a velocity of 36 km/h has a head on collision with a stationary ball of mass 3 kg. If after the collision, the two balls move together, find the loss in kinetic energy due to collision.

30. A ball moving with velocity 2 m/s collides head on with another stationary ball of double the mass. If the coefficient of restitution is 0.5, then what will be their velocities (in m/s) after collision ?

SOLUTIONS

Brief Explanations of Selected Questions

1 EXERCISE

FILL IN THE BLANKS :

1. total
2. 746
3. 20
4. $\sqrt{2}$
5. the same
6. zero
7. m_2/m_1
8. motion
9. 0°
10. Scalar
11. potential energy
12. energy
13. kinetic
14. negative

TRUE/FALSE :

1. True
2. True
3. False
4. False
5. True
6. True
7. True
8. False
9. True
10. False
11. False
12. True
13. False
14. False
15. True

MATCH THE COLUMNS :

1. (A) → (p); (B) → (q); (C) → (q); (D) → (s)
2. (A) → (q); (B) → (s); C → (p); (D) → (r)

VERY SHORT ANSWER QUESTIONS :

1. Work done against gravity is *mgh*, as no work is done in the horizontal displacement.
2. No, work has to be done to compensate the energy loss due to friction.
3. Since, the gravitational force is conservative in nature, so work done against it depends only on initial and final points, not on time.
4. Yes, he is doing work to oppose the river current.
5. $K.E = \frac{p^2}{2m}$ $\quad\because\ p$ is same, $K.E \propto \frac{1}{m}$

   ∴ Light body will have greater kinetic energy.
6. Distance = $\frac{K.E}{\text{Retarding force}}$

   Here the retarding force = frictional force.
   Since mass of the truck is more, so its frictional force will be more.
   ∴ The truck will come to rest at a lesser distance.
7. Yes, it is possible that a force is acting on a body but still the work done is zero. For example, consider a coolie carrying a load on his head across a level road. He exerts an upward force equal to the weight of the load. But the displacement of the load is in the horizontal direction. Thus the angle between force F and displacement s is 0º. Hence

   W = Fs cos 90° = 0.
8. When an object does work, it loses energy. When work is done on an object, the object gains energy.
9. The muscular energy obtained from food through chemical reactions gets stored in our body. We use this energy in throwing the ball. The muscular energy gets converted into kinetic energy of the ball.
10. The commercial unit of electric energy is kilowatt-hour (kWh) or Board of Trade (B.O.T.) unit. It is defined on the electric energy consumed by an appliance of power 1000 watt in one hour.

    1 kWh = 1 kW × 1 h = 1000 W × 1 h
    = 1000 Js^{-1} × 3600 s.
    or 1 kWh = 3600000 J = 3.6×10^6 J
    The electrical energy used in households, industries and commercial establishments is expressed in kilowatt hour. The energy used is expressed in terms of 'units' in our monthly bills. Here 1 unit means 1 kilowatt hour.

SHORT ANSWER QUESTIONS :

1. Positive work:
   W = F . s = Fs cosθ
   When θ is acute (> 90°), cos θ is positive. Hence work done is positive.
   For example :
   (i) When a body falls freely under the action of gravity, θ = 0°, cos θ = cos 0 = +1. Therefore, work done by gravity on a body falling freely is positive.
   (ii) When a lawn roller is pulled by applying a force along the handle at an acute angle, work done by the applied force is positive.

   Negative work :
   W = F . s = Fs cosθ
   ∴ When θ is obtuse (< 90°), cosθ is negative. Hence work done is negative.

   For example:
   (i) When a body is thrown up, its motion is opposed by gravity. The angle θ between gravitational $\vec{F}$ force and the displacement s is 180°. Fs cosθ = cos 180° = – 1, therefore, work done by gravity is negative.
   (ii) When brakes are applied on a moving vehicle, work done by the braking force is negative
2. When a car is moving on a flat road, it has to do work to overcome the friction of the road and air resistance but no work is done against the force of gravity. On the other hand, when the car is going up the hill, then in addition to

friction and air resistance, it has to do work against the force of gravity. Thus, a driver increases the speed of his car on approaching a hilly road to give more kinetic energy to the car so that it may go up against gravity.

3. All of these objects possess energies of the following forms :
(i) Potential
(ii) Both kinetic and potential
(iii) Both kinetic and potential
(iv) Potential
(v) Potential
(vi) Potential
(vii) Potential

4. The mechanical energy (K.E. + P.E.) of the falling water is converted into heat energy when water falls on the ground. Due to this heat energy, the temperature of water at the bottom of water-fall increases.

5. The heat energy produced due to combustion of coal converts water into steam. The heat energy of steam is converted into mechanical energy when it turns blades of a turbine. The mechanical energy so obtained is converted into electrical energy by the generators.

6. When we push immovable objects like a huge stone or a wall, we feel tired quickly, although the work done appears to be zero as there is no displacement. There is no doubt that the work done by us on the object is zero. However, the work done on our own body is not zero. Our muscles are stretched, our blood is displaced to the straining muscles and it is in making these displacements that energy is lost. It is because of this loss of energy that we feel tired.

7. (i) P.E. to K.E. and Electrical energy.
(ii) Chemical to electrical energy.
(iii) P.E. to K.E.
(iv) Chemical/Nuclear energy to K.E., sound energy.
(v) P.E., K.E. conversion.
(v) Mechanical energy to heat and sound energy.

2 EXERCISE

TEXT-BOOK QUESTIONS :

1. Work done = Force × Displacement
= 7 N × 8m = 56 Nm = 56 J

2. Work is said to be done whenever a force acts on a body and the body undergoes some displacement.

3. Work done = Force × Displacement

4. One joule of work is said to be done when a force of one newton displaces a body through a distance of 1 metre in its own direction.

5. Here, Force (F) = 140 N, Displacement (s) = 15 m
Work done, W = Fs = 140 N × 15m = 2100 J.

6. The energy possessed by an object by virtue of its motion is called its kinetic energy.

7. The kinetic energy possessed by an object of mass m moving with a uniform velocity v is

$$E_k = \frac{1}{2}mv^2$$

8. Mass of the object = m
Velocity of the object , $v = 5\ ms^{-1}$
K.E. = 25 J

$$K.E. = \frac{1}{2}mv^2$$

$$25 = \frac{1}{2} \times m \times (5)^2$$

or $m = 2kg$

(i) When velocity is doubled
velocity = 10 ms^{-1}, $m = 2$ kg

$$K.E. = \frac{1}{2}mv^2$$

$$= \frac{1}{2} \times 2 \times (10)^2 = 100\ J$$

(ii) When velocity is increased three times
velocity = 15 ms^{-1} , $m = 2$ kg

$$K.E. = \frac{1}{2}mv^2$$

$$= \frac{1}{2} \times 2 \times (15)^2 = 225\ J.$$

9. Power is defined as the rate of doing work or the rate of transfer of energy.

10. The power of an agent is one watt if it does work at the rate of 1 joule per second.

11. $P = \frac{W}{t} = \frac{1000J}{10s} = 100\ Js^{-1} = 100W$

12. The average power of agent may be defined as the total energy consumed by it divided by the total time taken.

$$\text{Average power} = \frac{\text{Total energy consumed}}{\text{Total time taken}}$$

$$P = \frac{W}{t}$$

TEXT-BOOK EXERCISE :

1. (i) Yes, Suma is doing work by pushing water in the backward direction.
(ii) No, because the force exerted by donkey in the upward direction is perpendicular to the horizontal displacement of the load.
(iii) Yes, work is done in lifting water against the force of gravity.
(iv) No, because the leaves of plants remain at rest during photosynthesis.
(v) Yes, engine is doine work in pulling the train. Both the applied force and displacement are in same direction.
(vi) No, because food grains remain at rest.
(vii) Yes, work is done by the wind in moving the sailboat.

2. Zero. This is because the net displacement of the object is in the horizontal direction while the force of gravity acts in the vertical downward direction. The vertical height is the same in the initial and final positions of the object hence displacement in vertical direction is zero.

3. First the battery converts chemical energy into electrical energy. Then the bulb converts this electrical energy into heat and light energy.

4. Here, m = 2 kg, u = $5 ms^{-1}$, v = $2 ms^{-1}$

Work done = Change in K.E.

or $W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}m(v^2 - u^2)$

$= \frac{1}{2}\times 20\times(2^2 - 5^2) = 10\times(4-25)$

$= -10\times 21 = -210$ J

The negative sign indicates the retarding nature of the applied force.

5. Displacement AB is horizontal.

Force of gravitation acts vertically downwards.

That is, force acts perpendicular to displacement.

$\therefore\ W = Fs\cos 90° = 0$

6. No. The law of conservation of energy is not violated. The loss in potential energy appears as an equal gain in kinetic energy of the object.

7. We use our muscular energy in pulling the bicycle. So, our muscular energy changes into kinetic energy. A part of the muscular energy is used in doing work against friction on the road. This part of the muscular energy changes into heat.

8. There is no transfer of energy between you and the rock. But you do work to expand and contract your muscles and to circulate blood faster than the normal ratio. Thus energy is spent on yourself.

9. 1 unit of energy = 1 kWh = 3.6×10^6 J

$\therefore$ 250 units of energy = $250 \times 3.6 \times 10^6$ J

= 9×10^8 J.

10. Here, m = 40 kg, g = $10 ms^{-2}$, h = 5 m

Potential energy of the object at a height of 5 m,

$E_p = mgh$

= 40 × 10 × 5 = 2000 J

When the object is half-way down (s = 2.5m), let its velocity be v.

Then $v^2 - u^2 = 2gs$

$v^2 - 0^2 = 2\times 10\times 2.5$

or $v^2 = 50$

$\therefore$ Kinetic energy of the object,

$E_k = \frac{1}{2}mv^2 = \frac{1}{2}\times 40\times 50 = 1000$ J

11. Zero. When the satellite moves around the earth, the force of gravity acts on it along the radius of its orbit while its direction of motion is along the tangent to the orbit at any point. Thus, force acts perpendicular to displacement. Hence, the work done on the satellite is zero.

12. Yes. For example, rain drops fall on the earth while the net force on them is zero.

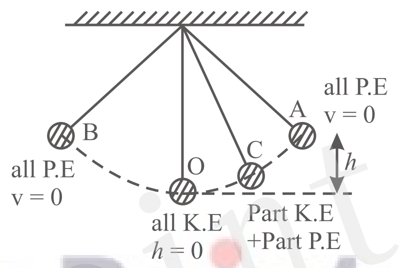
13. No. The bundle of hay remains stationary *i.e.*, displacement is zero. So, the work done is also zero.

14. Here P = 1500 W, t = 10 h

Energy used = P × t = 1500 W × 10 h

$= \frac{1500}{1000}$ kW × 10 h = 15 kWh.

15. **Conservation of energy during the osicllations of a simple pendulum. As** shown in figure, a simple pendulum consists of a spherical metal bob suspended by a thread from a fixed support. As the bob is displaced to end A, it gains potential energy. As it is released from rest, its kinetic energy begins to increase. At an intermediate position like C, the energy is partly kinetic and partly potential. At mean position O, the energy is totally kinetic. At the end B, again the energy becomes totally potential. Total mechanical energy (K.E. + P.E) remains constant at all points.



Conservation of energy in a simple pendulum

The energy of the body is gradually spent in doing work against the force of friction at the point of suspension and also against friction of air. The energy spent is converted into heat. There is no violation of the law of conservation of energy. Due to decrease in the energy of the bob, its amplitude of oscillation decreases with time and eventually the bob comes to rest.

16. Work done on object = Change in K.E. of the object

$= \frac{1}{2}mv^2 - \frac{1}{2}m(0)^2 = \frac{1}{2}mv^2$

17. Here, m = 1500 kg

v = 60 km/h = $\frac{60\times 1000\,m}{3600\,s} = \frac{50}{3}$ m/s

Work required to be done to stop the car,

W = Change in K.E. of the car

$= \frac{1}{2}mv^2 - \frac{1}{2}m(0)^2 = \frac{1}{2}mv^2$

$= \frac{1}{2}\times 1500\times\left(\frac{50}{3}\right)^2 = 208333.3$ J

18. (a) In this case force is perpendicular to displacement. (θ = 90°)

$W = Fs\cos 90° = 0$ (zero work done)

(b) In this case F and s are in same direction. (θ = 0°)

$W = Fs\cos 0° = Fs$ (positive work done).

(c) In this case F and s are in opposite direction. $(\theta = 180°)$

$W = Fs\cos 180°$

$= Fs(-1) = -Fs$ (negative work done).

19. Yes, the acceleration of the object would be zero when the several forces acting on the object add up to give a zero resultant force.

$$a = \frac{F}{m} = \frac{0}{m} = 0$$

20. Here, $P = 500\ W = \frac{500}{1000}\ kW = 0.5\ kW$

$t = 10\ h$

Energy consumed by four devices

$= 4(p \times t) = 4 \times 0.5\ kW \times 10h$

$= 20\ kWh.$

21. The kinetic energy of the object changes into heat and sound.

EXEMPLAR QUESTIONS :

1. Initial velocity = v, then $v' = 3\,v$

Initial kinetic energy $= \frac{1}{2}\,m\,v^2$

Final kinetic energy (K.E.) $= \frac{1}{2}mv'^2 = \frac{1}{2}m(3v)^2 = 9(\frac{1}{2}mv^2)$

(K.E) initials (K.E) final = 1 : 9

2. $F = 5N$

$W = F.S$

$W = 5 \times [1500 + 200 + 2000] = 18500J.$

3. Since, weight of the person on planet A is half that on the earth, acceleration due to gravity there, will be 1/2 that on the earth. Hence he can jump double the height with the same muscular force.

Or

The potential energy of the person will remain the same on the earth and on planet A

Thus, $m\,g_1h_1 = m\,g_2h_2$

if $g_1 = g$ then $g_2 \_\ g$, $h_1 = 0.4$

Then $h_2 = \frac{g_1h_1}{g_2} = \frac{g \times 0.4}{g/2}$

or $h_2 = 0.4 \times 2 = 0.8m$

4. $v^2 - u^2 = 2\,a.s$

This gives $s = \frac{v^2 - u^2}{2a}$

$F = m\,a$

we can write work done (W) by this force F as

$$W = ma\left[\frac{v^2 - u^2}{2a}\right] = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = (K.E)_f - (K.E)_i$$

5. $m\,g\,h = m \times 10 \times 10 = 100\ m\ J.$

Energy is reduced by 40% then the remaining energy is 60 m J.

Therefore, $60\ m = m \times 10 \times h'$ or $h' = 6\ m$

HOTS QUESTIONS :

1. If a moving truck or a car is brought to a stop then the change in momentum is brought to zero.

As both the truck and the car are traveling at the same speed but the truck is more heavier than the car therefore momentum of the truck is more and hence a greater force will be required to change its momentum to zero.

A moving car could require more stopping force if the time during which its momentum is brought to zero is reduced considerably. For example, if the time of impact is reduced by 10, then force of impact is extended to 10 times.

2. A punch with a bare fist is more forceful than with a boxing glove. A boxing glove allows a longer time of impact thus decreasing the force of impact whereas a punch with a bare fist has smaller time of impact and hence it has greater force of impact.

3. When the boxer is punching a heavy bag, the bag provides impulse to stop the punches and reduce the momentum to zero. The boxer's punch acts as action to the bag and the bag exerts an equal and opposite force of reaction to the boxer and thus stops the punches. Therefore the boxer can punch the bag for an hour without tiring.

While boxing with an opponent, punches are missed because the opponent bends sideways to protect himself and thus missed punches are in air. In this case air molecules will be providing impulse to stop the punches and that will be very small. There will be no enough impulse to bring the momentum to zero.

As a result the body of boxer will be pulled along with the punches and hence he will tire quickly.

4. Momentum is a vector quantity having both magnitude and direction whereas kinetic energy is a scalar quantity having only magnitude. Momentum that is directional is capable of being cancelled entirely.

The vector sum of the momenta of a flock of birds in flight can be zero because of birds flying in different directions in the flock.

Each flying bird has some kinetic energy and the algebraic addition of the kinetic energies of all birds in the flock cannot be zero.

5. Let initial kinetic energy, $E_1 = E$

Final kinetic energy, $E_2 = E + 300\%$ of $E = 4E$

As

$$P \propto \sqrt{E} \Rightarrow \frac{P_2}{P_1} = \sqrt{\frac{E_2}{E_1}} = \sqrt{\frac{4E}{E}} = 2$$

$\Rightarrow P_2 = 2P_1$

$\Rightarrow P_2 = P_1 + 100\%$ of P_1

i.e. momentum will increase by 100%.

6. $E = \frac{P^2}{2m}$ if P = constant then $E \propto \frac{1}{m}$

According to problem $m_1 > m_2$

$\therefore\ E_1 < E_2$

7. $P = \vec{F}.\vec{v} = ma \times at = ma^2t$ [as u = 0]

$$= m\left(\frac{v_1}{t_1}\right)^2 t = \frac{mv_1^2t}{t_1^2} \qquad [\text{as } a = v_1/t_1]$$

8. The gravitational potential energy it loses turns into kinetic energy, so

$(E_{grav})_{top} = (E_{kin})_{bottom}$ $mgh_{top} = \frac{1}{2}mv^2{}_{bottom}$

$$v = \sqrt{2gh} = \sqrt{2\,(9.8\ m/s^2)\,(0.15m)} = 1.7\ m/s$$

9. No, since work done W = mgh.

10. $m = \frac{E_{grav}}{gh} = \frac{20000J}{(9.8m/s^2)\,(12m)} = 170\ kg$

11. The kinetic energy of the body changes into its potential energy.

12. In the second case.

3 EXERCISE

SINGLE OPTION CORRECT :

1. (c) Work done (W) = Force (F) × displacement (d)
= 10 × 5 = 50 joule

2. (c) **3.** (c) **4.** (d) **5.** (d)

6. (d) Work done $= \int Fdx$

7. (a)

8. (c) **9.** (c) **10.** (d) **11.** (b) **12.** (b)

13. (d) **14.** (a) **15.** (b) **16.** (b) **17.** (a)

18. (c) **19.** (c) **20.** (b) **21.** (b) **22.** (d)

23. (d) **24.** (b) **25.** (c) **26.** (d) **27.** (d)

28. (b) **29.** (d) **30.** (a) **31.** (d)

32. (c) As $\frac{1}{2}m_A v_A^2 = \frac{1}{2}m_B v_B^2$

$$\frac{v_A}{v_B} = \sqrt{\frac{m_B}{m_A}}\ ;$$

$$= \frac{m_B}{m_A}\sqrt{\frac{m_A}{m_B}} = \sqrt{\frac{m_B}{m_A}} = \frac{1}{\sqrt{3}}$$

33. (d) As $E = \frac{p^2}{2m}$ $\therefore \frac{dE}{E} = 2\left(\frac{dp}{p}\right) = 2\times 5\% = 10\%$

MORE THAN ONE OPTION CORRECT :

1. (b, c) **2.** (c, d) **3.** (a, c)

4. (c, d) **5.** (a, d) **6.** (b, d)

7. (a, b, c, d)

$$x = u\cos\theta\ t$$

$$y = u\sin\theta t - \frac{1}{2}gt^2$$

$$E = \frac{1}{2}mu^2 - mgy$$

$$= \frac{1}{2}mu^2 - mg\ u\sin\theta t + \frac{1}{2}mg^2t^2$$

$$= \frac{1}{2}mu^2 - mg\tan\theta x + \frac{mg^2}{u^2\cos^2\theta}x^2$$

Also $E = \frac{P^2}{2m}$

Hence all the graphs are correct.

8. (b, c, d)

MULTIPLE MATCHING QUESTIONS :

1. (b) (A) → (t); (B) → (s); (C) → (p, r); (D) → (q)

PASSAGE BASED QUESTIONS :

1. (d) **2.** (a)
Mass of liquid pumped M = a × h × d
Height through which it is pumped
= 2h – h = h
∴ Work done = mgh
= (ahd) gh = dgh^2a

Power consumed $P = \frac{W}{t}$

$$= \frac{dgh^2a}{t}$$

$$= \frac{0.2\times 9.8\times (20)^2\times 2}{20}$$

$= 78.4\ JS^{-1}$

ASSERTION & REASON :

1. (c) **2.** (a) **3.** (a) **4.** (c) **5.** (d)

6. (d)

7. (a) $W = \vec{F}.\vec{s} = Fs\cos\theta$

if θ = 90°, W = 0

INTEGER/NUMERIC TYPE QUESTONS :

1. (a) W = F s cos 90° = zero

2. (c) W = F s cos θ = 10 × 2 cos 60° = 10 J.

3. (c) $W = \vec{F}.\vec{s} = (5\hat{i} + 2\hat{j}).(2\hat{i} + 3\hat{j}) = 10 + 6 = 16\ J.$

4. (d) Forward thrust, $F = \frac{P}{v} = \frac{100\times 746}{20} = 3730\,N.$

5. (b) Total energy at the time of projection

$$= \frac{1}{2}m\,v^2 = \frac{1}{2}\times 0.1(20)^2 = 20J$$

Half way up, P.E. becomes half the P.E. at the top i.e.

P.E. $= \frac{20}{2} = 10J$ ∴ K.E. = 20 – 10 = 10J.

6. (d) As $E = \frac{p^2}{2m}$ $\therefore \frac{dE}{E} = 2\left(\frac{dp}{p}\right) = 2\times 5\% = 10\%$

4 ADVANCED EXERCISE
BASED ON CONNECTING TOPICS

1. (b) ΔK.E = total work done on a particle only if the forces on the body are conservative.
2. (b) 3. (a) 4. (d)
5. (b) At the top of flight, horizontal component of velocity

$= u\cos 45° = u/\sqrt{2}$

$\therefore\ K.E. = \frac{1}{2}m\left(\frac{u}{\sqrt{2}}\right)^2 = \frac{1}{2}\left(\frac{mu^2}{2}\right) = \frac{1}{2}K.$

6. (c) Work done $= \frac{1}{2}\times\text{Stress}\times\text{Volume}\times\text{Strain}$

$= -\times\text{Force}\times\text{Extension} = \frac{1}{2}\times 200\times 1\times 10^{-3}$

$= 0.1\text{J}$

7. (b) If $\vec{L} = 0 \Rightarrow K$.E may or may not be zero.

If K.E = 0, $\vec{L} = 0$.

8. (c) $v = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{a\times b + 0}{a + c} = \frac{a(b)}{a+c}.$

9. (d) As $e^n = \left(\frac{h_n}{h_0}\right)^{1/2}$ $\therefore\ h_n = e^{2n}h_0 = e^{2\times 2}h = e^4 h.$

10. (d) Applying the principle of conservation of linear momentum, we get

$3m\times v = \sqrt{(m\times 60)^2 + (m\times 60)^2}$

$v = 20\sqrt{2}$ m/s

11. (b) Work done against gravity

$W_g = 50\times 10\times 30 = 15$ kJ

Work done against friction

$W = \mu mg\cos\theta\times s = 0.4\times 50\times 10\times\frac{4}{5}\times 50 = 8\,\text{kJ}$

Total work done $= W_g + W_f = 15$ kJ $+ 8$ kJ $= 23$ kJ

12. (b) $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$

$mv^2 = kx^2$ or $0.5\times(1.5)2 = 50\times x^2$

$\therefore\ x = 0.15$ m

13. (c) Fraction of energy transferred $= \frac{4\times 2}{(1+2)^2} = \frac{8}{9}$

14. (a, d) Net force acting on the system is zero, hence force as well as momentum of both the blocks are equal and opposite.

15. (c, d) Acceleration and velocity change in direction, hence are not uniform.

16. (a, b, c)

$K = \frac{1}{2}mv^2$ and $p = mv$

$\therefore\ K = \frac{1}{2}pv$ or $2K = pv$

or $v = \frac{2K}{p}$

or $p = \sqrt{2mK}$

17. (b, d)
18. (a) (A) → (p); (B) → (r,s); (C) → (q,s); (D) → (q,t)
19. (b) For maxiumum angle,

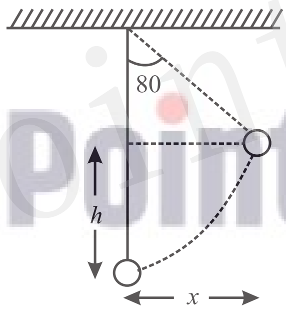
work done by F = gain in P.E of bob

or F_x = mgh

$F\,l\sin\theta_0 = mgl(1-\cos\theta_0)$

$\frac{1-\cos\theta_0}{\sin\theta_0} = \frac{F}{mg}$ or $\tan\theta_0/2 = \frac{F}{\text{mg}}$

or $\theta_0 = 2\tan^{-1}\left(\frac{F}{mg}\right)$



20. (a) Velocity is maximum when net force on the bob is zero.

Let the angle mode by the string with vertical at that instant be θ, then

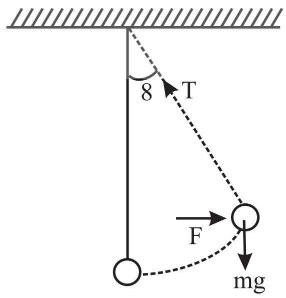
$T\cos\theta = mg - \theta$ and $T\sin\theta = F$

or $\theta = \tan^{-1}\frac{F}{mg}$

By work energy theorem,

$F\,l\sin\theta - mgl(1-\cos\theta) = \frac{1}{2}mv^2$

or $v = \left[\frac{u(\sqrt{F^2m^2g^2} - mg)}{m}\right]^{1/2}$



21. (d) At the position of maximum deflection,

$T = mg\cos\theta_0 + F\sin\theta_0$

using $\cos\theta_0 = \frac{1-\tan^2\theta_0/2}{1+\tan^2\theta_0/2}$ and $\sin\theta_0 = \frac{2\tan\theta_0/2}{W\tan^2\theta_0/2}$

We get

$T = mg$

22. (d) When frictional force is opposite to velocity, kinetic energy will decrease.

23. **(c)** $H = \frac{u^2}{2g} = \frac{10\times10}{2\times10} = 5m$

Initial height of C.M. = $\frac{m\times0+m\times20}{2m} = 10m$

Total height = 10 + 5 = 15m

24. **(b)** In elastic collision between two identical masses, the masses just exchange their velocities. So time would remain same as would had been it they had not collides.

$t_1 = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times20}{10}} = 2s$

$t_2 = \frac{2u}{g} = \frac{2\times20}{10} = 4s$

$\Delta t = t_2 - t_1 = 2s$

25. (c) For a system of two isolated sphere having non-zero initial kinetic energy, the complete kinetic energy can be converted to other forms of energy if the momentum of system is zero. This is due to the fact that for an isolated system, the net momentum remains conserved. If an isolated system has non-zero momentum. For the momentum to remain constant complete kinetic energy of the system cannot become zero. Hence statement 1 is true while statement 2 is false.

26. **(c)** Statement 2 contradicts Newton's third law and hence is false.

27. $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

From law of conservation of momentum

$1.2 \times 20 + 0 = 1.2\, v_1 + 1.2\, v_2$

$\therefore\ v_1 + v_2 = 20$ cm/sec ....(1)

$e = \frac{3}{5} = \frac{v_2 - v_1}{u_1 - u_2}$

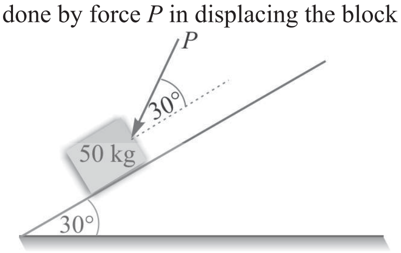
$\therefore\ v_2 - v_1 = 12$ cm/sec ....(2)

From eq^{n}s. (1) and (2)

$v_1 = 4$ cm/s and $v_2 = 16$ cm/s

$\therefore\ \Delta K = \frac{1}{2}m_1u_1^2 - \frac{1}{2}m_1v_1^2 - \frac{1}{2}m_2v_2^2 = 7.7 \times 10^{-3}$ J

28. Work done by force P in displacing the block by 7m



$W_1 = Fs\cos\theta$

$= P \times 7\times \cos30° = \frac{7\sqrt{3}}{2}P$ J .

$W_2 = mgh = 50 \times 9.8 \times 7 \sin30° = 1715.$

Using work-energy theorem, $W = \Delta$K.E.

or $W_1 + W_2 = \frac{1}{2}m(v_f^2 - v_i^2)$

or $\frac{7\sqrt{3}}{2}P + 1715 = \frac{1}{2}\times50\times(10^2 - 0^2)$

or $6.06\,P + 1715 = 2500$

$\therefore$ $P = 129.5$ N.

29. Apply conservation of momentum,

$m_1v_1 = (m_1 + m_2)v$

$v = \frac{m_1 v_1}{(m_1 + m_2)}$

Here $v_1 = 36$ km/hr = 10 m/s,

$m_1 = 2$ kg, $m_2 = 3$ kg

$v = \frac{10\times2}{5} = 4$ m/s

K.E. (initial) = $\frac{1}{2}\times2\times(10)^2 = 100$ J

K.E. (Final) = $\frac{1}{2}\times(3+2)\times(4)^2 = 40$ J

Loss in K.E. = 100 – 40 = 60 J

Alternatively use the formula

$-\Delta E_k = \frac{1}{2}\frac{m_1m_2}{(m_1+m_2)}(u_1 - u_2)^2$

30. Clearly $v_1 = 2$ ms^{-1}, $v_2 = 0$

$m_1 = m$ (say), $m_2 = 2m$

$v_1' = ?,\ v'_2 = ?$

$e = \frac{v_1' - v_2'}{v_2 - v_1}$ ....(i)

By conservation of momentum,

$2m = mv_1' + 2mv_2'$ ... (ii)

From (i), $0.5 = \frac{v_2' - v_1'}{2}$

$\therefore\ v_2' = 1 + v_1'$

From (ii), $2 = v_1' + 2 + 2\,v_1'$

$\Rightarrow\ v_1 = 0$ and $v_2 = 1$ ms^{-1}

Chapter 5

SOUND

INTRODUCTION

Sound is a form of energy that we hear. A vibrating object i.e., anything that moves back and forth, to-and-fro from side to side, in and out and up and down produces sound, as the object (vibrating) has a certain amount of energy. Sound requires material medium-a solid, a liquid or a gas to travel.

If there is no medium to vibrate then no sound is possible, sound cannot travel in a vacuum. Air is a poor conductor of sound compared with solids and liquids.

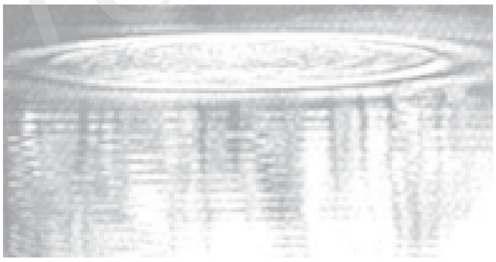
Sound is a longitudinal wave form travelling by forming compressions and rarefactions along the direction of propagation. Sound waves like other waves suffer reflection. Megaphone or speaking tube, ear trumpet or hearing aid or sound boards etc are based on reflection of sound.

Sound reflects from a smooth surface in the same way that light does – The angle of incidence is equal to the angle of reflection. Sometimes when sound reflects from the walls, ceiling and floor of a room, the surfaces are too reflective and the sound becomes Garbled. This is due to multiple reflections called reverberations.

PRODUCTION OF SOUND

Sound is a form of energy which produces a sensation of hearing in our ears.
Propagation of sound and light take place in the form of waves.
A source of vibration is normally a source of sound. When we pluck a string of guitar or sitar or veena it produces sound. Similarly vibrations of wings of bee or mosquito.
Sound is produced by vibrating source and is transmitted through a material medium producing sensation of hearing in our ears.
Let us think about an incidence of our daily life which is an experience of all of us. When we throw a pebble on the standstill surface of water then a disturbance is produced at that place. After some time the effect of this disturbance is seen on the other regions of the water surface. If a cork is placed on the surface of water, in this situation, we see that cork keeps on vibrating, up and down, at its place while the disturbance propagates ahead. Similarly if one end of a string is made to vibrate then we see that the pulses in the string propagate, speedily, one after the other.
If you sit on the shore and watch the waves roll in, you can see them pounding against the rocks and rolling the sand. They carry a lot of energy, transporting it from far out at sea. Yet a boat 100 metre from shore is not carried in with the waves. It just bobs up and down. Waves keep coming in to shore, but the water merely moves up and down, or back and forth. The waves deliver energy, not water, to the shore.
Similarly, we hear sounds from various sources like humans, birds, bells, machines, vehicles, televisions, radios etc.
From all everyday examples it is clear that we require concept that can explain transportation of energy from one place to other without transporting matter and hence the concept of wave comes.
The disturbance is called a wave and its propagation is called the wave motion.
ACTIVITY: To show that waves transfer energy from one part of medium to another part.

- Drop a piece of stone in a pond full of water. Water waves (ripples) will start in all the directions from the place where the stone was dropped, in the form of rings.
- Place a cork piece on stationary surface of water. As the water waves reach the cork piece, it begins to move up and down at the same place. The motion of cork piece also represents the motion of water molecules. Hence water molecules move up and down about their mean position and do not move forward along with the wave.

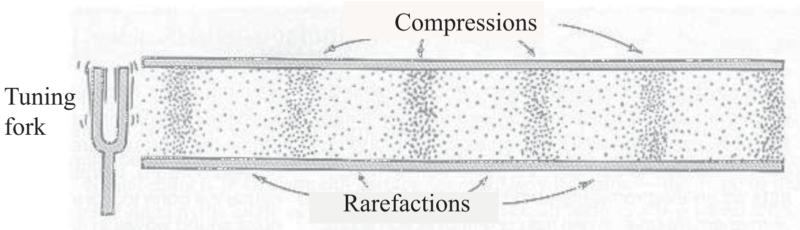


Ripples on the surface of water

- Up and down motion of water molecules indicates that they must have received energy from the waves travelling on water surface. This energy must have been generated at the place where the stone was dropped in water.
- It shows that waves transfer energy from one part of medium to the another part.

SOURCE OF SOUND AND ITS PROPAGATION

The motion of a vibrating source sets up waves in the surrounding medium.
When the prongs are in mean position, the air in their surroundings has normal density.
As the right prong moves out towards right, it pushes the air layers to the right. This produces a compression
The prong returns inwardly to mean position. The compression moves to the right. The air near the prong again has normal density.



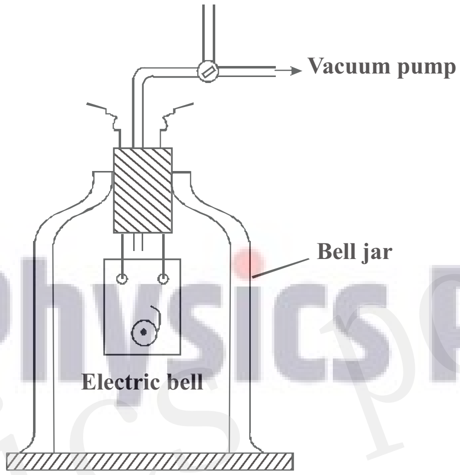
As the prong continues moving towards extreme left, vacating the space, density of air falls in the region and a rarefaction is produced. As the prong moves back to right extreme. it completes one vibration. Also the motion of the prong produces a new compression. This completes one wave.

Since one vibration of the prong has generated one wave in the medium (air), in one second as many waves will be generated as the number of vibrations that the tuning fork will make in one second. This number is called frequency of the tuning fork. (This number is engraved on the tuning fork near the bend). Hence we conclude that the wave frequency, (the number of waves being generated per second) equals frequency of the tuning fork.

Sound needs material medium to travel. If there is no medium solid, liquid or gas then no sound is possible. Sound cannot propagate in a vacuum.

ACTIVITY: To show Sound needs a material medium for its propagation

- An electric bell is suspended inside a bell jar by its leads.
- A pipe through the cork leads out to vacuum pump (pump which draws the air out of a vessel).



- The bell is connected to a battery through a key.
- The bell is started by closing the key. Initially when jar has normal air inside it, sound waves produced by the ringing bell heard outside the jar.
- The vacuum pump is started and the air from inside the jar is gradually drawn out. With decreased air inside the jar, sound heard becomes weaker and weaker but the striker can still be seen to be moving, after sometime no sound is heard. This suggests that sound needs material medium to travel.

WAVE AND ITS TYPES

Due to the vibratory motion of the particles of the medium a periodic disturbance is produced in a material medium. This is called a **wave**.

In the absence of medium solid, liquid or gas around the source, sound wave is not being propagated and light (electromagnetic) waves travel through the vacuum.

Types of Waves

On the basis of the requirement of medium, waves are of two types

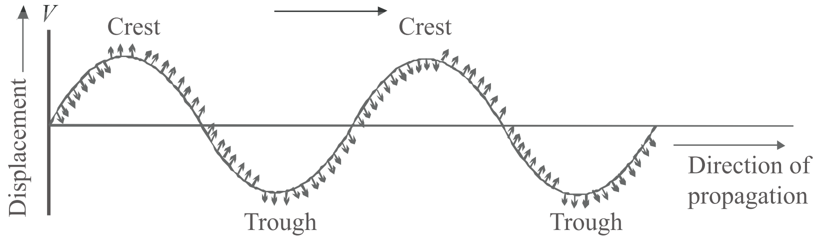
(i) Mechanical waves (ii) Electromagnetic waves

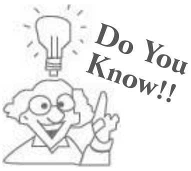
(i) Mechanical Waves

A mechanical wave is a periodic disturbance which requires a material medium for its propagation. The properties of these waves depend on the medium so they are known as elastic waves, such as sound-waves, water waves, waves in stretched string etc. On the basis of motion of particles the mechanical waves are classified into two parts.

(a) Transverse wave (b) Longitudinal wave

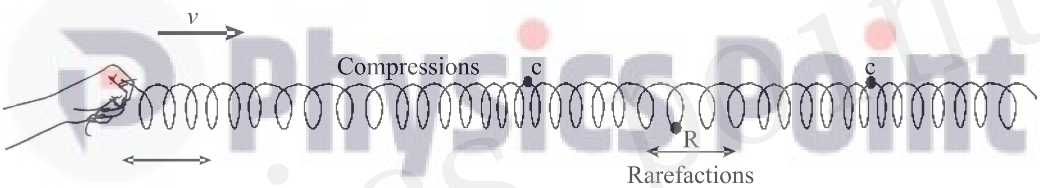
(a) Transverse wave : When the particles of the medium vibrate in a direction perpendicular to the direction of propagation of the wave, the wave is known as the transverse wave. For example, waves produced in a stretched string, waves on the surface. These waves travel in the form of crests and troughs. *These waves can travel in solids and liquids only.*

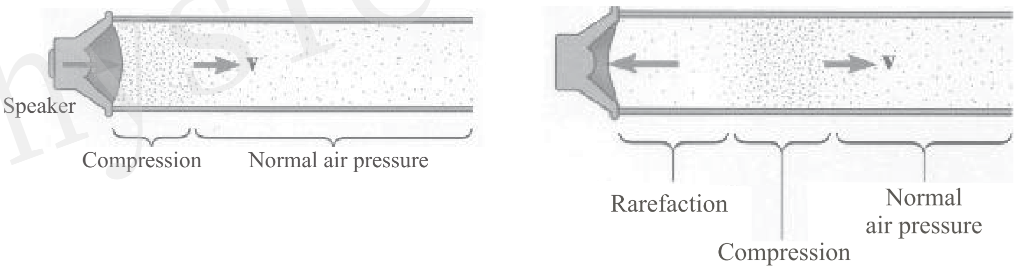




A periodic wave is that in which displacement repeats itself after a distance equal to the wavelength (λ) or after a time equal to the period (T) of the wave.

(b) Longitudinal wave : When the particles of the medium vibrate along the direction of propagation of the wave then the wave is known as the longitudinal wave. For example sound wave in air, waves in a solid rod produced by scrabbing etc.
These waves travel in the form of compressions and rarefactions. *These waves can travel in solids, liquids and gases.*





Speaker membrane expands, creating a region where the air molecules are packed closely together, a "compression". The air pressure in a compression is higher than normal.

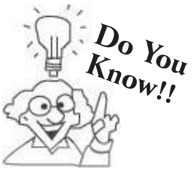
As the membrane moves back, a region is left behind where few molecules are located, a "rarefaction". Meanwhile, the compression moves forward.

(ii) Electromagnetic Waves

The waves which do not require medium for propagation are called electromagnetic waves. This means that these waves can travel through vacuum also. For example, light waves, X-rays, γ-rays, infrared waves, radio waves, microwaves, etc. These waves of transverse nature.

Difference between sound waves and electromagnetic waves

(a) Sound waves are longitudinal and electromagnetic waves are transverse.
(b) Sound waves travel at a speed of 340 m/s whereas electromagnetic waves travel at a speed of 3×10^8 m/s
(c) Sound waves do not pass through a vacuum but electromagnetic waves (light) do.

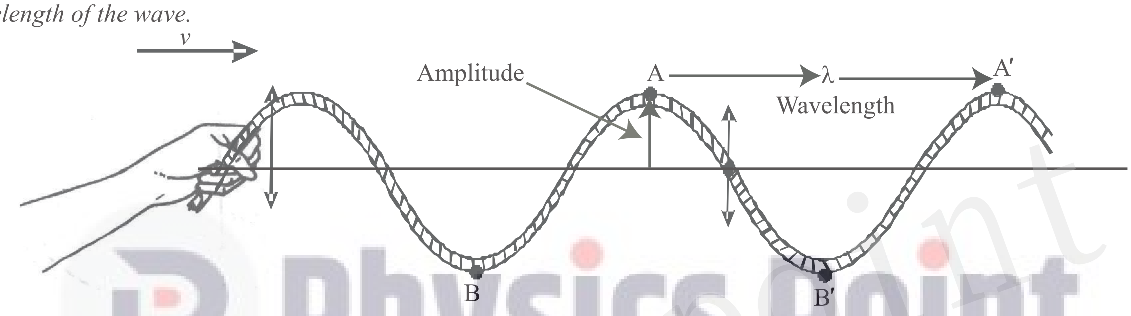


1. *For a wave motion to take place in a medium, the medium must possess the properties of elasticity and inertia.*
2. *A wave motion is not the bodily motion of the medium. In a wave motion, the transfer of energy (not the matter) takes place from one point of the medium to the other.*

CHARACTERISTICS OF A SOUND WAVE-FREQUENCY, AMPLITUDE AND SPEED

Travelling Waves

Consider the two points marked A and A' in figure. At the moment the picture was made, both were at the peak of crest. Similarly, points B and B' both are at the bottom of a trough at the same moment. The distance between A and A' or between B and B' is called the wavelength of the wave. *In a longitudinal wave, distance between two points of greatest compression or rarefaction is called the wavelength of the wave.*



As the crest travels from A to A' goes through one full cycle, from crest to trough and back to crest again. Time taken in one complete vibration (full cycle) is called its **time period**. This gives us an important relationship between period and wavelength. Since the wave travels one wavelength (λ, the Greek letter lambda) in the time of one period, the speed of wave (v) is given as

$$v = \lambda/T$$

Let us consider another term **frequency** defined as *the number of vibrations (or oscillation) completed by a particle in one second.* One oscillation is numerically equivalent to one complete wavelength. Thus the frequency of a wave may be regarded as the number of complete wavelengths traversed by the wave in one second. Remember that the frequency is characteristic of the source which produces the disturbance in the medium.

It is usually represented by ν (Greek letter Nu). Its **SI unit** is hertz (with unit symbol Hz).

The frequency ν of oscillation is characteristic of its source that produces the disturbance. Different sources produce oscillations of different frequencies. Thus, the wavelength λ of these source will be different. However these will be change in such a way that the product $\nu\lambda = V$, remains a constant in the given medium. The wave velocity remains the same for a given medium under the same physical conditions.

Let us consider another very important term **amplitude (A)** i.e., *the maximum displacement of the particles of medium from their normal position when a wave passes through it*. It defined as the relation between wave velocity, frequency and wavelength for a periodic wave.

The frequency ν of the wave is the reciprocal of its time period T, i.e., —

Wave velocity can be defined as the distance covered by a wire in one time period. Therefore,

$$\text{Wave velocity} = \frac{\text{distance covered}}{\text{time taken}} = \frac{\text{wavelength}}{\text{time taken}}$$

or $\qquad v = \lambda/T \qquad$ .... (1)

As $v = \dfrac{1}{T}$, eq. (1) connecting v and λ in terms of the frequency ν can be written as

$$v = \nu\lambda \qquad \text{........ (2)}$$

or Wave velocity = frequency × wavelength

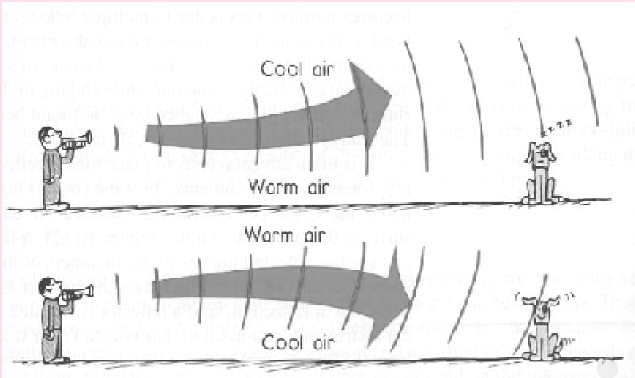
Thus, the wave velocity is the product of frequency and wavelength. It holds true for any periodic wave, both for transverse or longitudinal waves.

Knowledge ENHANCER

The speed with which a sound wave travels in air depends on the temperature. Waves travel faster in warmer air. The speed (in ms^{-1}) at any temperature can be found from this equation :

$$v_{air} = 331 + (0.6)T_C$$

where T_C is the Celsius temperature. Speed of sound is maximum in solids.



Sound travels faster in warm air.
Lower part of wavefront gets ahead of upper part, so front turns upward.

Do You Know!!

In a progressive or travelling wave, all particles of the medium have the same amplitude but different phases at a given time.

SPEED OF SOUND

Speed of sound through any medium depends upon elasticity and density of medium.

(i) In solids, $v = \sqrt{\frac{Y}{d}}$; Y = Young's modulus of elasticity, d = density of solid

(ii) In liquids, $v = \sqrt{\frac{B}{\rho}}$; B = Bulk modulus, ρ = density of liquid

(iii) In gases, $v = \sqrt{\frac{\gamma P}{\rho}}$ (Newton-Laplace formula)

$$v = \sqrt{\frac{\gamma RT}{M}} \; ; \; \gamma = \frac{C_P}{C_v}, \; M = \text{molecular mass}$$

Propagation of sound in a gas is adiabatic phenomenon.

Do You Know!!

The density of water vapour is less than that of the dry air. The moist air contains water vapour and not water as such. The moist air is less dense than dry air.

Factors Affecting the Speed of Sound

(a) **Temperature :** Speed of sound is directly proportional to the square root of absolute temperature $v \propto \sqrt{T}$.

$$\frac{v_1}{v_2} = \left(\frac{T_1}{T_2}\right)^{1/2}$$

If v_0 is the speed of sound at 0°C and v_t is speed of sound at t°C then

$$\frac{v_t}{v_0} = \left[\frac{t+273}{273}\right]$$

$$v_t = v_0\left(1+\frac{t}{273}\right)^{1/2} \Rightarrow v_t \simeq \left(1+\frac{t}{546}\right)$$

Change in speed of sound per 1°C = $\frac{v_t - v_0}{v_0} = \frac{1}{546} = 0.61\ \text{ms}^{-1}$

(b) **Pressure :** As pressure is changed, the density of gas changes in the same proportion, speed of sound remains unchanged.Thus speed of sound is independent of pressure.

(c) **Density :** Speed of sound is inversely proportional to the square root of density of the gas.

$$v = \sqrt{\frac{\gamma P}{\rho}} \Rightarrow v \propto \frac{1}{\sqrt{\rho}} \text{ or, } \frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

(d) **Humidity :** Humid air is lighter than dry air that is why speed of sound increase as humidity increases.

CHECK Point

The velocity of sound is generally greater in solids than in gases at N.T.P. why?

Solution

Both the elasticity and density of the solids are very large as compared to that of the gases. The effect of high value of elasticity of solids is to increase the speed of the sound, whereas the effect of density is to decrease. However, the effect of elasticity weighs heavier upon the effect of density and hence the speed of sound is greater in solids than in gases.

ILLUSTRATION : 1

If the period of small ripples on water is 0.1 s and their wavelength is 5 cm, what is the speed of the waves ?

SOLUTION :

We know that, $v = \nu\lambda$

$$v = \frac{1}{T} = \frac{1}{0.1}$$

λ = 5cm = 0.05 m (given)

$$\therefore \quad v = \frac{1}{0.1} \times 0.05 = 0.5\text{m/s}$$

ILLUSTRATION : 2

A wave pulse on a string moves a distance of 8m in 0.05s.

(a) Find the velocity of the pulse.

(b) What would be the wavelength of the wave on the same string if its frequency is 200 Hz ?

SOLUTION :

(a) Velocity of the wave pulse $v = \frac{\text{distance covered}}{\text{time}} = \frac{8\text{m}}{0.05\text{s}} = 160\,\text{m/s}$

(b) The periodic wave has the same velocity as that of the wave pulse on the same string.

Wavelength of a wave $\lambda = \frac{v}{\nu} = \frac{160\,\text{m/s}}{200\text{Hz}} = 0.8\text{m}$

Thus the wavelength of the periodic wave is 0.8m.

ILLUSTRATION : 3

The frequency of a source of sound is 100 Hz. How many times does it vibrate in a minute ?

SOLUTION :

Frequency of source of sound, $N = 100$ Hz
Time interval, $t = 1$ minute = 60s
Number of vibrations in 1 minute, $n = ?$
From relation, number of vibrations = vibrations per second × time
i.e., $n = vt$,
Putting values, we get, number of vibrations, $n = 100 \times 60 = 6000$

ILLUSTRATION : 4

25 waves pass through a point in 5 seconds. If the distance between one crest and the adjacent trough is 0.05m, calculate (a) the frequency (b) the wavelength (c) wave velocity.

SOLUTION :

(a) In 5 seconds, the number of waves produced = 25

$\therefore$ In 1 second, the number of waves produced = $\frac{25}{5} = 5$

$\therefore$ Frequency = 5 Hz

(b) Distance between two consecutive crests = one wavelength = λ .

$\therefore$ Distance between one crest and the adjacent trough = $\lambda/2$

Also the distance between one crest and the adjacent trough = 0.05 m.

$\therefore \quad \frac{\lambda}{2} = 0.05\,\text{m}$

Wavelength, $\lambda = 2 \times 0.05$ m = 0.10 m

(c) Wave velocity $v = \nu\lambda = 5 \times 0.10 = 0.5$ m/s

ILLUSTRATION : 5

How fast will a wave travel in air at a temperature of 15°C.

SOLUTION :

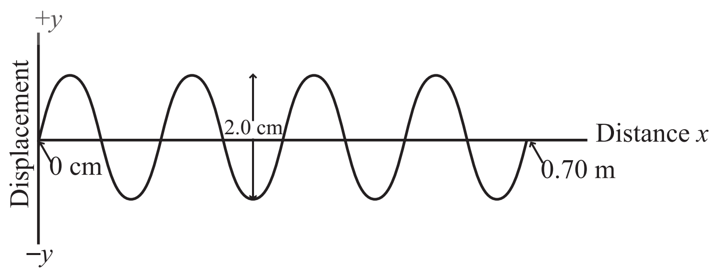
From knowledge enhancer section : $v_{air} = 331 + (0.6)T_e$

$\therefore \quad v_{air} = 331 + (0.6)(15)$

or, $\quad v_{\text{air}} = 340$ m/s

ILLUSTRATION : 6

A snapshot is taken of a periodic wave travelling on a string as shown below. What is the amplitude and wavelength of the wave?



SOLUTION :

The height of the wave is seen from the "snapshot" to be 2.0 cm. The amplitude is only one-half of this, so $A = 1.0$ cm.
The wave is seen to have your wavelengths spread over a distance of 0.70 m so the wavelength

$$\lambda = 0.70\text{ m}/4 = 0.175\text{ m}$$

ILLUSTRATION : 7

A wave on a string has a frequency of 440 Hz and a wavelength of 1.3m. How fast does the wave travel ?

SOLUTION:

As we know, $v = f\lambda = (440\text{ Hz})(1.3\text{m}) = 570$ m/s

ILLUSTRATION : 8

(a) Speed of sound in air is 332 m/s at NTP. What will be the speed of sound in hydrogen at NTP if the density of hydrogen at NTP is (1/16) that of air?

(b) Calculate the ratio of the speed of sound in neon to that in water vapour at any temperature. [Molecular weight of neon $= 2.02 \times 10^{-2}$ kg/mol and for water vapours $= 1.8 \times 10^{-2}$ kg/mol]

SOLUTION :

The velocity of sound in air is given by $v = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$

(a) In terms of density and pressure $\frac{v_H}{v_{air}} = \sqrt{\frac{P_H}{\rho_H} \times \frac{\rho_{air}}{P_{air}}} = \sqrt{\frac{\rho_{air}}{\rho_H}}$ [as $P_{\text{air}} = P_{\text{H}}$]

or $v_H = v_{\text{air}} \times \sqrt{\frac{\rho_{air}}{\rho_H}} = 332 \times \sqrt{\frac{16}{1}} = 1328\text{ m/s}$

(b) In terms of temperature and molecular weight

$$\frac{v_{Ne}}{v_w} = \sqrt{\frac{\gamma_{Ne}}{M_{Ne}} \times \frac{M_W}{\gamma_W}} \quad [\text{as } T_N = T_W]$$

Now as neon is monoatomic ($\gamma = 5/3$) while water vapours polyatomic ($\gamma = 4/3$)

so $\frac{v_{Ne}}{v_w} = \sqrt{\frac{(5/3)\times 1.8\times 10^{-2}}{(1/3)\times 2.02\times 10^{-2}}} = \sqrt{\frac{5}{4}\times\frac{1.8}{2.02}} = 1.055$

ILLUSTRATION : 9

The velocity of sound in hydrogen at 0°C is 1200 m/s. When some amount of oxygen is mixed with hydrogen, the velocity decreases to 500 m/s. Determine the ratio of H_2 to O_2 by volume in this mixture, given that the density of oxygen is 16 times that of hydrogen.

SOLUTION :

Given that, velocity of sound in hydrogen at 0ºC = 1200 m/s

$$\Rightarrow \quad 1200 = \sqrt{\frac{\gamma P}{\rho_H}} \quad ....(i)$$

Let there be x volume of H_2 and y volume of O_2, then,

$(x + y)\rho_{\text{mix}} = x\rho_{\text{H}} + y\rho_0 = x\rho_{\text{H}} + 16\,y\rho_{\text{H}}$

$$\Rightarrow \quad \rho_{\text{mix}} = \frac{(x+16y)}{x+y}\rho_H$$

$\therefore \quad 500 = \sqrt{\dfrac{\gamma P(x+y)}{(x+16y)\rho_H}}$ ....(ii)

Dividing eqs. (ii) by (i),

$$\frac{12}{5} = \sqrt{\frac{x+16y}{x+y}} \quad \text{or} \quad \frac{x}{y} = \frac{2.2}{1} \text{ (ratio of } H_2 \text{ to } O_2\text{)}$$

CHARACTERISTICS OF MUSICAL SOUND

Sound is characterised by three parameters :

(i) Pitch or frequency (ii) Loudness (iii) Quality or timbre

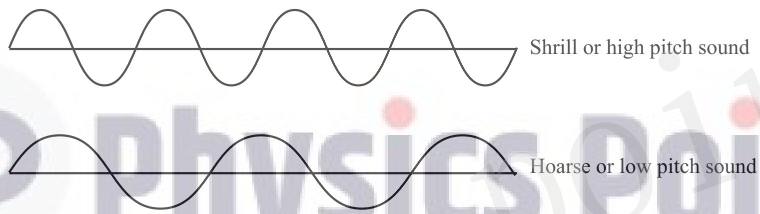
(i) Pitch or Frequency

Pitch is the sensation (brain interpretation) of the frequency of an emitted sound and is the characteristic which distinguishes a shrill (or sharp) sound from a grave (or flat) sound.

Faster the vibration of the source, higher is the frequency and higher is the pitch. Similarly low pitch sound corresponds to low frequency.

A high pitch sound is called a shrill sound (humming of a bee, sound of guitar).

A low pitch sound is called a hoarse sound (roar of a lion, car horn.)



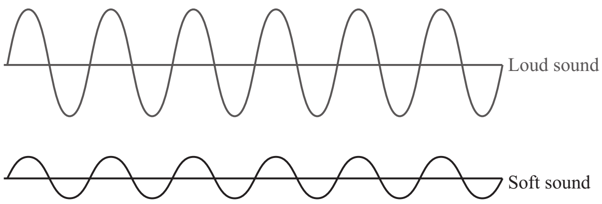
Examples : (a) Pitch of female voice is higher than male.

(b) The loudness being the sensation, depends upon the sensitivity of the listener's ear. Therefore, loudness of a sound of given intensity may be different for different listeners. Similarly, two sounds of equal intensity but different frequency may not appear to be equally loud even to the same listener because the sensitivity of the ear is different for different frequencies.

(c) In a tape-recorder or TV, bass and treble refer to low and high pitch respectively. So at bass (or woofer on), low pitch, i.e., grave sounds such as to 'tabla' or 'dholak' become loud while at treble, high pitch, i.e., shrill sounds such as of flute or 'ghoonghroo' become loud.

(ii) Loudness

Loudness or softness of a sound wave is the sensation that depends upon its amplitude. The loudness of sound is a measure of the sound energy reaching the ear per second. When we strike a table top with more force, it vibrates and produces loud sound waves which have more amplitude. When struck with smaller force, vibrating table top produces soft sound waves which have less amplitude. A loud sound wave carries more energy and can be heard at large distance. Reduction in amplitude at large distance, makes the sound soft. In other words loudness is the sensation received by the ear due to intensity [intensity of sound wave is the amount of sound energy (energy carried by sound wave) passing per second normally through unit area] of sound.



In a record player if a record of 50 revolution per minute RPM, pitch will increase and sound will become shriller. If the same record is played at 30 RPM, pitch will decrease and so sound will become grave.

The loudness of sound is measured in **'decibel, dB**. The loudness of sound of people talking quietly is about 65 dB, the loudness of sound in a very noisy factory is about 100 dB.

Intensity of sound is the time rate at which the sound energy flows through a unit area.

The intensity and loudness are not the same. Intensity depends on the energy per unit area of the wave and it is independent of the response of the ear, but the loudness depends on energy as well as on the response of the ear.

Sound waves of the same intensity but of different frequencies usually have different loudness.

Knowledge ENHANCER

(1) The energy transmitted by a wave depends upon the frequency as well as the amplitude. If the frequency of a note is doubled, twice as many compressions and rarefactions strike the ear each second and more energy is received. In fact the energy in a wave is proportional to both $(\text{frequency})^2$ and $(\text{amplitude})^2$.

(2) Loudness level of various sound :

S.No.	Sources of sound	Loudness level	Effect of sound on human ear
1.	Whispering	10 dB - 25 dB	Just audible
2.	Radio or T.V. at low volume	30 dB - 40 dB	Quite audible-comfortable sound
3.	Conversation	50 dB - 60 dB	Moderately loud sound
4.	Light vehicles	60 dB - 70 dB	Very loud sound
5.	Mixer-grinder/busy crossing	70 dB - 80 dB	Very loud but tolerable sound
6.	Motor cycle-heavy vehicle	90 dB - 105 dB	Noise, very loud and uncomfortable
7.	Lightning	120 dB - 130 dB	Very uncomfortable loud sound
8.	Jet aeroplane	130 dB and above	Painful sound
9.	Ordinary breathing	10 dB	In general not noticed

(iii) Quality or Timbre

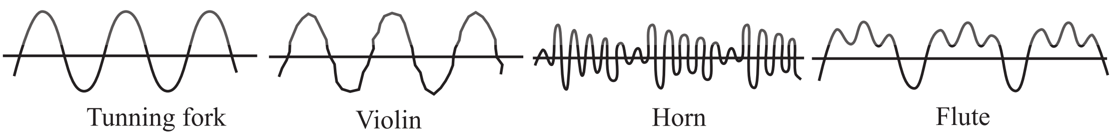
Quality or timbre of a sound wave is that characteristic which helps us in distinguishing one second from another having same pitch and loudness. We recognise a person (without seeing) by listening to his sound as it has a definite quality.

A pure sound of single frequency is called a **tone**.

An impure sound produced by mixture of many frequencies is called a **note**. It is pleasant to listen.

Notes of the same pitch played upon different musical instruments are distinguished from each other by their quality.

The quality of a note depends on the wave form. The waves produced by different instruments differ in their forms.



Knowledge ENHANCER

The ratio of frequencies of two notes sounded together is called their musical interval. The interval is said to be unison when the frequency ratio is 1 : 1. Some names are given below :

Name of interval	Frequency ratio
Unison	1 : 1
Octave	2 : 1
Major tone	9 : 8
Minor tone	10 : 9
Semi-tone	16 : 15

REFLECTION OF SOUND

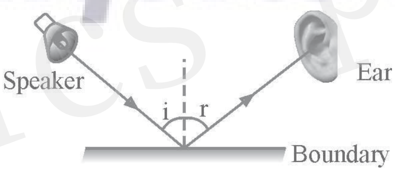
It is a common experience that when we shout into a well or inside an empty hall, or inside a dome, we hear our own sound after a short time. It happens because our sound is reflected from the walls.

When sound waves strike a surface, they return back into the same medium. This phenomenon is called reflection of sound.

Laws of Reflection of sound

(i) Angle of incidence $\angle i$ is equal the angle of reflection $\angle r$.

(ii) The incident wave, the reflected wave and the normal all lie in the same plane.



Uses of Reflection of Sound

(i) **Speaking tube or megaphone :** You must have seen in fairs or tourist spots, people using megaphones addressing a group of people. Megaphone is simply a horn-shaped tube. The sound waves are prevented from spreading out by successive reflections and are confined to the air in the tube. For the same reason, loud speakers also have horn-shaped openings.

(ii) The circular runway round the base of a dome is a whispering gallery. If someone whispers at a point close to the wall of the gallery, the sound gets reflected round the wall of the gallery. The sound can be distinctly heared at all points near the wall all around.

(iii) **Ear trumpet or hearing aid :** It is a device which is used by the persons who are hard of hearing. The sound waves received by the wide end of the trumpet are reflected into a much narrower area, leading it to the ear. This enhances the amplitude of vibrating layer of air inside the ear and helps in improving hearing.

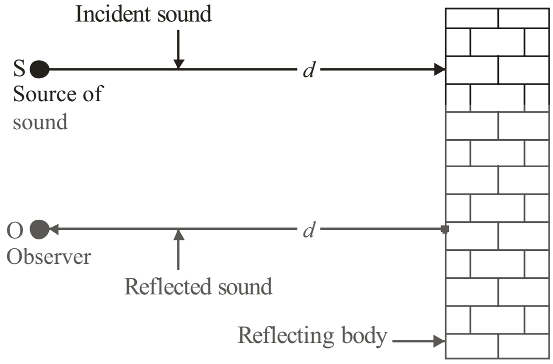
(iv) **Stethoscope :** It is an instrument used by the doctors for listening sound produced within the body, especially in the heart and lungs. In the stethoscope, the sound produced within the body of a patient to picked up by a sensitive diaphragm and then reaches the doctors ears by multiple reflection.

ECHO

The Phenomenon of *hearing back our own sound is called an echo*. It is due to successive reflection from the surfaces obstacles of large size.

Relation Between Speed of Sound, Time of Hearing Echo and Distance of Reflecting Body

If t is the time at which an echo is heard, d is the distance between the source of sound and the reflecting body and v is the speed of sound. The total distance travelled by the sound is $2d$.



Speed of sound, $v = \frac{2d}{t}$ or $d = \frac{vt}{2}$

The sensation of sound lasts or persists in our brain for 0.1s (1/10 sec) even after the source of sound has stopped vibrating.
Taking sound speed (v) to be 344 m/s.

$$d = \frac{344 \times 0.1}{2} = \frac{34.4}{2} = 17.2\,\text{m}$$

Thus, *for hearing distinct echoes, the minimum distance of the obstacle from the source of sound must be half of this distance i.e. 17.2 m from the source.*

Conditions for the formation of Echoes

(i) The minimum distance between the source of sound and the reflecting body should be 17.2 metres.

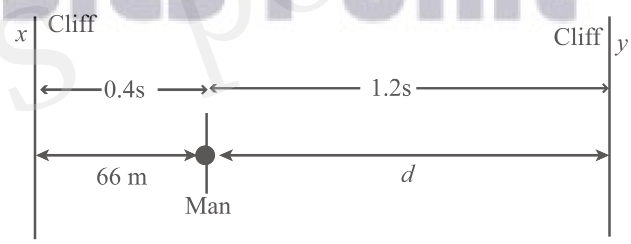
(ii) The wavelength of sound should be less than the height of the reflecting body.

(iii) The intensity of sound should be sufficient so that it can be heard after reflection.

ILLUSTRATION : 10

A man stands in between two cliffs *x* and *y*, such that he is at a distance of 66m from *x*. When he blows a whistle he hears first echo after 0.4s and second echo after 1.2s. Calculate : (i) speed of sound (ii) distance of cliff *y* from man.

SOLUTION :



(i) Speed of sound $v = \frac{2d}{t} = \frac{2 \times 66}{0.4} = 330\,\text{ms}^{-1}$

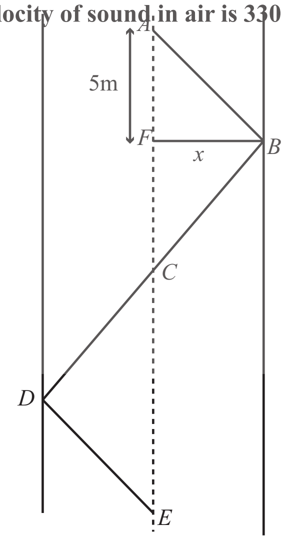
(ii) Distance of man from cliff

$$y - d = \frac{v \times t}{2} = 330 \times \frac{1.2}{2} = 198\,\text{m}$$

ILLUSTRATION : 11

A road runs mid-way between two parallel rows of building. A motorist moving with a speed of 36 km/h. sounds the horn. Find the distance between the two rows of building. When will he hear the echo second time ? Velocity of sound in air is 330 m/sec.

SOLUTION:



The situation is shown in figure.

Given the velocity of motorist = 36 km/h = 10 m/sec.

So he travels a distance of 10m in one second. He will hear the first echo after the sound had travelled 330m through the last path, i.e., reflected at point B.

From figure, $AB = \sqrt{(AF^2 + BF^2)} = \sqrt{(5^2 + x^2)}$

$\therefore \quad AB + BC = 2\sqrt{(5^2 + x^2)}$

But $\quad AB + BC = 330$ m

$\therefore \quad 2\sqrt{(5^2 + x^2)} = 330\,\text{m}$

Solving we get $x = 164.9$m

So the distance between the two rows of building = $2x$ = 329.8 m. He will hear the second echo at E after the sound had travelled a further distance of 330m, reflecting from the other row, i.e., 2 seconds after the horn is sounded.

REVERBERATION

Persistence of sound after its production is stopped, is called reverberation.

When a sound is produced in a big hall, its wave reflect from the walls and travel back and forth. Due to this energy does not reduce and the sound persist.

A short reverberation is desirable in a concert hall (where music is being played) because it gives 'life' to sound. But too much reverberation confuses the programmers and must be reduced.

The material having sound-absorbing properties is used for making the seats in a big hall or auditorium to reduce reverberations.

Panels made of sound-absorbing materials (like compressed fibre board) are put on the walls and ceiling of big halls and auditoriums to reduce reverberations.

RANGE OF HEARING

Normal human ears can hear the sound of frequency 20 Hz to 20000 Hz. Sound of frequency less than 20 Hz is called infrasonic. Sound of frequency greater than 20000 Hz is called **ultrasound**. Children under the age of five and dogs, owls can hear upto 25 kHz. Whales and elephants produce sound in the infrasonic range. Rhinoceroes make communication between themselves by using a frequency as low as 5 Hz.

ULTRASOUND

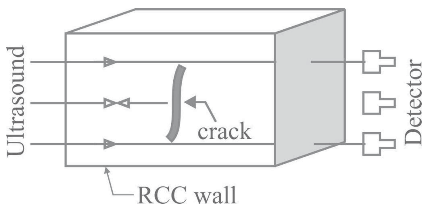
Frequencies higher than 20000 Hz are called ultrasound. Ultrasound can be produced by Galton's whistle. Some animals, such as dolphins can produce ultrasound. Bats can produce and hear ultrasound.

Applications of Ultrasound

On being high frequency waves, ultrasound possesses high intensity, and therefore can penetrate any solid or liquid medium.

1. Ultrasound can kill bacteria and therefore can be used for water purification.
2. To detect cracks in metal and in thick walls: Ultrasound can be used to detect cracks in walls of huge structure like atomic power plant. The cracks or holes inside the metal blocks or RCC walls which are invisible from outside reduces the strength of the structure.

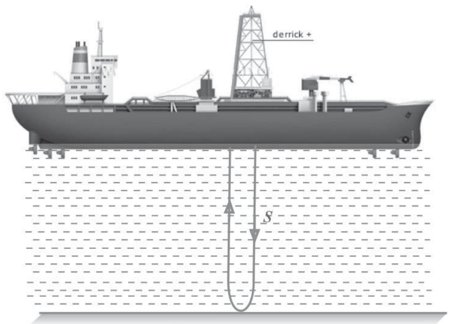
   Ultrasonic waves are allowed to pass through the walls and detectors are used to detect the transmitted waves .If there is a crack in the wall, the ultrasound gets reflected back indicating the presence of defect (see *Figure*).



3. Echocardiography : By making ultrasound of some specific intensity, these are made to reflect from various parts of the heart and form its image. This technique is known as echocardioigraphy.
4. Ultrasound may be used to break stones formed in the kidney. The crushed stone later get flushed out with urine.
5. Sonography : Ultrasonography is used for examination of the factor during frequency to detect congenial defects and growth abnormalities.

SONAR

SONAR stands for **SO**und **N**avigation **A**nd **R**anging. SONAR is a device which is used to find depth of sea or to detect the position of submarine hidden inside water. Sonar consists of a transmitter and a detector. They are installed in a ship (see *Figure*).



Sea bed

Ultrasound sent by the transmitter and received by the detector.

The transmitter produces ultrasonic waves and transmit them. These waves propagate through water and after striking from the object inside water, get reflected back and are recorded by the detector. The distance of the object (submarine etc) can be calculated by knowing the speed of sound in water and the time interval between transmission and reception of the ultrasound in water . The total distance travelled by ultrasound is $2d$.

$$\therefore \quad 2d = v \times t$$

or

$$d = \frac{vt}{2}$$

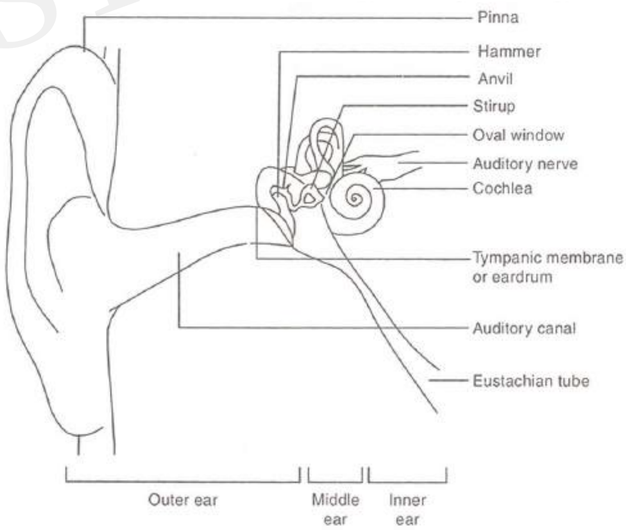
THE HUMAN EAR

It is a highly sensitive part of the human body which enables us to hear a sound. It converts the pressure variations in air with audiable frequencies into electric signals which travel to the brain via the auditory nerve.

The human ear has three main parts. Their auditory functions are as follows:

(i) **Outer ear:** The outer ear is called 'pinna'. It collects the sound from the surrounding.

The collected sound passes through the auditory canal. At the end of the auditory canal there is a thin membrane called the ear drum or tympanic membrane. When compression of the medium produced due to vibration of the object reaches the ear drum, the pressure on the outside of the membrane increases and forces the eardrum inward. Similarly, the eardrum moves outward when a rarefaction reaches. In this way the ear drum vibrates.



Auditory parts of the human ear.

(ii) **Middle ear:** The vibrations are amplified several times by three bones (the hammer, anvil and stirrup) in the middle ear which act as leavers. The middle ear transmits the amplified pressure variations received from the sound wave to the inner ear.

(iii) **Inner ear:** In the inner ear, the pressure variations are turned are turned into electrical signals by the cochlea. These electrical signals are sent to the brain via the auditory nerve, and the brain interprets them as sound.

CONNECTING TOPIC

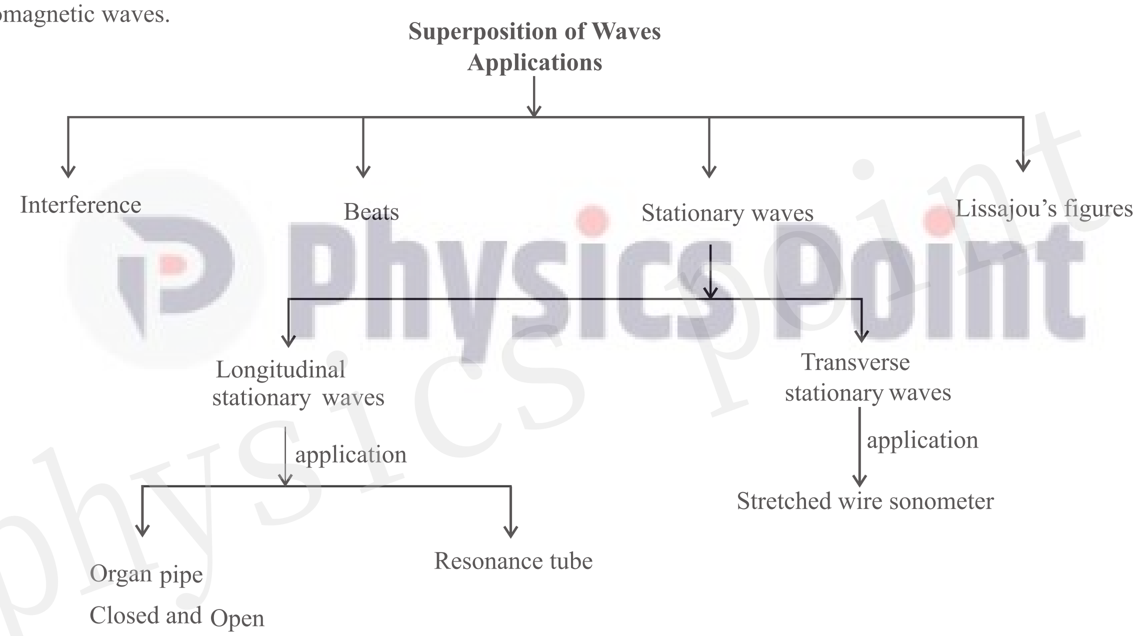
THE PRINCIPLE OF SUPERPOSITION OF WAVES

Two or more progressive waves can travel simultaneously in a medium without affecting the motion of one another. Therefore, the resultant displacement of each particle of the medium at any instant is equal to the vector sum of the displacements produced by the two waves separately, This principle is called 'principle of superposition'.

$$y = y_1 \pm y_2 \pm \ldots\ldots\ldots\ldots\ldots$$

$$\vec{y} = \vec{y}_1 + \vec{y}_2 + \ldots\ldots\ldots\ldots\ldots$$

It holds for all types of waves, provided the waves are not of very large amplitude. If waves are of very large amplitude, as laser waves, then this principle does not hold. When we listen to an orchestra, we receive a complex sound due to the superposition of sound waves of different characteristics produced by different musical instruments. Still we can recognize separately the sounds of different instruments. Similarly, our radio antenna is open to the waves of different frequencies transmitted simultaneously by different radio stations. But when we tune the radio of particular station, we receive the programme of that station only as if the other stations were silent. Thus, then principle of superposition holds not only for the mechanical waves but also for the electromagnetic waves.



INTERFERENCE OF WAVES

When two waves of equal frequency and nearly equal amplitude travelling in same direction having same state of polarisation in medium superimpose, then intensity is different at different points. At some points intensity is large, whereas at other points it is nearly zero.

Consider two waves

$y_1 = A_1 \sin(\omega t - kx)$ and $y_2 = A_2 \sin(\omega t - kx + \phi)$

By principle of superposition

$y = y_1 + y_2 = A \sin(\omega t - kx + \delta)$

where, $A^2 = A_1{}^2 + A_2{}^2 + 2A_1A_2 \cos\phi$, and $\tan\delta = \dfrac{A_2 \sin\phi}{A_1 + A_2\cos\phi}$

As intensity $I \propto A^2$

So, resultant intensity $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

For Constructive Interference (Maximum Intensity)

Phase difference, $\phi = 2n\pi$ or path difference $= n\lambda$

where n = 0, 1, 2, 3, ...

$\Rightarrow A_{max} = A_1 + A_2$ and $I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = (\sqrt{I_1} + \sqrt{I_2})^2$

For Destructive Interference (Minimum Intensity)

Phase difference, $\phi = (2n + 1)\pi$, or path difference = $(2n-1)\frac{\lambda}{2}$; where n = 0, 1, 2, 3, ...

$\Rightarrow A_{min} = A_1 - A_2$ and $I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2} = (\sqrt{I_1} - \sqrt{I_2})^2$

STATIONARY WAVES

The wave propagating in a bounded medium will be reflected at the boundary and produce a wave of the same kind travelling in the opposite direction. The superposition of the two waves gives rise to a stationary wave. Transmission of energy from stationary wave is not possible. Formation of stationary wave is possible only in bounded medium.

Do You Know!!

In a stationary wave, all particles of the medium have the same phase at a given instant but have different amplitudes.

Characteristics of the Stationary Waves

In the stationary waves, the disturbance (crests and troughs or compressions and rarefaction) does not move forward or backward. The disturbance or the energy is not transferred from particle to particle. The time period of periodic motions of all the particles of the medium is same, except those at the nodes. The amplitude of vibration of different particles is different. It is maximum at the antinodes and minimum or zero at the nodes. The nodes are permanently at rest and the velocity of particles is zero.

Nodes and antinodes occur alternately and the separation between any two consecutive nodes or antinodes is half the wavelength. Twice during one vibration, all the particles simultaneously pass through their mean positions with their maximum velocities. Also, twice during one vibration, all the particles simultaneously reach their extremes with maximum displacement and zero velocity. The direction of motion of the particles is reversed after half a vibration of half time period. The wavelength and frequency or period of stationary wave is the same as that of each of the component waves (direct and reflected waves). The pressure variation is maximum at the nodes and minimum at the antinodes. All the particles in one segment between two consecutive nodes are in phase and the phase of particles in each of the segment on either side of this segment is always opposite.

Types of Stationary Waves

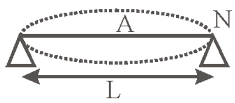
1. Transverse stationary wave **2.** Longitudinal stationary wave

Transverse Stationary wave (Stationary Waves in Strings)

A string of length L is stretched between two points. When the string is set into vibrations, a transverse progressive wave begins to travel along the string. It is reflected at the fixed end. The incident and the reflected waves interfere to produce a stationary transverse wave in which the ends are always nodes.

(a) In the simplest form, the string vibrates in one loop in which the ends are the nodes and the centre is the antinode. This mode of vibration is known as the fundamental mode and the frequency of vibration is known as the **fundamental frequency** or **first harmonic.**

$L = \frac{\lambda_1}{2}$ $\Rightarrow \lambda = 2L$



(a)

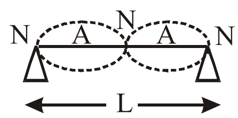
If f_1 is the fundamental frequency of vibration, then the velocity of transverse waves is given as,
$v = \lambda f_1$ or $f_1 = v/2L \Rightarrow v = 2Lf_1$

(b) The same string under the same conditions may also vibrate in two loops, such that the centre is also the node.

$\therefore L = \frac{\lambda_2}{2}$ $\Rightarrow \lambda_2 = L$

If f_2 is the frequency of vibrations, then the velocity of transverse waves is given as,
$v = \lambda_2 f_2 \therefore v = Lf_2$ or $f_2 = v/L = 2f_1$

The frequency f_2 is known as **second harmonic or first overtone.**



(b)

(c) The same string under the same conditions may also vibrate in three segments.

$\therefore \quad L = 3\frac{\lambda_3}{2} \quad \Rightarrow \quad \lambda_3 = \frac{2}{3}L$

(c)

If f_3 is the frequency in this mode of vibration, then,

$$v = \lambda_3 f_3 \;\therefore\; v = \frac{2}{3}Lf_3 \;\text{ or }\; f_3 = 3v/2L = 3f_1$$

The frequency f_3 is known as the **third harmonic** or **second overtone**. Thus a stretched string in addition to the fundamental node, also vibrates with frequencies which are integral multiples of the fundamental frequencies. These frequencies are known as harmonics.

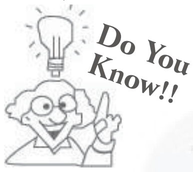
The *velocity of transverse wave in a stretched string is given as* $v = \sqrt{\frac{T}{\mu}}$ where T = tension in the string.

μ = linear density or mass per unit length of string.

If the string fixed at two ends, vibrates in its fundamental mode, then

$$v = 2Lf \;\therefore\; f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

μ = volume of unit length × density $= \pi r^2 \times 1 \times \rho = \pi \times \frac{D^2}{4} \times \rho$ where D = diameter of the wire, ρ = density.



When the string vibrates in one segment, the sound produced is called fundamental note. The string is said to vibrate in fundamental mode. The fundamental note is called first harmonic, and is given by $v_0 = \frac{v}{2\ell}$*, where v = speed of wave.*

Laws of Vibration of Stretched String

(i) **Law of length:** For a given string, under a given tension, the fundamental frequency of vibration is inversely proportional to the length of the string.

i.e, $n \propto \frac{1}{\ell}$ (T and m are constant)

(ii) **Law of tension :** The fundamental frequency of vibration of stretched string is directly proportional to the square root of the tension in the string, provided that length and mass per unit length of the string are kept constant.

i.e., $n \propto \frac{1}{\ell}$ (ℓ and m are constant)

(iii) **Law of mass :** The fundamental frequency of vibration of a stretched string is inversely proportional to the square root of its mass per unit length provided that length of the string and tension in the string are kept constant.

i.e., $n \propto \frac{1}{\sqrt{m}}$ (ℓ and T are constant)

Melde's experiment : In Melde's experiment, one end of a flexible piece of thread is tied to the end of a tuning fork. The other end passed over a smooth pulley carries a pan which can be loaded.

Transverse Arrangement

Case 1 : In a vibrating string of fixed length, the product of number of loops in a vibrating string and square root of tension is a constant or, $p\sqrt{T} =$ constant.

Case 2 : When the tuning fork is set vibrating such that the prong vibrates at right angles to the thread. As a result the thread is set into motion. The frequency of vibration of the thread (string) is equal to the frequency of the tuning fork. If length and tension are properly adjusted then, standing waves are formed in the string. (This happens when frequency of one of the normal modes of the string matched with the frequency of the tuning fork). Then, if p loops are formed in the thread, then the frequency of the tuning fork is given by, $n = \frac{p}{2\ell}\sqrt{\frac{T}{m}}$

Case 3 : If the tuning fork is turned through a right angle, so that the prong vibrates along the length of the thread, then the string performs only a half oscillation for each complete vibrations of the prong. This is because the thread sags only when the prong moves towards the pulley i.e. only once in a vibration.

Longitudinal Arrangement

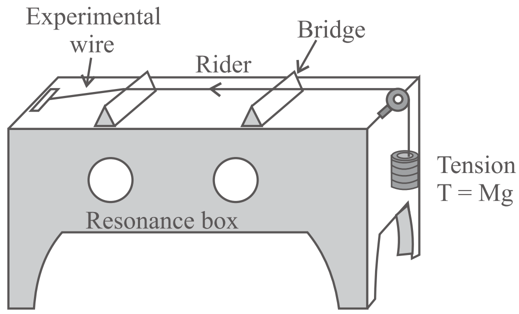
The thread performs sustained oscillations when the natural frequency of the given length of the thread under tension is half that of the fork. Thus if p loops are formed in the thread, then the frequency of the tuning fork is given by, $n = \frac{2p}{2\ell}\sqrt{\frac{T}{m}}$

Sonometer

Sonometer consists of a hollow rectangular box of light wood. One end of the experimental wire is fastened to one end of the box. The wire passes over a frictionless pulley at the other end of the box. The wire is stretched by a tension T.

The box serves the purpose of increasing the loudness of the sound produced by the vibrating wire. If the length of the wire between the two bridges is ℓ, then the frequency of vibration,

$$n = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$$



To test the tension of a tuning fork and string, a small paper rider is placed on the string. When a vibrating tuning fork is placed on the box, and if the length between the bridges is properly adjusted and when the two frequencies are exactly equal, the string quickly picks up the vibrations of the fork and the rider is thrown off the wire.

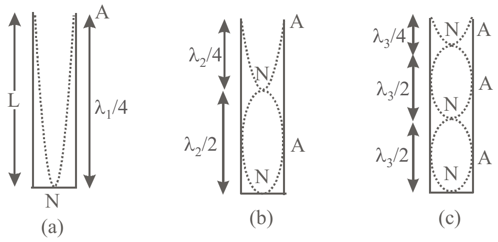
STATIONARY LONGITUDINAL WAVES AND AIR COLUMNS

When two longitudinal waves of same frequency and amplitude travel in a medium in opposite directions then by superposition, standing waves are produced.

These waves are produced in air columns in cylindrical tube of uniform diameter. These sound producing tubes are called **organ pipes.**

Vibration of Air Column in Closed Organ Pipe

The tube which is closed at one end and open at the other end is called closed organ pipe. On blowing air at the open end, a wave travels towards closed end from where it is reflected towards open end. As the waves reaches open end, it is reflected again. So, two longitudinal waves travel in opposite directions to superpose and produce stationary waves. At the closed end there is a node since particles does not have freedom to vibrates, whereas at open end there is an antinode because particles have greatest freedom to vibrate,



Hence on blowing air at the open end, the column vibrates forming antinode at free end and node at closed end. If ℓ is length of pipe and λ be the wavelength and v be the velocity of sound in organ pipe then,

Case (a) $L = \frac{\lambda}{4} \Rightarrow \lambda = 4L \Rightarrow n_1 = \frac{v}{\lambda} = \frac{v}{4L}$ Fundamental frequency or first harmonic.

Case (b) $L = \frac{3\lambda}{4} \Rightarrow \lambda = \frac{4L}{3} \Rightarrow n_2 = \frac{v}{\lambda} = \frac{3v}{4L}$ First overtone or third harmonic

Case (c) $L = \frac{5\lambda}{4} \Rightarrow \lambda = \frac{4L}{5} \Rightarrow n_3 = \frac{v}{\lambda} = \frac{5v}{4L}$ Second overtone or fifth harmonic.

When closed organ pipe vibrate in m^{th} overtone then

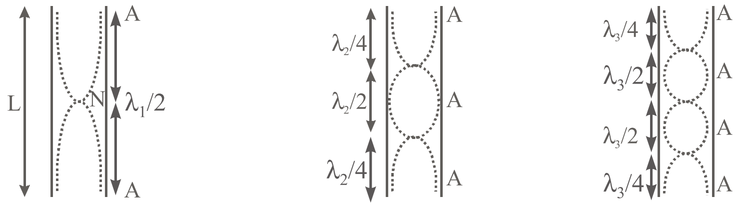
$$L = (2m+1)\frac{\lambda}{4} \quad \text{so, } \lambda = \frac{4L}{(2m+1)} \Rightarrow n = (2m+1)\frac{v}{4L}$$

Hence frequency of overtones is given by

$$n_1 : n_2 : n_3 .... = 1 : 3 : 5....$$

Vibration of Air Column in Open Organ Pipe

The tube which is open at both ends is called an open organ pipe. On blowing air at the open end, a wave travel towards the other end from waves travel in opposite direction to superimpose and produce stationary wave. Now the pipe is open at both ends by which an antinode is formed at open end. Hence on blowing air at the open end antinodes are formed at each end and nodes in the middle. If ℓ is length of the pipe and λ be the wavelength and v is velocity of sound in organ pipe then,



(a) (b) (c)

Case (a) $L = \frac{\lambda}{2} \Rightarrow \lambda = 2L \Rightarrow n_1 = \frac{v}{\lambda} = \frac{v}{2L}$ Fundamental frequency or first harmonic.

Case (b) $L = \frac{2\lambda}{2} \Rightarrow \lambda = \frac{2L}{2} \Rightarrow n_2 = \frac{v}{\lambda} = \frac{2v}{2L}$ First overtone or second harmonic.

Case (c) $L = \frac{3\lambda}{2} \Rightarrow \lambda = \frac{2L}{3} \Rightarrow n_3 = \frac{v}{\lambda} = \frac{3v}{2L}$ Second overtone or third harmonic.

Hence frequency of overtones are given by the relation

$n_1 : n_2 : n_3$ ....= 1 : 2 : 3.........

When open organ pipe vibrate in m^{th} overtone then

$$L = (m+1)\frac{\lambda}{4} \quad \text{so,} \quad \lambda = \frac{4L}{m+1} \Rightarrow n = (m+1)\frac{v}{2L}$$

End correction : Due to finite momentum of air molecules in organ pipes reflection takes place not exactly at open end but somewhat it so in an organ pipe antinode is not formed exactly at free-end but above it at a **distance e = 0.6r called end correction** or **Rayleigh correction** with r being the radius of pipe. So for closed organ pipe $L \to L + 0.6r$ while for open organ pipe $L \to L + 2 \times 0.6r$ (as both ends are open) so that,

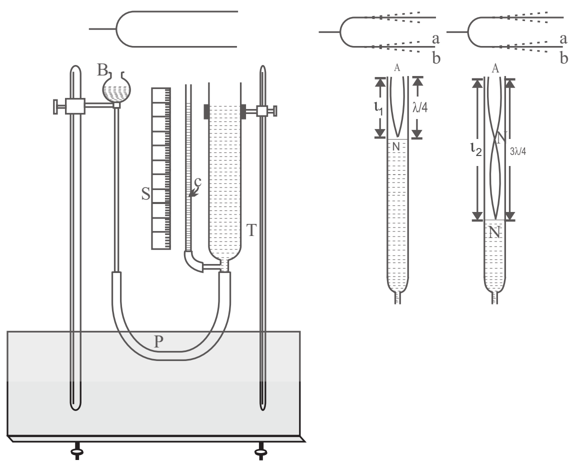
$$f_C = \frac{v}{4(L+0.6r)} \quad \text{while,} \quad f_0 = \frac{v}{2(L+1.2r)}$$

This is why for a given v and L narrower the pipe higher will the frequency or pitch and shriller will be the sound.

Resonance Tube

Determination of the speed of sound in air by resonance tube :

First of all the water reservoir B is raised until the water level in the tube T rises almost to the top of the tube. Then the pinch-cock P is tightened and the reservoir B is lowered. The water level in T stays at the top. Now a tuning fork is sounded and held over the mouth of tube . The pinch-cock P is opened slowly so that the water level in T falls and the length of the air-column increases. At a particular length of air-column in T, a loud sound is heard. This is the first state of resonance. In this position the following phenomenon takes place inside the tube.



(i) For first resonance

$\ell_1 = \lambda/4$ ...(1)

(ii) For second resonance

$\ell_2 = 3\lambda/4$ ...(2)

Substracting eq^n. (2) from eq^n. (1)

$\ell_2 - \ell_1 = \lambda/2$

$\therefore \quad \lambda = 2(\ell_2 - \ell_1)$

If the frequency of the fork be n and the temperature of the air-column be t°C, then the speed of sound at t°C is given by

$v_t = n\lambda = 2n(\ell_2 - \ell_1)$

The speed of sound wave at 0°C

$v_0 = (v_t - 0.61\, t)$ m/s.

End correction : In the resonance tube, the antinode is not formed exactly at the open end but slightly outside at a distance x. Hence the length of the air -column in the first and second states of resonance are $(\ell_1 + x)$ and $(\ell_2 + x)$ then,

(i) For first resonance, $\ell_1 + x = \lambda/4$ ... (1)

(ii) For second resonance, $\ell_2 + x = 3\lambda/4$ ... (2)

Substracting eq^n. (2) from eq^n. (1)

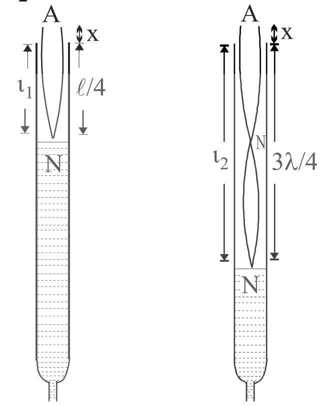
$$\ell_2 - \ell_1 = \lambda/2$$

$$\lambda = 2(\ell_2 - \ell_1)$$

Put the value of λ in eq. (1)

$$\ell_1 + x = \frac{2(\ell_2 - \ell_1)}{4} \Rightarrow \ell_1 + x = \frac{\ell_2 - \ell_1}{2}$$

or, $$x = \frac{\ell_2 - 3\ell_1}{2}$$



BEATS

When two sound waves of nearly same frequency are produced simultaneously, then the intensity of resultant sound wave increases and decreases with time. This change in the intensity of sound is called as the phenomenon of 'beats'.

The time interval between two successive beats is called **beat period** and the number of beats per second is called the **beat frequency.**

If f_1 and f_2 are the frequencies $(f_1 > f_2)$ of the two waves, then the beat frequency

$$b = f_1 - f_2$$

Important Features

1. At frequency difference greater than about 6 or 7 Hz, we no longer hear individual beats. For example, if you listen to a whistle that produces sounds at 2000 Hz and 2100 Hz, you will hear not only these tones but also a much lower 100 Hz tone.
2. If the frequency of a tuning fork is f and it produces Δf beats per second with a standard fork of frequency f_0, then

$$f = f_0 \pm \Delta f$$

If **on filing the arms** of an unknown fork, the beat frequency decreases, then

$$f = f_0 - \Delta f$$

This is because filing an arm of a tuning fork increases its frequency.

Similarly, if **on loading/ waxing** of the unknown fork, the beat frequency decreases, then the frequency of the unknown fork is $f = f_0 + \Delta f$. This is because loading/ waxing decreases the frequency of tuning fork.

Similarly, $f = f_0 + \Delta f$ if on filing beat frequency decreases

and $f = f_0 - \Delta f$ if loading/ waxing beat frequency increases.

Tuning fork : When tuning fork is sounded by striking its one end on rubber pad, then:

(a) The ends of prongs vibrate in and out while the stem vibrates up and down or vibrations of the prongs are transverse and that of the stem is longitudinal. Generally tuning fork produces fundamental note.

(b) At the free end of a fork antinodes are formed. At the place where stem is fixed antinode is formed. In between these antinodes, nodes are formed.

(c) Frequency of tuning fork

$$n \propto \left(\frac{t}{\ell^2}\right)\sqrt{\frac{E}{d}}$$

where,

t = thickness of tuning fork

ℓ = length of arm of fork

E = coefficient of elasticity for the material of fork

d = density of the material of fork.

Frequency of tuning fork decreases with increase in temperature.

Uses of Beats Phenomenon

(i) Determination of frequency - If we know the frequency n_1 of a tuning fork, then we can determine the exact frequency of another fork of nearly equal frequency by the phenomenon of beats. For this, both the tuning forks are sounded together and the beats are heard. Suppose, x beats are heard in 1 second. Then the frequency of the second fork will be either $(n_1 + x)$ or $(n_1 - x)$.

Now one prong of this fork is loaded with a small wax so that its frequency is slightly lowered. Again, the two forks are sounded together and beats are heard. If the number of beats per second decreases then it means that the new (lowered) frequency of the second tuning forks is more nearer to the frequency of the first tuning fork. This would happen if the frequency of the second tuning fork is higher than the frequency of the first fork. Hence the frequency of the second fork is $(n_1 + x)$. On the other hand, if on loading with wax, the number of beats per second increases, then the frequency of the second fork is $(n_1 - x)$.

(ii) Tuning of musical instruments - The musicians make use of beats for tuning their instruments. They sound two instruments, one by one, and adjust the frequency of one in such a way that its sound appears to them similar to the sound of the other instrument. Thus they make the frequencies of the two instruments nearly equal. Then they sound the two instruments simultaneously and hear beats. Now they adjust the frequency of one in such a way that the number of beats per second goes on decreasing slowly until the beats disappear. Now the frequency of the two instruments are exactly equal.

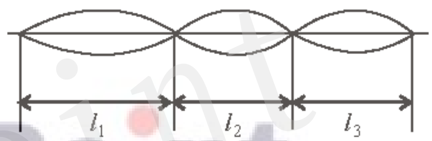
ILLUSTRATION : 12

The length of a sonometer wire between two fixed ends is 1.10 m. Where should the two bridges be placed to divide the wire into three segments whose fundamental frequencies are in the ratio of 1 : 2 : 3?

SOLUTION :

Let l_1, l_2, l_3 be the lengths of three segments.

Given $l_1 + l_2 + l_3 = 1.10$ .... (i)



From relation $n = \dfrac{1}{2l}\sqrt{\left(\dfrac{T}{\mu}\right)}$

If tension T and mass per unit length m are fixed, then $n \propto 1/l$ so nl = constant

$n_1 l_1 = n_2 l_2 = n_3 l_3$

$\therefore \quad l_1 : l_2 : l_3 \equiv \dfrac{1}{n_1} : \dfrac{1}{n_2} : \dfrac{1}{n_3} \equiv \dfrac{1}{1} : \dfrac{1}{2} : \dfrac{1}{3} \equiv \dfrac{6:3:2}{6} \equiv 6:3:2$

$\therefore \quad l_1 = 6k, l_2 = 3k, l_3 = 2k$, k being a constant.

From Eq. (i)

$6k + 3k + 2k = 1.10$ or $11k = 1.10$

$\therefore \quad k = 1.10/11 = 0.1$

$\therefore \quad l_1 = 6 \times 0.1 = 0.6$ m; and $l_2 = 3 \times 0.1 = 0.3$ m $l_3 = 2 \times 0.1 = 0.2$ m

Therefore the bridges must be placed at distance 0.6 m and (0.6 + 0.3) = 0.9 m

ILLUSTRATION : 13

For a certain organ pipe, three successive resonance frequencies are observed at 425, 595 and 765 Hz respectively. Taking the speed for sound in air to be 340 m/s (a) Explain whether the pipe is closed at one end or open at both ends (b) determine the fundamental frequency and length of the pipe.

SOLUTION :

(a) The given frequencies are in the ratio 425 : 595 : 765, i.e., 5 : 7 : 9
Clearly these are odd integers so the given pipe is closed end pipe.

(b) It is clear that the frequency of 5th harmonic (which is third overtone) is 425 Hz
so, $425 = 5f_c$ or, $f_c = 85$ Hz

Further as $f_c = \dfrac{v}{4L} \Rightarrow L = \dfrac{v}{4f_c} = \dfrac{340}{4 \times 85} = 10$ m

ILLUSTRATION : 14

A column of air and a tuning fork produces 4 beats per second when sounded together. The tuning fork gives the lower note. The temperature of air is 15ºC. When the temperature falls to 10ºC, the two produce 3 beats per second. Find the frequency of the tuning fork.

SOLUTION :

Let the frequency of the tuning fork be n Hz
Then frequency of air column at 15ºC = $n + 4$
Frequency of air column at 10ºC = $n + 3$
According to $v = n\lambda$, we have

$v_{15} = (n+4)\lambda$ and $v_{10} = (n+3)\lambda$ $\therefore$ $\frac{v_{15}}{v_{10}} = \frac{n+4}{n+3}$

The speed of sound is directly proportional to the square-root of the absolute temperature.

$$\therefore \quad \frac{v_{15}}{v_{10}} = \sqrt{\frac{15+273}{10+273}} = \sqrt{\frac{288}{283}}$$

$$\therefore \quad \frac{n+4}{n+3} = \sqrt{\frac{288}{283}} = \left(1+\frac{5}{283}\right)^{1/2} \Rightarrow 1 + \frac{\quad}{\quad} = 1 + 1/2 \times \frac{5}{283} = 1 + \frac{5}{566}$$

$$\Rightarrow \quad \frac{1}{n+3} = \frac{5}{566} \Rightarrow n + 3 = 113 \Rightarrow n = 110 \text{ Hz}$$

ILLUSTRATION : 15

A string of length 25 cm and mass 2.5 g is under tension. A pipe closed at one end is 40 cm long. When the string is set vibrating in its first overtone and the air in the pipe in its fundamental frequency, 8 beats per second are heard. It is observed that decreasing the tension in the string decreases the beat frequency. If the speed of sound in air is 320 ms^{-1}, find the tension in the string.

SOLUTION :

Fundamental frequency of the string $n = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$ $\Rightarrow$ $n = \frac{1}{2 \times 0.25}\sqrt{\frac{T}{10^{-2}}} = 20\sqrt{T}$

The fundamental frequency of a closed pipe $n' = \frac{c}{4\ell}$ $\therefore$ $n' = \frac{320}{4 \times 0.40} = 200\text{Hz}$

The frequency of the first overtone of the string = $2n = 2 \times 20\sqrt{T} = 40\sqrt{T}$

Since there are 8 beats / $s, 2n - n' = 8$ or $40\sqrt{T} - 200 = 8$

Since on decreasing the tension, the beat frequency decreases,
$2n$ is definitely greater than n'. $\therefore$ $40\sqrt{T} - 200 = 8$ or T =27.04 N

ILLUSTRATION : 16

Two tuning forks A and B sounded together give 6 beats per second. With an air resonance tube closed at one end, the two forks give resonance when the two air columns are 24 cm and 25 cm respectively. Calculate the frequencies of forks.

SOLUTION :

Let the frequency of the first fork be f_1 and that of second be f_2.

We then have, $f_1 = \frac{v}{4 \times 24}$ and $f_2 = \frac{v}{4 \times 25}$

We also see that $f_1 > f_2$ $\therefore$ $f_1 - f_2 = 6$ …(i) and $\frac{f_1}{f_2} = \frac{24}{25}$ …(ii)

Solving eqs. (i) and (ii), we get f_1 = 150 Hz and f_2 = 144 Hz

ACOUSTIC DOPPLER EFFECT (DOPPLER EFFECT IN SOUND)

The apparent change in the frequency of sound when the source of sound, the observer and the medium are in relative motion is called Doppler effect.

While deriving the expressions for apparent frequency we make the following assumptions:

(i) The velocity of the source, the observer and the medium are along the line joining the positions of the source and the observer.

(ii) The velocity of the source and the observer is less than velocity of sound.

Doppler effect takes place both in sound and light. In sound it depends on whether the source or observer or both are in motion while in light it depends on whether the distance between source and observer is increasing or decreasing.
When the distance between the source and listener is increasing, the apparent frequency decreases and vice-versa.

Case I : When Source is Moving and Observer and Medium are at Rest

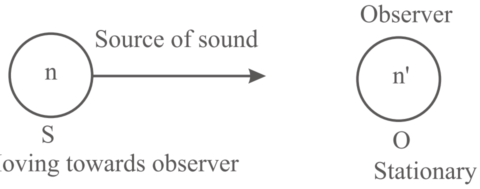
Let n be the frequency of sound emitted by the source. Then n waves will be emitted by the source in one second v is the velocity of sound then,

Wavelength $\lambda = \frac{v}{n}$

(i) When source is moving towards stationary observer: Let the source start moving towards the observer with velocity v_s. After one second, the n waves will be crowded in $(v - v_s)$. Now the observer shall feel that he is listening sound of wavelength λ' and frequency n'

Now apparent wavelength $\lambda' = \frac{v - v_s}{n}$
Apparent frequency,

$$n' = \frac{v}{\lambda} = \frac{v}{\left(\frac{v - v_s}{n}\right)} = n\left(\frac{v}{v - v_s}\right)$$



So, as the source of sound approaches the observer the apparent frequency n' becomes greater than the actual frequency n $(n' > n)$

(ii) When source is moving away from stationary observer: For this situation n waves will be crowded in $(v + v_s)$.

So, *apparent wavelength* $\lambda' = \frac{v + v_s}{n}$
Apparent frequency,

$$n' = \frac{v}{\lambda'} = \frac{v}{\left(\frac{v + v_s}{n}\right)} = n\left(\frac{v}{v + v_s}\right)$$

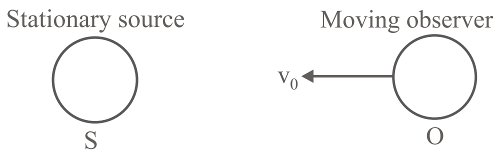
So, apparent frequency n' becomes less than actual frequency n (n' < n)

Case II : When Observer is Moving and Source and Medium are at Rest

Let the source (S) and medium (m) are in rest and observer is moving with velocity v_0.

(i) When observer is moving towards stationary source : When observer O moves towards S with velocity v_0, it will cover v_0 distance in one second. So the observer has received not only the n waves but also received additional number of Δn waves.

So, total waves received by observer in one second
i.e., Apparent frequency,
n' = actual waves (n) + additional waves (Δn)

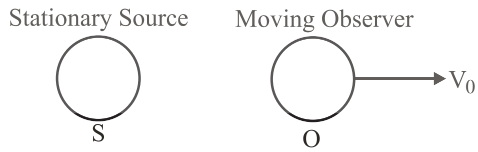


$$n' = \frac{v}{\lambda} + \frac{v_0}{\lambda} = \frac{v + v_0}{(v/n)} = n\left(\frac{v + v_0}{v}\right) \qquad \left(\because \lambda = \frac{v}{n}\right)$$

Hence apparent frequency is greater than actual frequency $(n' > n)$

(ii) When observer is moving away from stationary source:

For this situation n waves will be crowded in $(v - v_0)$.
When observer moves away from source with v_0 velocity then he will get Δn waves less than real number of waves.
So, total number of waves received by observer
i.e., Apparent frequency n' = actual waves (n)
– reduction in number of waves (Δn)



$$v = \frac{2d}{t} = \frac{2 \times 66}{0.4} = 330\,\text{ms}^{-1} \quad \left(\because \lambda = \frac{v}{n}\right)$$

Hence apparent frequency in this case is less than actual frequency $(n' < n)$.

General formula for doppler effect

$$n' = n\left[\frac{v \pm v}{v \quad v}\right] \qquad \text{...(i)}$$

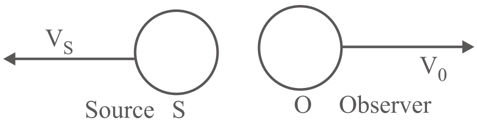
If medium (air) is also moving with v_m velocity in direction of source and observer then velocity of sound relative to observer will be $v \pm v_m$ (–ve sign, if v_m is opposite to sound velocity). So,

$$n' = n\left(\frac{v \pm v_m \pm v_0}{v \pm v_m \mp v_s}\right)$$

[On replacing v by $v \pm v_m$ in equation (i)]

Case-III : When Both Source and Observer are Moving and Medium is at rest

(i) When observer and source are moving away from each other.



Then, $n' = \left(\frac{v - v_0}{v + v_s}\right)n$

Hence apparent frequency is less than actual frequency (n′ >n).

(ii) If both source and observer are moving towards each other.

When, $n' = \left(\frac{v + v_0}{v - v_s}\right)n$

Therefore actual frequency is less than apparent frequency.

(iii) If source is moving towards observer which is moving away from source then,

Apparent frequency $n' = \left(\frac{v - v_0}{v - v_s}\right)n$

If $v_0 < v_s$ then, $n' > n$

$v_0 = v_s$ then $n' = n$

$v_0 > v_s$ then $n' < n$

(iv) If observer is moving towards source which is moving away from observer then,

Apparent frequency $n' = \left(\frac{v + v_0}{v + v_s}\right)n$

If $v_0 > v_s$ then, $n' > n$

$v_0 = v_s$ then $n' = n$

$v_0 < v_s$ then $n' < n$

Doppler's Effect in Reflection of Sound (Echo)

When the sound is reflected from the reflector the observer receives two notes one directly from the source and other from the reflector. If the two frequencies are different then superposition of these waves result in beats and by the beat frequency we can calculate speed of the source.

If the source is at rest and reflector is moving towards the source with speed u, then apparent frequency heard by reflector

$$n_1 = \left(\frac{v+u}{v}\right)n$$

Now this frequency n_1 acts as a source so that apparent frequency received by observer

$$n_2 = \left(\frac{v}{v-u}\right)n_1 = \left(\frac{v}{v-u}\right) \times \left(\frac{v+u}{v}\right)n = \left(\frac{v+u}{v-u}\right)n$$

If u << v then,

$$n_2 = n\left(1+\frac{u}{v}\right)\left(1-\frac{u}{v}\right)^{-1} = n\left(1+\frac{u}{v}\right)^2 = n\left(1+\frac{2u}{v}\right)$$

Beat frequency $\Delta n = n_2 - n = \left(\frac{2u}{v}\right)n$

So speed of the source $u = \frac{v}{2}\left(\frac{\Delta n}{n}\right)$

ILLUSTRATION : 17

A train approaching a hill at a speed of 40km/h sounds a whistle of frequency 580 Hz when it is at a distance of 1km from a hill. A wind with a speed of 40Km/h is blowing in the direction of motion of the train. Find the frequency of the whistle as heard by an observer on the hill. (velocity of sound in air = 1200 km/h)

SOLUTION :

According to Doppler's effect, the apparent frequency when both source and observer move along the same direction is

$$n' = \frac{(v-w)-v_0}{(v+w)-v_s}$$

Velocity of observer $v_0 = 0 \quad \therefore\ n' = \dfrac{(v+w)}{v+w-v_s}\, n$

Given $v = 1200$ km/h, $w = 40$ km/h, $v_s = 40$ km/h and $n = 580$ Hz

$\therefore \quad n' = \dfrac{1200+40}{(1200+40)-40} \times 580 = 599.33 \text{ Hz} = 600 \text{ Hz}$

ILLUSTRATION : 18

A source of sound of frequency 256Hz is moving rapidly towards a wall with a velocity of 5m/s. How many beats per second will be heard if sound travels at a speed of 330 m/s by an observer behind the source?

SOLUTION :

When the source S is between the wall (W) and the observer (O).

For direct sound the source is moving away from the observer, therefore the apparent frequency

$n'' = \dfrac{v}{v+v_s}\, n = \dfrac{330}{330+5} \times 256 = 252.2$

and frequency of reflected sound $n' = \dfrac{v}{v-v_s}\, n = \dfrac{330}{330-3} \times 256 = 259.9$

Number of beats/sec $= n' - n'' = 259.9 - 252.2 = 7.7$

ILLUSTRATION : 19

A siren is fitted on a car going towards a vertical wall at a speed of 36 km/hr A person standing on the ground, behind the car, listens to the siren sound coming directly from the source as well as that coming after reflection from the wall. Calculate the apparent frequency of the wave (a) coming directly from the siren to the person and (b) coming after reflection. Take the speed of sound to be 340 m/s.

SOLUTION :

(a) Here the observer is at rest with respect to the medium and the source is going away from the observer. The apparent frequency heard by the observer is, therefore,

$$\nu' = \frac{v}{v+v_s}\,\nu = \frac{340}{340+10} \times 500 \text{ Hz} = 486 \text{ Hzt}$$

(b) The frequency received by the wall is

$$\nu'' = \frac{v}{v-v_s}\,\nu = \frac{340}{340-10} \times 500 = 515 \text{ Hz}$$

The wall reflects this sound without changing the frequency. Thus, the frequency of the reflected wave as heard by the ground observer is 515 Hz.

SIMPLE HARMONIC MOTION (S.H.M.)

Oscillatory motion in which the acceleration of the particle is directly proportional to the displacement and directs towards a fixed point in a direction opposite to displacement is called simple harmonic motion abbreviated as S.H.M.

If a particle performs oscillatory motion such that its acceleration (a) and displacement (x) are related as below

$$a \propto -x,$$

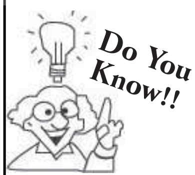
then the motion of particle is simple harmonic.

The force (F) acting on the particle is obviously proportional to x and directs in opposite to it.i.e.,

$$F \propto -x$$

or $\quad F = -kx$, where k is a constant.

This force F is known as the restoring force as it always restore the position of the particle.



An oscillatory motion is always periodic i.e., the motion that repeats itself in equal intervals of time but a periodic motion may not be oscillatory.

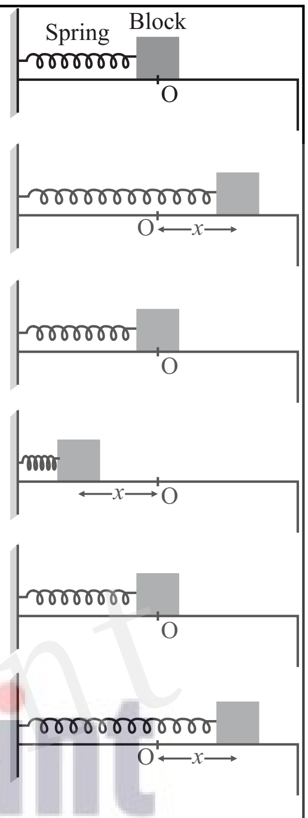
Example of S.H.M.

Let us consider the figure on the right. It consists of a spring having no mass (although not possible practically) attached to a block of mass *m*. Such a system is known as spring-mass system. The system is placed on a smooth table. The free end of the spring is fixed.

The block is now pulled a distance *x* towards right and released. It will go towards left through the point *O* (known as mean position) upto a distance exactly equal to *x* and again move towards the right. The block again move through *O* upto a distance *x* from it and come back and so forth. It will perform *a* to and fro motion about *O*.

We see that block always experiences a restoring force towards the fixed point *O*. Thus, the motion of the block is S.H.M.

There are many other systems which perform S.H.M.s. Some of them are (i) clock pendulum, (ii) oscillating liquid in a U-tube, (iii) oscillating block in a liquid, (iv) oscillating frictionless piston fitted in a cylinder filled with ideal gas, etc.



CHECK Point

Is a bouncing ball an example of simple harmonic motion ? Is the daily movement of a student from home to school and back simple harmonic motion?

Solution

The bouncing ball is not an example of simple harmonic motion. The position of the ball is not described by a sinusoidal function. The daily movement of a student is also not simple harmonic motion because the student stays at a fixed position-school-for a long period. If this motion were sinusoidal, the student would move more and more slowly as she approached her desk, and, as soon as she sat down at the desk, she would start to move back towards home again.

Equation of S.H.M.

The equation of S.H.M. represents the displacement (x) of the particle at any time (t).

It is generally given by

$$y = A\sin(\omega t + \phi) \quad \text{or} \quad y = A\cos(\omega t + \phi)$$

or $$x = A\sin(\omega t + \phi) \quad \text{or} \quad x = A\cos(\omega t + \phi)$$

Here, A = amplitude and ω = angular frequency

ϕ = phase constant or initial phase

Terms Related to S.H.M.

Amplitude (A): It is the maximum distance on the either side of the mean position of oscillating particle. It is represented by A, its **S.I. unit** is metre m.

Phase: Phase of a vibrating particle at any instant is the state of the vibrat ing particle regarding its displacement and direction of vibration at that particular instant.

The *cosine* in equation $x = A\cos(\omega t + \phi_0)$ gives the phase of oscillation at time t.

It is denoted by ϕ.

$$\therefore \quad \phi = \omega t + \phi_0 \qquad \text{or} \quad \phi = 2\pi\frac{t}{T} + \phi_0$$

It is clear that phase ϕ is a function of time t. The phase of a vibrating particle can be expressed in terms of fraction of the time period that has elapsed since the vibrating particle left its mean position in the positive direction.

Again, $\phi - \phi_0 = \omega t = \frac{2\pi}{T}t$

So, the phase change in time t is $\frac{2\pi t}{T}$.

The phase change in T second will be 2π which actually means 'no change in phase'. Thus, time period may also be defined as the time interval during which the phase of the vibrating particle changes by 2π.

Velocity

The displacement of a particle executing S.H.M. is given by

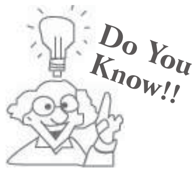
$$x = A\sin(\omega t + \phi)$$

Hence, velocity $v = \frac{dx}{dt} = A\omega\cos(\omega t + \phi)$

or $\quad v = A\omega t\cos(\omega t + \phi) \qquad$ or $\quad v = \omega\sqrt{A^2 - x^2}$

At mean position, $x = 0 \Rightarrow v_{max} = \omega t$

At extreme position, $x = \text{A} \Rightarrow v = 0$



The velocity of the particle executing S.H.M. decreases, as it moves from the mean to extreme position.

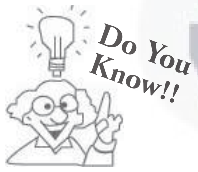
Acceleration

If $\quad x = A\sin(\omega t + \phi) \quad v = \frac{dx}{dt} = A\omega\cos(\omega t + \phi)$

$$a = \frac{dv}{dt} = -A\omega^2 \sin(\omega t + \phi) = -\omega^2 x$$

At mean position, $x = 0 \Rightarrow a = 0$

At extreme position, $x = A \Rightarrow |a_{max}| = \omega^2 A$



The acceleration of a particle executing S.H.M. increases as it moves from the mean position to extreme position.

Time period : It is the time taken by the oscillating particle to complete one oscillation. It is represented by T.

Kinetic energy : A particle executing SHM possesses kinetic energy by virtue of its motion.

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(\text{A}^2 - \text{x}^2) \qquad (v = \omega\sqrt{A^2 - x^2})$$

At mean position, $x = 0$

$$(K.E)_{max} = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}KA^2 \qquad (k = m\omega^2)$$

At extreme position, $x = A$, K.E. = 0

Potential energy : A particle executing SHM possesses potential energy due to its displacement from its mean position.

$$P.E = \frac{1}{2}k\,x^2 \qquad \Rightarrow \qquad P.E = \frac{1}{2}m\omega^2 x^2 \quad (k = m\omega^2)$$

At mean position, $x = 0 \Rightarrow$ P.E. = 0

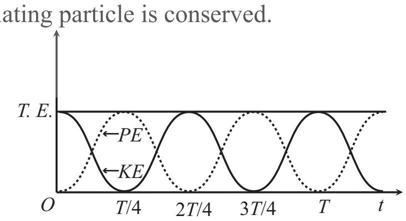
At extreme position, $x = A \quad \Rightarrow \quad (P.E)_{max} = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}k\omega^2$

Total mechanical energy : An oscillating particle has both kinetic and potential energy at any instant. The sum of kinetic and potential energy remains constant. Thus the total mechanical energy of oscillating particle is conserved.

$$T.E. = K.E. + P.E. \Rightarrow T.E = \frac{1}{2}m\omega^2(A^2 - x^2) + \frac{1}{2}m\omega^2 x^2$$

$$\text{T.E} = \frac{1}{2}m\omega^2\text{A}^2 = \frac{1}{2}k\text{A}^2 = \text{constant}$$

$$T.E. = (K.E.)_{max} = (P.E.)_{max}$$



From this graph it is evident that $P.E$ and $K.E$ complete one oscillation in $\frac{T}{2}$ time hence frequency of oscillation of $K.E.$ or $P.E$ is twice $(2f)$ that of oscillating particle (f).

ILLUSTRATION : 20

A particle in SHM is described by the displacement equation, $x = A \sin(\omega t + \phi)$; $\omega = \frac{2\pi}{T}$. If at $t = 0$, position of particle is 1 cm and its initial velocity is π cms^{-1}, find its amplitude & initial phase. The frequency is $\frac{1}{2}s^{-1}$.

SOLUTION :

At $t = 0, x = 1$

$f = \frac{1}{2}s^{-1}$, $\omega = 2\pi f \Rightarrow \omega = \pi$ rads^{-1}

$1 = A \sin \phi$ .... (i)

At $t = 0, v = \pi$ cms^{-1}

$\frac{dx}{dt} = v = A\omega \cos(\omega t + \phi)$

$\pi = A \times \pi \cos(0 + \phi)\pi = A \times \pi \cos(0 + \phi)$

$1 = A \cos \phi$ .... (ii)

From equations (i) & (ii)

$\frac{A \sin\phi}{A\cos\phi} = 1 \Rightarrow \tan\phi = 1$

$\phi = \frac{\pi}{4}$ or $\frac{5\pi}{4}$

Also, $A^2 \sin^2 \phi + A^2 \cos^2 \phi = 2$

$A = \sqrt{2}$cm

ILLUSTRATION : 21

A simple harmonic motion is represented by $x = 5\sin(10t + \pi/4)$. Write its amplitude, angular frequency, frequency, time period and initial phase. Displacement is in metres and time in seconds.

SOLUTION :

Comparing $x = 5\sin(10t + \pi/4)$ with $x = A\sin(\omega t + \phi)$

$A = 5$ m, $\omega = 10$ rad./s

$2\pi f = 10 \Rightarrow f = \frac{5}{\pi}s^{-1}$

$T = \frac{1}{f} = \frac{\pi}{5}$s; $\phi = \frac{\pi}{4}$ rad.

ILLUSTRATION : 22

A particle executes SHM with amplitude 20 cm and time period 4 *s*. Find the minimum time required for the particle to move between two points 10 cm on either side of the mean position.

SOLUTION :

Given $A = 20$ cm; $T = 4$ s

Let the equation of SHM be $x = A \sin \omega t$

$x = 10$, $10 = 20\sin\frac{2\pi}{4}t \Rightarrow \frac{1}{2} = \sin\frac{\pi t}{2}$

$\sin\frac{\pi}{6} = \sin\frac{\pi t}{2} \Rightarrow t = \frac{1}{3}s$

The required time $= 2t = \frac{2}{3}s$

MISCELLANEOUS SOLVED EXAMPLES

1. A narrow pulse (for example, a short pip by a whistle) is sent across a medium.

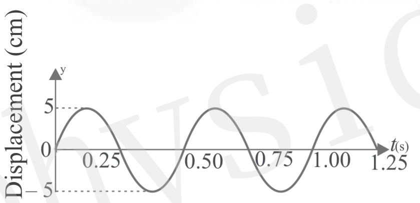
(a) Does the pulse have a definite (i) frequency, (ii) wavelength, (iii) speed of propagation?

(b) If the pulse rate is 1 after every 20 s, (that is the whistle is blown for a split of second after every 20s), is the frequency of the note produced by the whistle equal to 1/20 or 0.05 Hz.

Sol. (a) A narrow pulse does not have a definite wavelength or frequency. But being a sound, it has a definite speed.

(b) The frequency of the note produced by the whistle is not equal to 1/20 or 0.05 Hz, it is only the frequency of pulse repetation.

2. A wave is travelling along the x -axis, whose displacement-time graph is shown in *Fig.* Find period and frequency of wave.



Sol. The time period of wave

$T = 0.50$ s

The frequency of oscillation

$f = \frac{1}{T} = \frac{1}{0.50} = 2\ Hz$

3. A source of wave produces 40 crests and 40 troughs in 0.4 second. Find the frequency of the wave.

Sol. The total number of waves produced in 0.4 s is 40.

$\therefore$ The frequency of wave, $f = \frac{n}{t} = \frac{40}{0.4} = 100$ Hz

4. A boat at anchor is rocked by waves whose consecutive crests are 100 m apart. The wave velocity of the moving crests is 20 m/s. What is the frequency of rocking of the boat?

Sol. Given, wavelength of wave, $\lambda = 100$ m

and wave velocity, $v = 20$ m/s

$\therefore$ Frequency of rocking of boat = frequency of wave

or $f = \frac{v}{\lambda} = \frac{20}{100} = 0.20$ Hz

5. A longitudinal wave is produced on a toy slinky. The wave travels at a speed of 30 cm/s and the frequency of the wave is 20 Hz. What is the minimum separation between the consecutive compressions of the slinky?

Sol. Given, speed of wave, $v = 30$ cm/s

and frequency of wave, $f = 20$ Hz

$\therefore$ Wavelength of wave, $\lambda = \frac{v}{f} = \frac{30}{20} = 1.5$ cm

Thus the separation between the consecutive compressions $= 1.5$ cm.

6. What is the phase difference between the particles 1 and 2 located as shown in *Fig.*

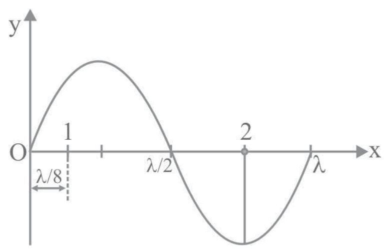
Sol. The distance between the particles

$\Delta x = \left(\frac{\lambda}{2} - \frac{\lambda}{8}\right) + \frac{\lambda}{4}$

$= \frac{5\lambda}{8}$

$\therefore \Delta\phi = \frac{2\pi}{\lambda}.\Delta x$

$= \frac{2\pi}{\lambda} \times \frac{5\lambda}{8} = \frac{5\pi}{4}$



SOLVED EXAMPLES BASED ON CONNECTING TOPICS

7. A wave of frequency 500 cycles/s has a phase velocity of 360 m/s . (a) How far apart are two points 60° out of phase? (b) What is the phase difference between two displacements at a certain point at times 10^{-3} s apart?

Sol. (a) Given $\Delta\phi = 60° = \frac{\pi}{3}$ rad and $\lambda = \frac{v}{f} = \frac{360}{500} = 0.72$ m

We know that,

$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$

$\therefore \Delta x = \Delta\phi\frac{\lambda}{2\pi} = \frac{\pi}{3} \times \frac{0.72}{2\pi} = 0.12$m

(b) Phase difference with time is given by

$\Delta\phi = \frac{2\pi}{T}.\Delta t$

Here $T = \frac{1}{f} = \frac{1}{500} = 0.002$s

$\therefore \Delta\phi = \frac{2\pi}{0.002} \times 10^{-3} = 3.14$ rad $= 180°$

8. A transverse harmonic wave on a string is described by y (x, t) = 3.0 sin (36t + 0.018x $+3.0\sin\left(36t+0.018x+\frac{\pi}{4}\right)$), where x, y are in cm and t in s. The positive direction of x is from left to right.

(i) Is this a travelling or a stationary wave? If it is travelling, what are the speed and direction of its propagation.

(ii) What are its amplitude and frequency?

(iii) What is the initial phase at the origin?

(iv) What is the least distance between two successive crests in the wave?

Sol. Given $y = A\sin(\omega t + kx + \phi_\circ)$ ...(i)

The standard equation of a harmonic wave travelling along negative x - direction is

$y = \phi_0$ ...(ii)

On comparing equations (i) and (ii), we have

ω = 36 rad/s, k = 0.018/ m

and $\frac{\pi}{4} = \frac{\omega}{k}$ rad

(i) $v = \frac{36}{0.018} = \frac{\omega}{2\pi}$

= 2000 cm/s = 20 m/s

(ii) A = 3.0 cm

$f = \frac{36}{2\pi} = \phi 0 = 5.73\ s^{-1}$

(iii) Initial phase $\frac{\pi}{4} = \frac{2\pi}{k}$ rad

(iv) Least distance between two successive crests

$= \lambda = \frac{2\pi}{k} = \frac{2\pi}{0.018} = 349.0$ cm = 3.49 m

9. A tuning fork of unknown frequency gives 4 beats with a tuning fork of frequency 310 Hz. It gives the same number of beats on filing. Find the unknown frequency.

Sol. The unknown frequency of the tuning fork can be

$= 310 \pm 4$

or f = 314 or 306 Hz

Suppose f = 314 Hz

On filing, let it becomes = 318 Hz.

When it sounded together with a fork of frequency 310 Hz, beats frequency will be more than 4 per second. Therefore unknown frequency can not be 314 Hz.

Now suppose f' = 306 Hz.

On filing, let it becomes = 314 Hz.

When it sounded again with a fork of frequency 310 Hz it gives 4 beats per second.

So unknown frequency must be 306 Hz.

10. A set of 56 tuning forks are arranged in series of increasing frequencies. Each fork gives 4 beats/s with preceding one. If the frequency of last fork is 2 times that of the first, what is the frequency of 40th fork?

Sol. Suppose the frequency of the first fork is f, then frequency of the last fork

$= f + (56-1)\times 4 \quad = f + 220$

Given $(f + 220) = 2f$

$\therefore$ f = 220 Hz

Now the frequency of the 40th fork

= f + (40 – 1) × 4

= 220 + 39 × 4 = 376 Hz.

11. A bat is fitting about in a cave, navigating via ultrasonic bleeps. Assume that the sound emission frequency of the bat is 40 kHz. During one fast sweep directly towards a flat wall surface, the bat is moving at 0.03 times the speed of sound in air. What frequency does the bat hear reflected off the wall?

Sol. The frequency of the sound reflected from the wall as perceived by the bat

$= f\frac{v+v_o}{v-v_s} = 40\left(\frac{v+0.03v}{v-0.03v}\right)$

= 42.47 kHz

12. A source emitting sound at frequency 4000 Hz, is moving along the y - axis with a speed of 22 m/s . A listener is situated on the ground at the position (660 m, 0). Find the frequency of the sound received by the listener at the instant the source crosses the origin. Speed of sound in air = 330 m/s .

Sol. Let sound produced at P will reach the listener at the instant when source crosses the origin.

Time of motion of sound from P to listener

= time of motion of source from P to origin

or $\frac{y}{22} = \frac{\sqrt{y^2+660^2}}{330}$ $\left[\cos\theta = \frac{44}{\sqrt{44^2+660^2}} = \frac{44}{661.5}\right]$

After solving

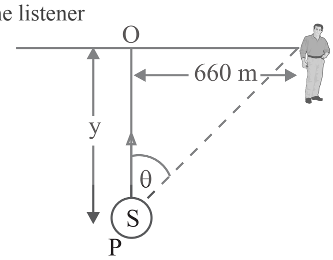
y = 44 m

The frequency heard by the listener

$f' = f\frac{v}{v - v_s\cos\theta}$

$= 4000\frac{330}{330-22\cos\theta}$

= 4018 Hz



1 EXERCISE

Fill in the Blanks :

DIRECTIONS: *Complete the following statements with an appropriate word/term to be filled in the blank space(s).*

1. Sound waves having the frequency .... are audible to human being.
2. The wavelength of a sound from a tuning fork of frequency 330Hz is nearly .... cm.
3. Velocity of sound in vacuum is ....
4. The total energy E of sound is related to their frequency as ....
5. Longitudinal waves cannot passed through ....
6. The note of the lowest frequency in a musical reach is called ....
7. Loudness of a note increases with the increase in ....
8. The frequency of the particles oscillating in a medium is .... the frequency of waves in the medium.
9. A transverse wave is made up of ....
10. A longitudinal wave is made up of ....
11. Wave motion involves the transport of ....
12. Velocity = frequency × ....
13. The functions which can be represented by a sine or a cosine function are called .... functions.

True/False :

DIRECTIONS: *Read the following statements and write your answer as true or false.*

1. Longitudinal waves are produced in all the three states.
2. Bells are made of metal and not of wood because the sound is not conducted by metals but is radiated.
3. The rate of transfer of energy in a wave depends directly on the square of the wave amplitude and square of the wave frequency.
4. Velocity of sound in air at the given temperature decreases with increase in pressure.
5. The pitch of the sound as detected by the observer is independent of original frequency.
6. Velocity of sound in air at the given temperature decreases with increase in pressure.
7. High pitch of sound has a high frequency.
8. Longitudinal waves consist of crests and troughs.
9. Pitch of the voice of a child is higher than that of a boy.
10. A pure sine wave of sound is called melody.
11. The total energy of a simple harmonic oscillator is constant.
12. Potential energy of simple harmonic oscillator at the mean position is always zero.

Match The Columns :

DIRECTIONS: *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column I have to be matched with statements (p, q, r, s) in column II.*

1. Match the columns

Column I	Column II
(A) high pitch	(p) Faint sound
(B) low pitch	(q) Loud sound
(C) small amplitude	(r) High frequency
(D) large amplitude	(s) Low frequency

Very Short Answer Questions :

DIRECTIONS: *Give answer in one word or one sentence.*

1. What is transferred by wave motion from one place to another, matter or energy?
2. A stone is dropped on the surface of water in a pond. Name the type of waves produced.
3. Define wavelength.
4. Why we cannot hear an echo in a small room?
5. Why are the longitudinal waves also called pressure waves?
6. What is the range of human audibility?
7. What is an echo?
8. State one important application of ultrasound.
9. What are sound waves ?
10. What is meant by reverberation ?
11. What are ultrasonic waves ?
12. Explain how the principle of echo is used by
    (a) the bat during its flight at night.
    (b) the dolphin to locate small fish as its prey.

Short Answer Questions :

DIRECTIONS: *Give answer in 2-3 sentences.*

1. State two properties of the medium required for wave propagation.
2. Sound can make a light spot dance. Describe a simple experiment to illustrate this fact.
3. Derive a relation between wavelength, frequency and velocity of a wave.
4. What is the difference between an echo and a reverberation ?

5. What is SONAR ? What is the basic principle of its working ? Explain its use.
6. Explain how bats use ultrasound to catch a prey?
7. Define audible, ultrasound and infrasound waves and also the range of hearing in human.
8. Bats use ultrasound to fly at night without colliding with other objects and do search their prey, explain.
9. Why soldiers are asked to break their steps while crossing a temporary bridge of rope?
10. A girl swinging suddenly stands up on the swing. What is the influence on the time period and frequency?

Long Answer Questions

DIRECTIONS: *Give answer in four to five sentences.*

1. Describe some practical applications from daily life based on multiple reflection of sound.
2. How can a longitudinal wave be represented graphically?
3. Explain how the human ear works.
4. (i) Which property of wave determines loudness of sound ?
   (ii) Write two points of differences between longitudinal and transverse waves. Give one example of each type.
   (iii) Why are ceilings of concert-halls made curved?

2 EXERCISE

Text-Book Questions :

1. How does the sound produced by a vibrating object in a medium reach your ear?
2. Explain how sound is produced by your school bell.
3. Why are sound waves called mechanical waves ?
4. Suppose you and your friend are on the moon. Will you be able to hear any sound produced by your friend ?
5. Which wave property determines (a) loudness, (b) pitch ?
6. Guess which sound has a higher pitch: guitar or car horn ?
7. What are the wavelength, frequency, time period and amplitude of a sound wave ?
8. How are the wavelength and frequency of a sound wave related to its speed ?
9. Calculate the wavelength of a sound wave whose frequency is 220 Hz and speed is 440 m/s in a given medium.
10. A person is listening to a tone of 500 Hz sitting at a distance of 450 m from the source of sound. What is the time interval between successive compressions from the source ?
11. Distinguish between loudness and intensity of sound.
12. In which of three media: air, water or iron, does sound travel the fastest at a particular temperature?
13. An echo returned in 3s. What is the distance of the reflecting surface from the source, given that the speed of sound is 342 ms^{-1} ?
14. Why are the ceilings of concerts halls curved ?
15. What is the audible range of the average human ear?
16. What is the range of frequencies associated with
    (a) Infrasound ? (b) Ultrasound ?
17. A submarine emits a sonar pulse, which returns from an obstacle underwater cliff in 1.02 s. If the speed of sound in salt water is 1531 m/s, how far away is the cliff ?

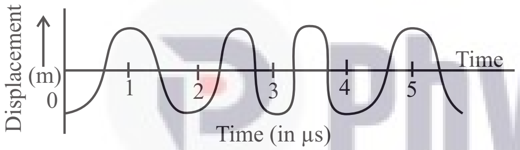
Text-Book Exercise :

1. What is sound and how is it produced ?
2. Describe with the help of a diagram, how compressions and are refactions are produced in air near a source of sound.
3. Cite an experiment to show that sound needs a material medium for its propagation.
4. Why is sound wave called a longitudinal wave ?
5. Which characteristic of the sound helps you to identify your friend by his voice while sitting with others in a dark room ?
6. Flash and thunder are produced simultaneously. But thunder is heard a few seconds after the flash is seen, why ?
7. A person has a hearing range from 20 Hz to 20 kHz . What are the typical wavelengths of sound waves in air corresponding to these two frequencies? Take the speed of sound in air as 344 m s^{-1}.
8. Two childrens are at opposite ends of an aluminium rod. One strikes the end of the rod with a stone. Find the ratio of times taken by the sound wave in air and in aluminium to reach the second child. (Speed of sound in air and aluminium is 346 ms^{-1} and 6420 ms^{-1} respectively)
9. The frequency of a source of sound is 100 Hz. How many times does it vibrate in a minute ?
10. Does sound follow the same laws of reflection as light does ? Explain ?
11. When a sound is reflected from a distant object, an echo is produced. Let the distance between the reflecting surface and the source of sound production remains the same. Do you hear echo sound on a hotter day ?
12. Give two practical applications of reflection of sound waves.
13. A stone is dropped from the top of a tower 500 m high into a pond of water at the base of the tower. When is the splash heard at the top? Given, g = 10 m s^{-2} and speed of sound = 340 m s^{-1}.

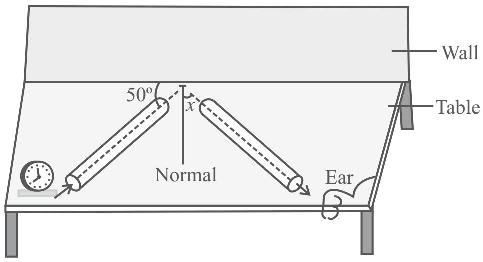
14. A sound wave travels at a speed of 339 m s^{-1}. If its wavelength is 1.5 cm, what is the frequency of the wave ? Will it be audible ?
15. What is reverberation ? How can it be reduced ?
16. What is loudness of sound ? What factors does it depend on ?
17. Explain how bats use ultrasound to catch a prey.
18. How is ultrasound used for cleaning ?
19. Explain the working and application of a sonar.
20. A sonar device on a submarine sends out a signal and receives an echo 5 s later. Calculate the speed of sound in water if the distance of the object from the submarine is 3625 m.
21. Explain how defects in a metal block can be detected using ultrasound.

Exemplar Questions :

1. The given graph (Fig.) shows the displacement versus time relation for a disturbance travelling with velocity of 1500 m^{S-1}. Calculate the wavelength of the disturbance.



2. A girl is sitting in the middle of a park of dimension 12 m × 12 m. On the left side of it there is a building adjoining the park and on right side of the park, there is a road adjoining the park. A sound is produced on the road by a cracker. Is it possible for the girl to hear the echo of this sound?
Explain your answer.
3. Why do we hear the sound produced by the humming bees while the sound of vibrations of pendulum is not heard?
4. For hearing the loudest ticking sound heard by the ear, find the angle x in the fig.



Hots Questions :

1. When a light or sound source moves toward you, is there an increase or a decrease in the wave speed?
2. The echo of our sound is not heard in a small room, but is heard distinctly in a big hall. Explain it.
3. An observer notes that there is a 6 second interval between seeing a flash of lightning and hearing the clap of thunder. How far away is the storm ?
4. Find the frequency of minimum distance between compression and rarefaction of a wire. If the length of the wire is 1m and velocity of sound in air is 360 m/s .

3 EXERCISE

Single Option Correct :

DIRECTIONS: *This section contains multiple choice questions. Each questions has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.*

1. Which of the following is carried by the waves from one place to another ?
(a) mass (b) velocity
(c) wavelength (d) energy

2. A thunder clap is heard 5.5 second after the lightening flash. The distance of the flash is
(velocity of sound in air = 330 m/s)
(a) 1780 m (b) 1815 m
(c) 300 m (d) 3560 m

3. The velocity of sound is largest in
(a) water (b) air
(c) metal (d) vacuum

4. The ratio of the speed of a body to the speed of sound is called
(a) Sonic index (b) Doppler ratio
(c) Mach number (d) Refractive index

5. The velocity of sound in a tube containing air at 27°C and a pressure of 76 cm. of mercury is 330 m/s. What will be the velocity of sound when the pressure is increased to 100cm of mercury and temperature is kept constant
(a) 330 m/s (b) $\frac{100}{76} \times 330$ m/s
(c) $\frac{76}{100} \times 330$ m/s (d) $\sqrt{\frac{100}{76}} \times 330$ m/s

6. A wave of frequency 1000 Hz travels between *X* and *Y*, a distance of 600m in 2 sec. How many wavelengths are there in distance *XY*
(a) 3.3 (b) 300
(c) 180 (d) 2000

7. The speed of sound of a wave of frequency 200 Hz in air is 340 m/s. The speed of sound of wave of frequency 400 Hz in same air is
(a) 340 m/s (b) 680 m/s
(c) 170 m/s (d) 3×10^8 m/s

8. Ultrasonic waves have frequency
(a) below 20 Hz
(b) between 20 and 20,000 Hz
(c) only above 20,000 Hz
(d) only above 20,000 MHz

9. A source produces 50 crests and 50 troughs in 0.5 seconds. What is the frequency of the wave ?
(a) 100 Hz (b) 150 Hz
(c) 50 Hz (d) 125 Hz

10. A person has the audible range from 20 Hz to 20×10^3 Hz. Find the wavelength range corresponding to these frequencies. Take velocity sound as 340 m/s.
(a) 15×10^{-3} m (b) 11×10^{-3} m
(c) 17×10^{-3} m (d) 15×10^{-8} m

11. One hertz is equivalent to
(a) one cycle per second
(b) one second
(c) one meter per second
(d) one second per meter

12. If you are at open-air concert and someone's head gets between you and the orchestra, you can still hear the orchestra because
(a) sound waves pass easily through a head
(b) a head is not very large compared with the wavelength of the sound
(c) the sound is reflected from the head
(d) the wavelength of the sound is much smaller than the head

13. An underwater explosion is caused near the sea-shore. There are two observers, *X* under water and *Y* on land, each at a distance of 1 km from the point of explosion
(a) *X* will hear the sound earlier
(b) *Y* will hear the sound earlier
(c) Both will hear the sound at the same time.
(d) *Y* will not hear the sound at all

14. In a long spring which of the following type of waves can be generated
(a) Longitudinal only
(b) Transverse only
(c) Both longitudinal and transverse
(d) Electromagnetic only

15. A source of frequency 500 Hz emits waves of wavelength 0.2m. How long does it take the waves to travel 300 m
(a) 75 s (b) 60 s
(c) 12 s (d) 3 s

16. Human ears can sense sound waves travelling in air having wavelength of
(a) 10^{-3} m (b) 10^{-2} m
(c) 1m (d) 10^2 m

17. Which of the following statements is wrong ?
(a) Sound travels in straight line
(b) Sound is form of energy
(c) Sound travels in the form of waves
(d) Sound travels faster in vacuum than in air

18. Voice of a friend is recognized by its
(a) pitch (b) quality
(c) intensity (d) velocity

19. A sound wave travels from east to west in which direction do the particles of air move ?
(a) east-west (b) north-south
(c) up and down (d) none of these
20. The unit of quantity on which loudness of sound depends is
(a) metre (b) Hz
(c) metre/second (d) second
21. The frequency of a wave travelling at a speed of 500 ms^{-1} is 25Hz. Its time period is
(a) 20s (b) 0.05 s
(c) 25s (d) 0.04 s
22. A periodic wave is produced on a stretched string. Note the following quantities associated with the wave and the string
I. frequency II. tension in the string
III. wavelength IV. linear density of the string
Which of the quantities influences the speed of the waves?
(a) I only (b) III and IV only
(c) I and II only (d) I, II, III and IV
23. What is the speed of a 150 Hz wave whose wavelength is measured to be 0.30 m
(a) 45 m/s (b) 5.0×10^2 m/s
(c) 0.0020 m/s (d) It cannot be found
24. A 440 Hz sound wave travels with a speed of 340 m/s. What is the wavelength of the wave?
(a) 1.5×10^5 m (b) 0.77m
(c) 1.3m (d) 1.1m
25. A wave travels through a medium with a speed of 340m/s. If the frequency of the wave is doubled, what happens to the wave speed ? What happens to the wavelength
(a) remains constant, doubles
(b) doubles, reduced by a factor of two
(c) remains constant, reduced by a factor of two
(d) reduced by a factor of two, remains constant
26. The velocity of sound in any gas depends upon
(a) wavelength of sound only
(b) density and elasticity of gas
(c) intensity of sound waves only
(d) amplitude and frequency of sound
27. If the amplitude of sound is doubled and the frequency reduced to one fourth, the intensity of sound at the same point will be
(a) increasing by a factor of 2
(b) decreasing by a factor of 2
(c) decreasing by a factor of 4
(d) unchanged
28. With the propagation of a longitudinal wave through a material medium, the quantities transmitted in the propagation direction are
(a) energy, momentum and mass
(b) energy
(c) energy and mass
(d) energy and linear momentum
29. What is the effect of humidity on sound waves when humidity increases?
(a) speed of sound waves is more
(b) speed of sound waves is less
(c) speed of sound waves remains same
(d) speed of sound waves becomes zero

More than One Option Correct :

DIRECTIONS: *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which One or More may be correct.*

1. Which of the following are transmitted by a wave ?
(a) Momentum (b) Energy
(c) Amplitude (d) Velocity
2. A mechanical wave propagates in a medium along the *x*-axis. The particles of the medium
(a) may move on the *y*-axis
(b) may move on the *x*-axis
(c) must move on the *x*-axis
(d) must move on the *y*-axis
3. The velocity of sound is affected by change in
(a) pressure (b) wavelength
(c) medium (d) temperature
4. Transverse mechanical waves can travel in
(a) stretched string (b) hydrogen gas
(c) iron rod (d) water
5. When a wave is refracted into another medium, which of the following will change ?
(a) Frequency (b) Phase
(c) Amplitude (d) Velocity
6. Which of the following waves are electromagnetic ?
(a) Light waves (b) Sound waves
(c) Microwaves (d) X-rays
7. Which of the following statements are correct ?
(a) Pitch of sound depends on its frequency.
(b) Loudness of sound depends on its intensity.
(c) Sound travels faster in air than in solids.
(d) Speed of sound in air varies with temperature.
8. Which of the following waves are transverse ?
(a) Light waves in air
(b) Sound waves in air
(c) Waves on stretched string
(d) Waves on surface of water

Multiple Matching Questions :

DIRECTIONS : *Following question has four statements (A, B, C and D) given in Column I and five statements (p, q, r, s, t) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in column I with entries in column II.*

1. Match the following

Column - I	Column - II
(A) Sound waves	(p) Electromagnetic
(B) Light waves	(q) Mechanical
(C) Micro waves	(r) Transverse
(D) Waves on surface of water	(s) Longitudinal
	(t) Travel faster in solids than in air.

	A	B	C	D
(a)	p,q	q	r, s	q, r
(b)	q, s, t	p, r	p, r	q, r
(c)	p, s	q	r, s, t	r
(d)	r, s	p, q, r	s, t	p

Passage Based Questions :

DIRECTIONS: *Study the given paragraph(s) and answer the following questions.*

PASSAGE

On the basis of wave motion, a wave can be categorized as transverse or longitudinal waves. In a transverse wave particles of the medium vibrate in a direction perpendicular to the direction of propagation of the wave, while in a longitudinal, particles of the medium vibrate in the direction of propagation of the wave. Electromagnetic waves are always transverse while mechanical waves may be transverse as well as longitudinal. Again, transverse mechanical waves can propagate through solids and on the surface of liquids but not in the bulk of liquids or gases. On the other hand, longitudinal waves can travel through any material medium.

1. Which of the following waves can never be transverse ?
(a) Sound waves through air
(b) Light waves through air
(c) Waves set up on a stretched string
(d) Waves set up on the surface of water when a stone is hit on the surface of water.

2. A mechanical wave is propagating through a solid rod. The wave may be
(a) transverse (b) longitudinal
(c) both (a) and (b) (d) none of these.

3. A transverse mechanical waves can propagate through a medium which has elasticity of shape. Which property of liquid makes propagation of transverse wave on the surface of the liquid ?
(a) bulk modulus (b) surface tension
(c) viscosity (d) pressure

Assertions & Reason :

DIRECTIONS: *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are correct and **Reason** is the correct explanation of Assertion.
(b) If both **Assertion** and **Reason** are correct, but **Reason** is not the correct explanation of **Assertion**.
(c) If **Assertion** is correct but **Reason** is incorrect.
(d) If **Assertion** is incorrect but **Reason** is correct.

1. Assertion : On a rainy day sound travel slower than on a dry day.
Reason : When moisture is present in air the density of air increases.

2. Assertion : Two persons on the surface of moon cannot talk to each other.
Reason : There is no atmosphere.

3. Assertion : Waves produced in a cylinder containing a liquid by moving its piston back and forth are longitudinal waves.
Reason : In longitudinal waves, the particle of the medium oscillate parallel to the direction of propagation of the wave.

4. Assertion : The longitudinal waves are called pressure waves.
Reason : Propagation of longitudinal waves through a medium involves changes in pressure and volume of air, when compression and rarefaction are formed.

Integer/Numeric type Questions :

DIRECTIONS : *Following are integer based/Numeric based questions. Each question, when worked out will result in one integer or numeric value.*

1. Ocean waves of time period of 10s have a speed of 15m/s. What is the wavelength of these. Also find the horizontal distance between a wave crest and the adjoining wave trough. You may assume the waves as harmonic.

2. A man standing in a gorge between two large cliffs gives a short sharp shout. He hears two echoes, the first after 1 second and the next after 1½ seconds. The speed of sound is 340 m/s. What is the distance between the cliffs.

3. A wave pulse on a string moves a distance of 16m in 0.1s. Calculate the velocity of the pulse and what would be the wavelength of the wave on the same string if its frequency is 200 Hz ?

4. In a ripple tank, 10 full ripples are produced in one second. The distance between a trough and a crest is 5 cm. Calculate :
(i) the frequency,
(ii) the wavelength and
(iii) the velocity of the ripples

5. The wheel of a siren has k = 30 holes and rotates at a frequency n = 600 rpm. Find the wavelength of the sound produced by the siren if the velocity of sound is v = 340m/s,

4 ADVANCED EXERCISE
BASED ON CONNECTING TOPICS

DIRECTIONS (Qs. 1-12) : *This section contains multiple choice questions. Each questions has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.*

1. A standing wave is represented by $y = A \sin(100t) \cos(0.01x)$, where y and A are in millimetre, t in seconds and x is in metre. Velocity of wave is
(a) 10^4 m/s (b) 1 m/s
(c) 10^{-4} m/s (d) not derivable from above data

2. For production of beats the two sources must have
(a) different frequencies and same amplitude
(b) different frequencies
(c) different frequencies, same amplitude and same phase
(d) different frequencies and same phase

3. A closed organ pipe (closed at one end) is excited to support the third overtone. It is found that air in the pipe has
(a) three nodes and three antinodes
(b) three nodes and four antinodes
(c) four nodes and three antinodes
(d) four nodes and four antinodes

4. A wave of frequency 100 Hz is sent along a string towards a fixed end. When this wave travels back after reflection, a node is formed at a distance of 10 cm from the fixed end of the string. The speeds of incident (and reflected) waves are
(a) 5 m/s (b) 10 m/s
(c) 20 m/s (d) 40 m/s

5. Two waves are said to be coherent, if they have
(a) same phase but different amplitude
(b) same frequency but different amplitude
(c) same frequency, phase & amplitude
(d) different frequency, phase and amplitude

6. Two waves having the intensities in the ratio of 9 : 1 produce interference. The ratio of maximum to the minimum intensity is equal to
(a) 2 : 1 (b) 4 : 1
(c) 9 : 1 (d) 10 : 8

7. Velocity of sound measured in hydrogen and oxygen gas at a given temperature will be in the ratio
(a) 1 : 1 (b) 2 : 1
(c) 1 : 4 (d) 4 : 1

8. The ratio of fundamental frequency of an organ pipe opened at both ends to that of the organ pipe closed at one end is
(a) 1 : 1 (b) 1.5 : 1
(c) 2 : 1 (d) 3 : 1

9. A cylindrical tube open at both ends, has a fundamental frequency f in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of air column is now
(a) $f/2$ (b) f
(c) $3f/4$ (d) $2f$

10. Two identical stringed instruments have frequency 100 Hz. If tension in one of them is increased by 4% and they are sounded together then the number of beats in one second is
(a) 1 (b) 8
(c) 4 (d) 2

11. Two whistles A and B produces notes of frequencies 660 Hz and 596 Hz respectively. There is a listener at the mid-point of the line joining them. Now the whistle B and the listener start moving with speed 30 m/s away from the whistle A. If speed of sound be 330 m/s, how many beats will be heard by the listener?
(a) 2 (b) 4
(c) 6 (d) 8

12. A source producing sound of frequency 170 Hz is approaching a stationary observer with a velocity 17 ms^{-1}. The apparent change in the wavelength of sound heard by the observer is (speed of sound in air = 340 ms^{-1})
(a) 0.1 m (b) 0.2 m
(c) 0.4 m (d) 0.5 m

DIRECTIONS (Qs. 13-19) : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which One or More may be correct.*

13. Apparent frequency of sound to an observer is different from its actual frequency if
(a) source of sound moves towards stationary listener.
(b) listener moves towards stationary source of sound.
(c) both source and listener move with same speed in same direction.
(d) both source and listener move with same speed towards each other.

14. Which of the following are true for a particle executing simple harmonic motion ?

(a) Speed of the particle is maximum at mean position.

(b) Speed of the particle is zero at extreme position.

(c) Acceleration of the particle is maximum at extreme position

(d) Acceleration of the particle is zero at mean position.

15. Which of the following motions are oscillatory ?

(a) Vibrations of a loaded spring.

(b) Motion of the earth around the sun.

(c) Motion of a ball bouncing from the ground without loss of energy.

(d) Motion of minute's hand of the clock.

16. In case of a simple harmonic motion

(a) restoring force is always directed towards the mean position.

(b) restoring force is maximum at the mean position.

(c) kinetic energy is maximum at the mean position.

(d) kinetic energy is zero at the extreme position.

17. Which of the following statements are correct ?

(a) Every oscillatory motion is simple harmonic.

(b) Every oscillatory motion is periodic.

(c) Every simple harmonic motion is oscillatory.

(d) Every simple harmonic motion is periodic.

18. Total energy of a particle executing simple harmonic motion

(a) is constant

(b) depends on mass of the particle.

(c) depends on frequency of the particle.

(d) depends on amplitude of the particle.

19. Which of the following represent periodic motion ?

(a) Oscillations of a simple pendulum.

(b) Motion of the moon around the earth.

(c) Motion of second's hand of the clock.

(d) Vibration of a loaded spring.

DIRECTIONS (Qs. 20-21) : *Following question has four statements (A, B, C and D) given in Column I and four & five statements (p, q, r, s, t) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in column I with entries in column II.*

20. Match the columns.

	Column - I		**Column - II**
(A)	A car moving along a circular track with increasing speed.	(p)	S.H.M.
(B)	Motion of a planet around the sun.	(q)	Oscillatory but not S.H.M.
(C)	The motion of piston on an automobile engine	(r)	Periodic but not oscillatory
(D)	The motion of a simple pendulum with small amplitude.	(s)	Non-periodic

	A	B	C	D
(a)	s	r	q	p
(b)	p, q, r, s	q	p, q, r, s	p, q, r, s
(c)	r, s	p	q, r	s
(d)	p,	q, r	r	s

21. Match the following for an oscillatory motion.

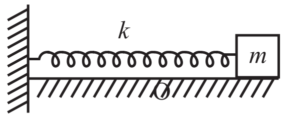
	Column - I		**Column - II**
(A)	Kinetic energy	(p)	Maximum at extreme position
(B)	Potential energy	(q)	Maximum at mean position
(C)	Total energy	(r)	Minimum at extreme position
(D)	Acceleration	(s)	Minimum at mean position
		(t)	Constant

	A	B	C	D
(a)	p,q	q	r, s	q, r
(b)	p, t	p, q, r	s	r, t
(c)	p, s	q	r, s, t	r
(d)	q, r	p, s	t	p, s

DIRECTIONS (Qs. 22-24) : *Study the given paragraph(s) and answer the following questions.*

PASSAGE

A block of mass m is attached to a spring of spring constant k and is placed on a horizontal frictionless surface with other end of the spring fixed as shown. Now the spring is compressed slightly and released so that the block starts oscillating about the initial position. Then,



22. The kinetic energy of the block is
(a) maximum at extreme position
(b) zero at extreme position
(c) minimum at mean position
(d) zero at mean position

23. The total energy of the spring block system is
(a) constant
(b) increasing continuously
(c) decreasing continuously
(d) first increasing then decreasing

24. The force on the block is always directed
(a) towards the mean position
(b) away from the mean position
(c) perpendicular to the line of oscillation
(d) none of these.

DIRECTIONS (Qs. 25-27) : *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are correct and **Reason** is the correct explanation of Assertion.
(b) If both **Assertion** and **Reason** are correct, but **Reason** is not the correct explanation of **Assertion**.
(c) If **Assertion** is correct but **Reason** is incorrect.
(d) If **Assertion** is incorrect but **Reason** is correct.

25. **Assertion :** Ultrasonic waves are longitudinal waves of frequency greater than 20,000 Hz.
Reason : The maximum frequency of audible sound waves is 20,0000 Hz

26. **Assertion :** In simple harmonic motion, the velocity is maximum when acceleration is minimum.
Reason : Displacement and velocity of SHM differ in phase by $\pi/2$.

27. **Assertion :** A particle performing SHM while crossing mean position is having minimum potential energy, this minimum PE could be non-zero.
Reason : In equilibrium position, the net force experienced by the particle is zero, hence PE would be zero at the mean position.

DIRECTIONS (Qs. 28-32) : *Following are integer based/ Numeric based questions. Each question, when worked out will result in one integer or numeric value.*

28. A simple harmonic oscillator has an amplitude A and time period T. Find the time required by it to travel from $x = A$ to $x = \frac{A}{2}$.

29. A spring has a force constant k, and a mass m is suspended from it. The spring is cut in half and the same mass is suspended from one of the halves. Is the frequency of vibration the same before and after the spring is cut? How are the frequencies related?

30. A particle undergoes simple harmonic motion having time period T. What will be the time taken in 3/8th oscillation?

31. Two particles are executing S.H.M. of same amplitude and frequency along the same straight line path. They pass each other when going in opposite directions, each time their displacement is half of their amplitude. What is the phase difference between them ?

32. A body of mass 5 gram is executing S.H.M. about a fixed point O. With an amplitude of 10 cm, its maximum velocity is 100 cm/s. At what distance its velocity will be 50 cm s^{-1}?

SOLUTIONS

Brief Explanations of Selected Questions

1 EXERCISE

FILL IN THE BLANKS :

1.	500 cycles/second	**2.**	100
3.	0	**4.**	$E \propto v^2$
5.	vacuum	**6.**	tone
7.	amplitude	**8.**	the same as
9.	crests and troughs		
10.	compressions and rarefactions		
11.	energy	**12.**	wavelength
13.	harmonic		

TRUE/FALSE :

1.	True	**2.**	False	**3.**	True	**4.**	False
5.	True	**6.**	False	**7.**	True	**8.**	False
9.	True	**10.**	False	**11.**	True	**12.**	False

MATCH THE COLUMNS :

1. (A) → (r); (B) → (s); (C) → (p); (D) → (q)

VERY SHORT ANSWER QUESTIONS :

1. Energy.

2. Transverse waves.

3. The distance travelled by a wave during the time a particle of the medium completes one vibration, is called 'wavelength' of the wave.

4. The minimum distance between a source of sound and the obstacle to produce an echo is 17 m. As the length of a room is generally less than 17 m, so we do not hear an echo.

5. Longitudinal waves travel in a medium as series of alternate compressions and rarefactions i.e., they travel as variations in pressure and hence they are called pressure waves.

6. Sounds waves having frequencies between Hz and 20,000 Hz can be heard by the human ear. This range of frequency is called 'human audible range'.

7. Echo is the phenomenon of repetition of sound due to its reflection from the surface of a large obstacle.

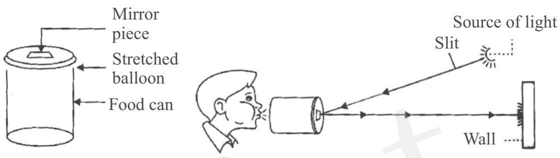
8. Ultrasound is used in echocardiography or ultrasonography.

SHORT ANSWER QUESTIONS :

1. (i) Medium must be elastic so that the medium particles have the tendency to return back to their original positions after the displacement.

(ii) Medium must have the inertia so that its particles have the capacity to store the energy.

2. Take a tin can. Remove both ends to make it a hollow cylinder. Take a balloon and stretch it over the can, then wrap a rubber band around the balloon. Take a small piece of mirror. Use a drop of glue to stick the piece to the balloon. Allow the light through a slit to fall on the mirror. After reflection the light spot is seen on the wall, as shown in figure given below. Shout directly into the open end of the can and observe the dancing light spot on the wall.



The sound wave incident on the back of the mirror makes set it into vibration. As a result, the reflected spot of light appears to dance on the wall.

3. Since wavelength is the distance travelled by the wave during the time a particle of the medium completes one vibration, therefore if l be the wavelength and T the time-period, then the wave travels a distance l in time T. Hence,

$$\text{Wave velocity} = \frac{\text{Distance}}{\text{Time}} \text{ or } v = \frac{\lambda}{T}$$

or

Wave velocity = frequency × wavelength.

The wave velocity in a medium remains constant under the same physical conditions.

4. An echo is produced when sound reflected from a distant obstacle comes back after an interval of second or more. In an echo, the original and reflected sounds are heard separately. Reverberation on the other hand consists of successive reflections which follow echo other so quickly that they cannot produce separate echoes.

5. SONAR is the short from of Sound Navigation and Ranging. It is used to locate under water bodies or enemy submarine etc. SONAR works on the principle of finding time take by ultrasonic sound to travel back to the ship after hitting the target.

It is given by $t = \frac{2d}{v}$ where v is the velocity of wave in the media and d is the distance of the object from the ship. It has a transmitter and receiver to send and receive ultrasonic waves.

6. Bats search out prey and fly in dark night by emitting and detecting reflections of ultrasonic waves. The high-pitched ultrasonic squeaks of the bat are reflected from the obstacles or prey and return to bat's ear.

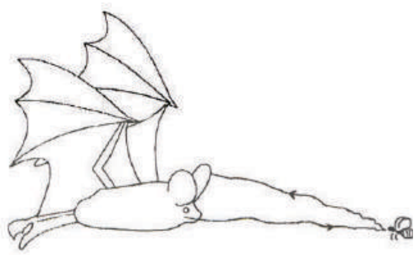


Fig. Ultrasound is emitted by bat and reflected back by the prey or obstacle.

The nature of reflections tells the bat where the prey is.

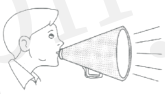
9. Resonance may occur where the frequency of vibration of the stepping of soldiers becomes equal to that of the bridge, so the bridge will vibrate with larger amplitude and may collapse.

10. The girl can be considered as an extended body. As the girl stands up on the swing the separation '*d*' between the point of suspension and the centre of gravity decreases since the time period is inversely proportional to $\sqrt{d}$, time period increases and frequency decreases

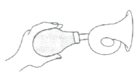
Long Answer Questions :

1. Uses of multiple reflection of sound :

(i) **Megaphones and musical instruments.**
Megaphones or loud hailers, horns, musical instruments such as trumpets and shehanais, are all designed to send sound in a particular direction without spreading it in all directions, as shown in Fig. In these instruments, a tube followed by a conical opening reflects sound successively to guide most of the sound waves from the source in the forward direction towards the audience.



Megaphone



Horn

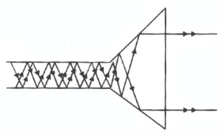


Fig. Multiple reflection of sound in a megaphone or horn.

(ii) **Stethoscope.**
In a stethoscope, the sound of a patient's heart bead is guided along the tube of the stethoscope to the doctor's ears by multiple reflections of sound, as shown in figure

***Fig.** Multiple reflections of sound in the tube of a stethoscope*

(iii) **Curved ceilings concert halls.**
The ceilings of concert halls, conference halls and cinema halls are made curved so that sound after reflection reaches all corners of the hall, as shown in figure.

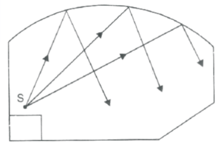


Fig. Curved ceiling of a conference hall

(iv) **Sound boards.**
In large halls or auditorium, large concave wooden boards are placed behind the speaker, as shown in figure. The speaker stands at the focus of this concave reflecting surface. After reflection, the sound is spread evenly towards the audience. This makes the speech readily audible even at a distance.

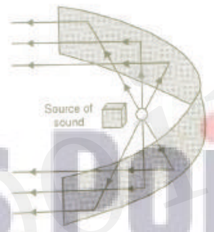


Fig. Sound board used in a big hall.

2. When a longitudinal wave passes through a medium, the particles of the medium alternately come closer together and move away from one another so that there are alternate regions of increased and decreased density. These regions are called compressions and rarefactions respectively. Figure (a) and (b) represent the density and pressure variations, respectively as a sound wave propogates through a medium. Figure (c) is the graphical representation of such a wave. It represents the variations of density as well as the pressure of the medium at a given time with distance, above and below the average value of density and pressure.

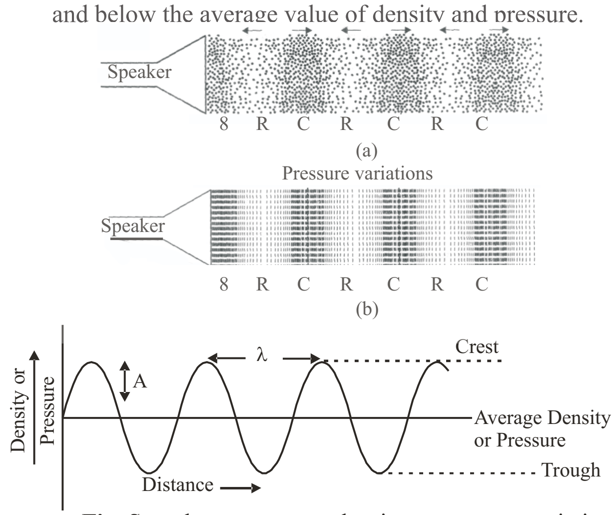


Fig. Sound propagates as density or pressure variations as shown in (a), (b), (c) represents graphically the density and pressure variations

The distance between two successive compressions (*C*) or two rarefactions (*R*) is called wavelength. It is usually represented by λ (lambda).

3. **Human ear**: It is a highly sensitive part of the human body which enables us to hear a sound. It converts the pressure variations in air with audible frequencies into electric signals which travel to the brain via the auditory nerve.

The human ear has three main parts. Their auditory functions are as follows.

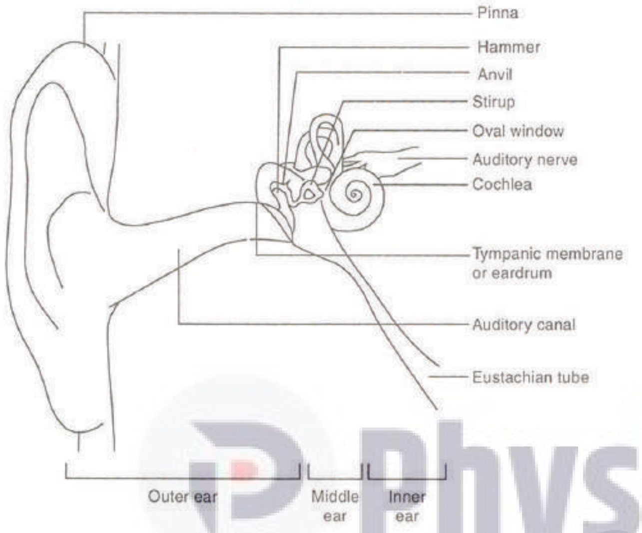


Fig. *Auditory parts of the human ear.*

(i) **Outer ear.** The outer ear is called 'pinna'. It collects the sound from the surrounding. The collected sound passes through the auditory canal. At the end of the auditory canal there is a thin membrane called the ear drum or tympanic membrane. When compression of the medium produced due to vibration of the object reaches the ear drum, the pressure on the outside of the membrane increases and forces the eardrum inward. Similarly, the eardrum moves outward when a rarefaction reaches. In this way the ear drum vibrates.

(ii) **Middle ear.** The vibrations are amplified several times by three bones (the hammer, anvil and stirrup) in the middle ear which act as levers. The middle ear transmits the amplified pressure variations received from the sound wave to the inner ear.

(iii) **Inner ear.** In the inner ear, the pressure variations are turned into electrical signals by the **cochlea**. These electrical signals are sent to the brain via the **auditory nerve**, and the brain interprets them as sound.

4. (i) Amplitude of sound waves determines loudness of sound.

(ii)

	Longitudinal		Transverse wave
1.	In these waves, displacement of the particles of the medium is at right angles to the direction of propagation of the waves.	1.	In these waves displacement of the particles of the medium is in the direction of propagtion of the waves.
2.	These waves travel in the form of crests and troughs. Example: Sound waves are longitudinal waves	2.	These waves travel in the form of compression and rarefactions. Example: The water waves formed on the surface of water in pond are transverse waves

(iii) The ceiling of concert halls are made curved so that sound, after reflection from the ceiling , reaches all the parts of the hall.

2 EXERCISE

Text-Book Questions :

1. The outer ear pinne collects the sound produced by a vibrating body. The collected sound passes though the auditory canal. When a compression of the medium reaches the eardrum the pressure on the outside of the membrane increases and forces the eardrum inward. Similarly, the eardrum moves outward when a rarefaction reaches it. In this way eardrum vibrates. The vibrations are amplified several times by three boxes (the *hammer, crinol* and *stirrup*) in the middle ear. The middle ear transmits the amplified pressure variations received from the sound wave to the inner ear. In the inner ear, the pressure varations are turned into electrical signals by the cochlea. These electrical signals are sent to the brain in the auditory nerve and brain interprets them as sound. Thus the sound produced by a vibrating object in a medium reach our ear.

2. When the school bell is hit by a hammer, it begins to vibrate and produces sound. If we touch the bell gently just after hitting it, we will feel vibrations. We can say that sound is produced by a vibrating body.

3. Sound needs a material medium like air, water, steel, etc. for its propagation. It is characterised by the motion of the particles of the medium. So, sound waves are called mechanical waves.

4. No. The moon has no atmosphere or material medium through which sound can travel.

5. (a) Amplitude of the wave determines the loudness.

(b) Frequency of the wave determines the pitch.

6. The sound of a guitar is shriller than that of a car horn. So, the sound of a guitar has a higher pitch.

7. **Wavelength (λ):** It is defined as the distance between two consecutive compressions or rarefactions of a sound wave.

Frequency (ν). The number of complete oscillations per unit time is called frequency.

Time period (T). The time taken by two consecutive compressions or rarefactions to cross a fixed point is called the time period of the sound wave.

Amplitude (A). The magnitude of the maximum disturbance in the medium on either side of the mean value is called the amplitude of the wave.

8. Wave speed = Frequency × Wavelength

or $v = \nu\lambda$

9. Wavelength $= \frac{\text{Wave speed}}{\text{Frequency}} = \frac{440\,\text{ms}^{-1}}{220\,\text{s}^{-1}} = 2\,\text{m}$

10. Time interval between two successive compressions,

$$T = \frac{1}{\nu} = \frac{1}{500\,\text{Hz}} = \frac{1}{500\,\text{s}^{-1}} = 0.002\,\text{s}$$

11.

	Loudness		Intensity of sound
1.	Loudness is a subjective quantity. It depends upon the sensitivity of the human ear. A sound may be loud for a person but the same sound may be feable for another person who is hard of hearing.	1.	Intensity of a soundis an objective physical quantity. It does not depend on the sensitivity of a human ear.
2.	Loudness cannot be measured as a physical quantity because it is just sensation which cannot be felt.	2.	Intensity of sound can be measured as a physical quantity.

12. Sound travels fastest through iron at a particular temperature at a speed of 5950 ms^{-1}.

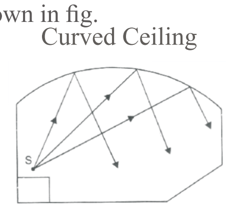
13. Speed of sound, $v = 342\ \text{ms}^{-1}$

Time taken for hearing the echo, $t = 3$ s

Distance of the reflecting surface,

$$d = \frac{v \times t}{2} = \frac{342 \times 3}{2} = 513\ \text{m}.$$

14. The ceilings of the concert halls are made curved so that sound after reflecting from the ceiling reaches all corners of the hall, as shown in fig.



Curved ceiling of a concert hall

15. For an average human ear, the audible range of frequency extends from 20 Hz to 20,000 Hz.

16. (a) Sound of frequency less than 20 Hz is called infrasound.

(b) Sound of frequency higher than 20 kHz is called ultrasound.

17. Time between transmission and detection of sonar pulse,

$t = 1.02$ s

Speed of sound in salt water,

$v = 1531\ \text{ms}^{-1}$

Distance of the cliff = d (say)

Then the distance travelled by sound = $2d$

But

$2d$ = Speed of sound × time = vt

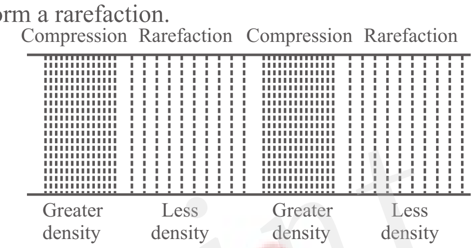
= 1531 × 1.02 m

$$\therefore \quad d = \frac{1531 \times 1.02}{2} = 780.81\,\text{m}$$

Text-Book Exercise :

1. Sound is a form of energy which produces a sensation of hearing in our ears. Sound is produced due to vibration of different materials.

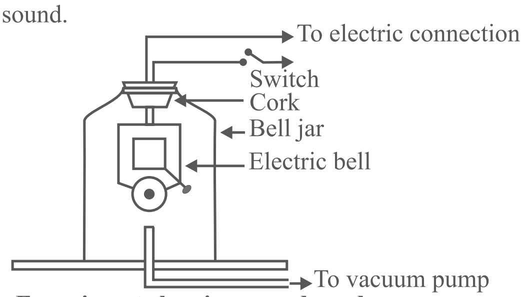
2. When a longitudinal wave passes through air, then some of the particles of air get crowded together and form compression, whereas other particles go farther apart and form a rarefaction.



Compressions and rarefactions produced in air

3. *Sound is a mechanical wave which needs a material medium to travel*. It can travel through air, water, steel, etc. But cannot travel through vacuum. This can be demonstrated by the following simple experiment.

Experiment. Suspend an electric bell inside a glass bell jar by passing the connecting wires through an airtight cork fitted at the mouth of the jar. Place the jar over a disc which has a pipe connected to a vacuum pump, as shown in Fig. When we turn on the switch, we hear the sound of the bell. Now, we pump out the air from the jar gradually with the help of the vacuum pump. The sound becomes fainter. When most of the air has been removed, we hear a very feeble sound.



Experiment showing sound needs a medium to travel

When there is air inside the jar, sound travels through it to the wall of the jar. This makes the wall to vibrate which in turn sends sound to us. When air is removed, sound from the bell cannot travel to the wall of the jar.

4. Sound waves travel through a medium in the form of compressions and rarefactions. The particles of the medium thus move to and from along the direction of propagation of the sound wave. That is why, the sound waves are called longitudinal waves.

5. It is the pitch of sound wave that makes us to identify the voice of our friend while sitting in a dark room.

6. The speed of sound (344 ms^{-1}) is much smaller than the speed of light (3×10^8 ms^{-1}) . So thunder is heard a few seconds after the flash is seen.

7. Speed of sound, v = 344 ms^{-1}

Frequencies, v_1 = 20 Hz and v_2 = 20 kHz = 20,000 Hz

Speed = wavelength × frequencies

$$v = \lambda \times v$$

$$\therefore \quad \lambda = \frac{v}{v}$$

Wavelength of sound waves corresponding to 20 Hz.

$$\lambda_1 = \frac{v}{v_1} = \frac{344}{20} = 17.2 \text{ m.}$$

Wavelength of sound waves corresponding to 20 kHz,

$$\lambda_2 = \frac{v}{v_2} = \frac{344}{20,000} = 0.0172 \text{ m}$$

8. Let length of the aluminium rod = d metre

Speed of sound in air = 346 ms^{-1}

Using the formula, time = $\frac{\text{distance}}{\text{speed}}$

Time taken by sound in air, t (air) = $\frac{d}{346}$ s

Speed of sound in aluminium = 6420 ms^{-1}

Time taken by sound in aluminium rod,

$$t(\text{aluminium}) = \frac{d}{6420} \text{ s}$$

Ratio of times taken by sound wave in air and aluminium,

$$\frac{t_{air}}{t_{aluminium}} = \frac{d}{346} \times \frac{6420}{d} = \frac{6420}{346} = 18.55.$$

9. Frequency of source = 100 Hz = 100s^{-1}

∴ Number of vibrations in 1s = 100

Number of vibrations in 1 minute or 60 s = 100 × 60 = 6000

10. Yes, sound follows the same laws of reflection as light does. The directions in which sound is incident and reflected make equal angles with the normal to the reflecting surface. The incident wave, the reflected wave and the normal at the point of incidence all lie in the same plane.

11. Time = $\frac{\text{Distance}}{\text{Speed}}$

i.e., Time is inversely proportional to the speed. In any medium, as we increase the temperature the speed of sound increases. Thus on a hot day due to high temperature the speed of sound increases. Hence the time decreases. So we can hear the echo sooner.

12. (i) In a stethoscope, the sound of a patient's heartbeat is guided along the tube of the stethoscope to the doctor's ears by multiple reflections of sound.

(ii) Reflection of sound plays an important role in the designing of concert halls.

13. First we find the time taken by the stone to reach the base of the tower. We use the relation.

$$s = ut + \frac{1}{2}gt^2$$

But, s = 500 m, u = 0, g 10 ms^{-2}

$$\therefore \quad 500 = 0 + \frac{1}{2} \times 10 \times t^2$$

or $t^2 = \frac{500}{5} = 100$

or $t = 10$ s

Time taken by sound to travel from base of the tower to its top,

$$t' = \frac{\text{Distance}}{\text{Speed of sound}} = \frac{500 \text{ m}}{340 \text{ ms}^{-1}} = 1.47 \text{ s}$$

Total time after which the splash is heard

$$= t + t' = 10 + 1.47 = 11.47 \text{ s}$$

14. Speed of sound wave, v = 339 ms^{-1}

wavelength, λ = 1.5 cm = 0.015 m

Frequency, $v = \frac{v}{\lambda} = \frac{339}{0.015} = 22,600$ Hz

As this frequency is higher than 20,000 Hz, so it is not audible.

15. The persistence of sound in a big hall or auditorium due to the repeated reflections of sound is called reverberation.

Reverberation can be reduced by following methods:

(i) By covering the walls and roofs of the halls with sound absorbing materials.

(ii) By providing heavy curtains with folds.

16. Loudness is a physiological response of the ear to the intensity of sound. It distinguishes between a loud sound and a low sound.

Loudness depends on two factors:

(i) Intensity of sound which is directly proportional to the square of amplitude of the sound wave.

(ii) Sensitivity of the ear.

17. Bat can produce and receive ultrasonic waves. During its flight, a bat emits ultrasonic waves. The bat receives back these waves after being reflected by the obstacle in its path. From the nature of the reflected waves, the bat gets information where the obstacle or prey is and what is its size. Propoises also use ultrasound for navigation and location of food even in total darkness.

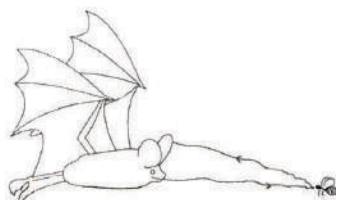


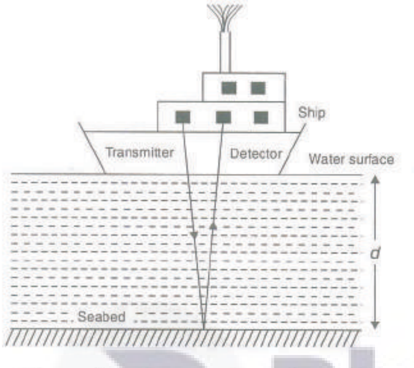
Fig. Ultrasound is emitted by bat and it is reflected back by the prey

18. The object to be cleaned is placed in a cleaning solution and ultrasonic waves are sent into the solution. Due to the high frequency, the particles of dust, grease and dirt get detached and the object gets thoroughly washed.

19. **SONAR:** The acronym **SONAR** stands for **Sound Navigation and Ranging**. Sonar is a device that uses ultrasonic waves to measure the distance, direction and speed of underwater objects.

Principle: It uses the phenomenon of echoes in determining the sea-depth and locating the presence of under -water objects.

Working: A strong beam of ultrasonic waves is sent from a transmitter mounted on the ship. The beam is reflected from the seabed and is received by an under-water detector which is also mounted on the ship. The time interval between transmission and reception of the ultrasonic signal is noted.



Ultrasound sent by a transmitter and received by a detector.

If ultrasonic waves travel with speed v in sea-water and time t is elapsed between the transmission and reception of the ultrasound signal, then sea depth will be

$$d = \frac{v \times t}{2}$$

The SONAR method is also called **echo ranging**. This technique is used to determine the depth of the sea and to locate underwater hills, valleys, submarine, icebergs, sunken ship etc.

20. Time between transmission and detection $t = 5$s
Distance of the sonar from the submarine, $d = 3625$ m
Total distance covered by sound
$= 2d = 2 \times 3625 = 7250$ m

Speed of sound, $v = \frac{2d}{t} = \frac{7250}{5} = 1450 \text{ ms}^{-1}$

21. **Ultrasonic detection of defects in metals:** Metallic components are used in the construction of big structures like buildings, bridges, machines, scientific equipments, etc. The cracks or holes inside the block reduce the strength of the structure. Such defects are not visible from outside. Ultrasonic waves can be used to detect such defects.

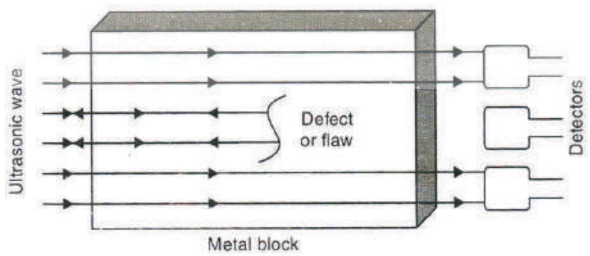


Fig. Ultrasound is reflected back from the defective locations inside a metal block.

EXEMPLAR QUESTIONS :

1. From the graph
Time period, $T = 2 \times 10^{-6}$ s.
Frequency, $\nu = 1/T = 5 \times 10^5$ Hz.
Wavelength, $\lambda = \upsilon/\nu = 5 \times 10^5$m.

2. If the time gap between the original sound and reflected sound received by the listener is around 0.1 s, only then the echo can be heard.
The minimum distance travelled by the reflected sound wave for the distinctly listening the echo
= velocity of sound × time interval
$\simeq 344 \times 0.1$
$\simeq 34.4$ m
But in this case the distance travelled by the sound reflected from the building and then reaching to the girl will be (6 + 6) = 12 m, which is much smaller than the required distance. Therefore, no echo can be heard.

3. Humming bees produce sound by vibrating their wings which is in the audible range. In case of pendulum the frequency is below 20 Hz which does not come in the audible range.

4. $\angle i = \angle r$, so $x = 90^o - \angle r = 90^o - 50^o = 40^o$

HOTS QUESTIONS :

1. Neither! It is the frequency of a wave that undergoes a change when the source is moving, not the wave speed.

2. There are two basic conditions for an echo to be heard. One, the obstacle should be rigid and large in size. Second. the obstacle should be at least at a distance of 17 metres from the source. Since length of a room is less than that of a hall (i.e., less than 17 metres) so no echo is heard in small room.

3. The light travels at 3.0×10^8 m/s and thus may be regarded as reaching the observer instantaneously. The sound takes 6 seconds and the distance it travels is given by speed × time. Sound travels at 340 m/s so distance = 340 m/s × 6s = 2040 m. Thus the storm is 2040m from the observer.

4. The minimum distance between compression and rarefaction of the wire

$$\ell = \frac{\lambda}{4}$$

∴ Wavelength

Now by $v = n\lambda \Rightarrow n = \frac{360}{4 \times 1} = 90 \text{ sec}^{-1}$

3 EXERCISE

SINGLE OPTION CORRECT :

1. (d)	**2.** (b)	**3.** (c)	**4.** (c)	**5.** (a)
6. (d)	7. (a)	**8.** (c)	**9.** (a)	**10.**(c)
11. (a)	**12.** (b)	**13.** (a)	**14.** (c)	**15.**(d)
16. (c)	**17.** (d)	**18.** (b)	**19.** (a)	**20.**(a)
21. (d)	**22.** (c)	**23.** (a)	**24.** (b)	**25.**(b)

26. (b) Velocity of sound in any gas depends upon density and elasticity of gas.

27. (c) Intensity $\propto$ (amplitude)^2 and also intensity $\propto$ (frequency)^2.

Therefore, intensity becomes $\frac{2^2}{4^2} = \frac{1}{4}$th

28. (b) With the propagation of a longitudinal wave, energy alone is propagated.

29. (a) Velocity of sound = $\sqrt{\frac{\gamma RT}{M}}$

When water vapour are present in air average molecular weight of air decreases and hence velocity increases.

More than one Option Correct :

1. (a, b)
A wave transports momentum and energy.

2. (a, b)
The particles of the medium will move along x-axis if the wave is longitudinal and along y-axis if the wave is transverse.

3. (a, c, d)

4. (a, c)
Transverse mechanical waves can travel in solids or on the surface of liquids only.

5. (c, d) **6.** (a, c, d)

7. (a, b, d) **8.** (a, c, d)

Multiple Matching Questions :

1. (b) (A) $\to$ (q, s, t); (B) $\to$ (p, r); (C) $\to$ (p, r); (D) $\to$ (q, r).

Passage based Questions :

1. (a) A mechanical wave in air can never be transverse.

2. (c) In solids, mechanical waves can be transverse as well as longitudinal.

3. (b) Due to surface tension, free surface of liquids tends to have minimum surface area.

Assertion & Reason :

1. (d) **2.** (a) **3.** (b) **4.** (a)

Integer/Numeric type Questions :

1. 75 m

2. 425 m.

3. 0.8 m

4. (i) 10 Hz (ii) λ = 30 cm (iii) 3 m/s

5. 1.13 m

4 ADVANCED EXERCISE
BASED ON CONNECTING TOPICS

1. (a) The wave equation is $y = A \sin(\omega t)\cos(kx)$;
$c = \omega/k = 100/0.01 = 10^4$ m/s.

2. (b) For production of beats different frequencies are essential. The different amplitudes affect the minimum and maximum amplitude of the beats and different phases affect the time of occurrence of minimum and maximum.

3. (d) Third overtone has a frequency $7n$, which means $L = \frac{7\lambda}{4}$ = three full loops + one half loop, which would make four nodes and four antinodes.

4. (c) As fixed end is a node, therefore, distance between two consecutive nodes $= \frac{\lambda}{2} = 10\,\text{cm}$

λ = 20 cm = 0.2 m

As $v = n\lambda$ $\therefore$ $v = 100 \times 0.2 = 20$ m/s

5. (c) The waves, whose frequencies, phases and amplitudes are same at a given time or at a given place in space are known as coherent waves.

6. (b) As intensity of wave $\propto$ (amplitude)^2

$$\frac{I_1}{I_2} = \frac{9}{1} = \frac{a_1^2}{a_2^2} \Rightarrow \frac{a_1}{a_2} = \frac{3}{1}$$

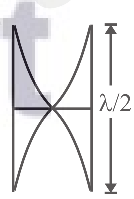
$$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)} = \frac{16}{4} \Rightarrow \text{ratio is } 4:1.$$

7. (d) Velocity of sound $\propto \frac{1}{\sqrt{\text{Density of gas}}}$

$$\therefore \frac{v_0}{v_H} = \sqrt{\frac{\rho_H}{\rho_0}} = \sqrt{\frac{1}{16}} = \frac{1}{4} \Rightarrow \frac{v_H}{v_0} = \frac{4}{1}$$

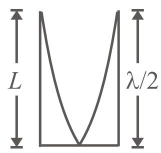
8. (c) The fundamental frequency of an organ pipe open at both ends is

$$v_0 = \frac{v}{2L} \quad \text{....(i)}$$



The fundamental frequency of an organ pipe closed at one end is

$$v_c = \frac{v}{4L} \quad \text{....(ii)}$$



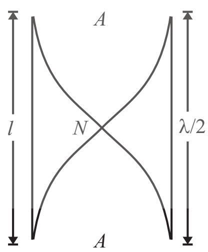
Dividing equation (i) by (ii)

$$\frac{v_0}{v_c} = \frac{v}{2L} \times \frac{4L}{v} = \frac{2}{1}$$

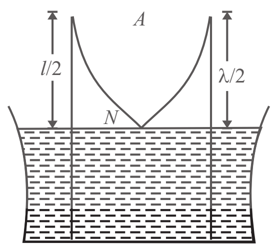
9. (b) In fundamental mode,

$$\frac{\lambda}{2} = l \Rightarrow \lambda = 2l$$

$$\therefore f = \frac{v}{\lambda} = \frac{v}{2l} \quad \text{....... (1)}$$



Fundamental mode



Half length dipped in water

In half length dipped in water mode,

$$\frac{l}{2} = \frac{\lambda}{4} \Rightarrow \lambda = 2l \qquad \therefore f' = \frac{v}{\lambda} = \frac{v}{2l} = f$$

10. (d) Frequency of vibration in tight string

$n = \frac{p}{2l}\sqrt{\frac{T}{m}} \Rightarrow n \propto \sqrt{T}$

$\Rightarrow \frac{\Delta n}{n} = \frac{\Delta T}{2T} = \frac{1}{2}\times(4\%) = 2\%$

$\Rightarrow$ Number of beats = $\Delta n = \frac{2}{100}\times n = \frac{2}{100}\times 100 = 2$

11. (b) For observer, note of *B* will not change due to zero relative motion.
Observed frequency of sound produced by *A*

$= 660\frac{(330-30)}{330} = 600\,\text{Hz}$

$\therefore$ No. of beats = 600 – 596 = 4

12. (a) $\lambda = \frac{v}{n} = \frac{340}{170} = 2m,\ n' = \frac{340}{340-17}\times 170$

n' = 178.9 Hz

Now $\lambda' = \frac{v}{n'} = \frac{340}{178.9} = 1.9$

$\Rightarrow \lambda - \lambda' = 2 - 1.9 = 0.1$

13. (a, b, d)
Apparent frequency of sound is different from actual frequency if there is relative motion between the source of sound and the observer.

14. (a, b, c, d)

15. (a, c)
Oscillatory motions are to and fro about a mean position.

16. (a, c, d) **17.** (b, c, d)

18. (a, b, c, d)
Total energy of a simple harmonic oscillator is given by E = ½ $m\omega A^2$

19. (a, b, c, d)

20. (a) (A) →(s); (B) → (r); (C) → (q); (D) → (p)

21. (d) (A) → (q, r); (B) → (p, s); (C) → (t); (D) → (p, s).

22. (b) Kinetic energy of an oscillating body is zero at extreme position and maximum at mean position.

23. (a) Total energy of an oscillating body is constant. It keeps on changing between kinetic and potential energies.

24. (a)

25. (b) **26.** (b)

27. (c) At mean position, net force acting on the particle is zero but *P.E.* can be non-zero but is minimum.

28. For S.H.M., $x = A\sin\left(\frac{2\pi}{T}t\right)$

When x = A, $A = A\sin\left(\frac{2\pi}{T}.t\right)$

$\therefore \sin\left(\frac{2\pi}{T}.t\right) = 1 = \sin\left(\frac{\pi}{2}\right)$

or t = (T/4)

where $x = \frac{A}{2}, \frac{A}{2} = A\sin\left(\frac{2\pi}{T}.t\right)$

or $\sin\frac{\pi}{6} = \sin\left(\frac{2\pi}{T}t\right)$

or $t = (T/12)$

Now, time taken to travel from $x = A$ to $x = A/2$ is equal to $T/4 - T/12$ = T/6

29. $f = \frac{1}{2\pi}\sqrt{\left(\frac{k}{m}\right)}$

Let l be the length of spring and Δl be the elongation when loaded with mg.

Then $k = \frac{mg}{\Delta l/2} = 2k$

$\therefore f' = \frac{1}{2\pi}\sqrt{\left(\frac{k'}{m}\right)} = \frac{1}{2\pi}\sqrt{\left(\frac{2k}{m}\right)}$

Now $\frac{f}{f'} = \sqrt{\left(\frac{k}{2k}\right)} = \frac{1}{\sqrt{2}}$

30. Time to complete 1/4th oscillation is $\frac{T}{4}$ s.

Time to complete $\frac{1}{8}$ th vibration from extreme position is obtained from

$y = \frac{a}{2} = a\cos\omega t = a\cos\frac{2\pi}{T}t \text{ or } t = \frac{T}{6}s$

So time to complete 3/8th oscillation

$= \frac{T}{4} + \frac{T}{6} = \frac{5T}{12}$

31. $y = a\sin(\omega t + \phi)$; when $y = a/2$, then $\frac{a}{2} = a\sin(\omega t + \phi)$

or $\sin(\omega t + \phi) = \frac{1}{2} = \sin\frac{\pi}{6}$ or $\sin\frac{5\pi}{6}$

So phase of two particles is –π/6 and 5 π/6 radians.
Hence phase difference = (5 π/6) – π/6 = 2 π/3

32. $v_{max} = 100 = a\omega$; $\omega = 100/a = 100/10 = 10$ rad/s

$v^2 = \omega^2(a^2 - y^2)$ or 50^2

$= 10^2(10^2 - y^2)$ or $25 = 100 - y^2$

or $y = \sqrt{75} = 5\sqrt{3}$ cm.

Chapter

6

Units and Measurements

INTRODUCTION

The process of comparing an unknown physical quantity with respect to a known quantity is known as measurement. When we say that the length of our bedroom is 10 feet it implies that the bedroom is 10 times the known quantity 'foot' (feet is the plural of foot). So, measurement of any physical quantity consists of two parts – (i) a numerical value and (ii) the known quantity. The known quantity is called the unit of that physical quantity.

Physical quantities (anything which can be measured) are of two types-fundamental e.g., mass, length, time, temperature, electric current etc. and derived e.g., velocity, acceleration, force etc. A derived quantity can be expressed in terms of fundamental quantities. A derived quantity expressed in terms of powers of fundamental quantities is said to have 'dimensions'.

Measurement of a physical quantity has in general, inaccuracy or errors. In this chapter we will learn measurement, units, dimensions and errors in measurement.

PHYSICAL QUANTITIES

Quantities which can be measured are called physical quantities. Velocity, acceleration, force, area, volume, pressure, etc. are some examples of physical quantities.

Kinds of Physical Quantities

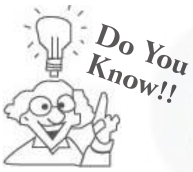
There are two kinds of physical quantities

(i) **Fundamental physical quantities :** Fundamental physical quantities are those which do not depend on other quantities and also independent of each other. They are seven in number *viz*; length, mass, time, thermodynamic temperature, electric current, luminous intensity and amount of substance.

(ii) **Derived physical quantities :** Derived physical quantities are those which are derived from fundamental physical quantities. For example, velocity is derived from the fundamental quantities length and time, hence it is a derived physical quantity.

UNITS

To measure a physical quantity it is compared with a standard quantity. This standard quantity is called the unit of that quantity.
For example, to measure the length of a desk, it is compared with the standard quantity known as 'metre'. Thus, 'metre' is said to be the unit of length.
The starting point in any study of measurement is an understanding of the need for universal standardisation of the units. In early times, man used body parts as a standard of measure. With progress, these rough inaccurate measurements were discarded for accurate systems of measurement. Also, as world trade increased, there developed a need for universal standards of units of measurement with opportunity for all countries to adopt such established standards.



The unit named to commemorate a scientist is not written with capital initial letter. For example, the unit of force is written as newton (and not as Newton), the unit of current is written as ampere (and not as Ampere) etc.
The symbols for units named after scientists are usually the first initial letter of their names in capital. For example, N for newton, A for ampere, J for joule etc.

Types of Units

There are two types of units :

(i) Fundamental units and (ii) Derived units

(i) **Fundamental units:** Fundamental units are those units which cannot be derived from any other unit, and they cannot be resolved into any basic or fundamental unit. Example : Length, mass, time, temperature, luminous intensity, electric current and amount of substance.

(ii) **Derived units:** Any unit which can be obtained by the combination of one or more fundamental units are called derived unit. Example : Area, speed, density, volume, momentum, acceleration, force etc.

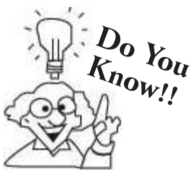
Systems of Units

Depending upon the units of fundamental physical quantities, there are four main systems of units, namely

1. **CGS** (Centimeter, Gramme or Gram, Second)
2. **FPS** (Foot, Pound, Second)
3. **MKS** (Meter, Kilogram, Second)
4. **SI** (Systeme Internationale d′ Unites)

The first three of these systems recognize only three fundamental quantities i.e. length (L), mass (M) and time (T) while the last one recognizes seven fundamental quantities. i.e. length (L), mass (M), time (T), electric current (I or A), thermodynamic temperature (K or θ), amount of substance (mol) and luminous intensity (I_v).

An international organization, the **Conference Generale des Poids et Measures, or CGPM** is internationally recognized as the authority on the definition of units. In English, this body is known as *"General Conference on Weights and Measure"*. **The Systeme International de Unites, or SI system** of units, was set up in 1960 by the CGPM.



The units do not have plural forms. For example, it is wrong to write a force of 10 newton (or 10N) as 10 newtons (or 10Ns) or a mass of 10 kg as 10 kgs.
No full stop is put between the symbols for units. For example, it is wrong to write newton metre as N.m. One should write it as Nm.

Characteristics of a Standard Unit

A standard unit must have following features to be accepted world wide. It should

(i) have a convenient size.
(ii) be very well defined.
(iii) be *independent of time and place*.
(iv) be easily available so that all laboratories can duplicate and use it as per requirement.
(v) be *independent of physical conditions* like temperature, pressure, humidity etc.
(vi) be easily *reproducible*.
(vii) be universally accepted.

Advantages of SI Unit System

(i) *Coherent System :* All the derived units are obtainable directly from the basic units.
(ii) *Rational System :* Only one unit for one physical quantity.
(iii) *Metric System :* This makes calculations easier.

Disadvantages of SI Unit System

As it is a coherent system, all the derived units are not practical e.g., 1 coulomb (unit of electric charge), 1 farad (unit of electric capacitance), 1 bel (unit of loudness of sound) are too large units to be practical.

Fundamental or Base Units of SI System

The following table shows the seven fundamental units of S.I. System.

S. No.	Physical quantity	Unit of measurement	Symbol for unit
1.	Length	metre	m
2.	Mass	kilogram	kg
3.	Time	second	s
4.	Electric current	ampere	A
5.	Temperature	kelvin	K
6.	Luminous intensity	candela	cd
7.	Amount of substance	mole	mol

Supplementary Units of SI System

The following table shows the two supplementary units of SI. System.

S.No.	Physical quantity	Unit of measurement	Symbol for unit
1.	Plane angle	radian	rad
2.	Solid angle	steradian	sr

Derived Units

The units of derived quantities which depend on above fundamental units for their measurements are called derived units. E.g., unit of acceleration -m/s^2, of momentum - kg m/s etc.

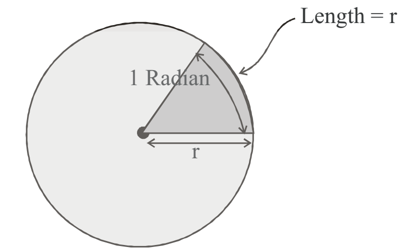
Definitions of the 7 SI Base Units and 2 Supplementary Units

1. **meter (m) (17th CGPM, 1983):** The meter is the length of the path travelled by light in vacuum during a time interval of 1/299,792,458 of a second.
2. **kilogram (kg) (3rd CGPM, 1901):** The kilogram is the unit of mass. It is equal to the mass of the international prototype of the kilogram. The international prototype of the kilogram is kept at the International Bureau for Weights and Measures (BIPM), at Severe, near Paris, France, and is made of a platinum (90%) and iridium (10%) alloy. It has a density of approximately 21,500 kg/m^3 and is shaped as a cylinder, with height and diameter equal to 39 mm.
3. **second(s) (13th CGPM, 1967):** The second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom.
   It is also 1/86,400 of a mean solar day. (There are 60 seconds in a minute, 60 minutes in an hour, and 24 hours in a day;
   1 day = 60 × 60 × 24 = 86,400 seconds)
4. **ampere (A) (9th CGPM, 1948):** The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to 2×10^{-27} newton per meter of length.
5. **kelvin (K) (13th CGPM, 1967):** The kelvin, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.

6. **Candela (Cd) :** The candela is the luminous intensity, in a given direction of a source that emits monochromatic radiation of frequency 540 × 10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian.
7. **Mole (Mol):** The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon – 12.

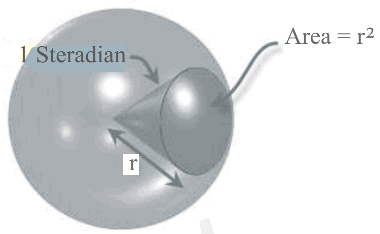
Supplementary Units

1. **Radian (rad):** The radian is the plane angle between two radii of a circle that cut off on the circumference an arc equal in length to the radius.



2. **Steradian (sr):** The steradian is the solid angle that, having its vertex at the center of a sphere, cuts off an area of the surface of the sphere equal to that of a square with sides of length equal to the radius of the sphere.

   The solid angle is very similar to the plane angle with the only difference being that it is three-dimensional rather than just two-dimensional. It can be described as the ratio of the area of the surface of the cone over the radius of the sphere.
   Imagine a sphere with a radius of 1 metre. A cone that covers an area of $1m^2$ on the surface of the sphere encloses a solid angle of 1 stearadian. A full sphere has a solid angle of 4π steradian.



PREFIXES FOR SI UNITS

In Physics we have to deal from very small (micro) to very large (macro) magnitudes. To express such large and small magnitudes simultaneously we use following prefixes:

Prefixes for powers of ten:

Multiple of 10	Prefix	Symbol	Sub -multiple	Prefix	Symbol
10^{24}	yotta	Y	10^{-1}	deci	d
10^{21}	zetta	Z	10^{-2}	centi	c
10^{18}	exa	E	10^{-3}	milli	m
10^{15}	peta	P	10^{-6}	micro	μ
10^{12}	tera	T	10^{-9}	nano	n
10^{9}	giga	G	10^{-12}	pico	p
10^{6}	mega	M	10^{-15}	femto	f
10^{3}	kilo	k	10^{-18}	atto	a
10^{2}	hecto	h	10^{-21}	zepto	z
10	deca	da	10^{-24}	yocto	y

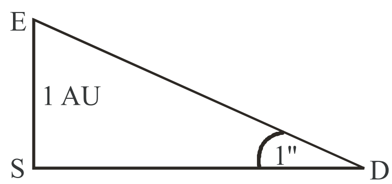
When a prefix is placed before the symbol of unit, the combined prefix and symbol should be considered as one new symbol which can be raised to a positive or negative power without any bracket, e.g., km^3 means $(10^3\ m)^3$ but never $10^3\ m^3$.

Practical Units of Length

Astronomical unit, AU: The average distance between the sun and the earth about 149 598 000 km is called 1 AU.

Parsec: The method of parallax gives rise to a natural distance unit that astronomers call the parsec. The parsec is defined to be the distance at which a star would have a parallax angle equal to one second of arc.

1 Parsec = 3.08568025 × 1016 m.



Light Year : The light year is the distance travelled by light in one year. All electromagnetic waves travel at a speed of 299,792,458 ms^{-1} and –an average year being 365.25 days.

One light year is 299,792,458 × $10^8 ms^{-1}$ × (365.25 × 24 × 60 × 60) s = 9.46073 × 10^{15} m. or 9.46073 × 10^{12} km.

Angstrom: An angstrom is a unit of length used to measure small lengths such as the wavelengths of light, atoms and molecules. One angstrom ,1 Å =10^{-10}m.

Fermi: A unit of length used to measure nuclear distance = 10^{-15} meter, 1 fermi = 10^{-15}m.

SI DERIVED UNITS WITH SPECIAL NAMES

Physical quantity	SI Unit			
	Name	**Symbol**	**Expression in terms of other units**	**Expression in terms of SI base units**
Frequency	hertz	Hz		s^{-1}
Force	newton	N		kg m s^{-2} or kg m/s^2
Pressure, stress	pascal	Pa	N/m^2 or N m^{-2}	kg m^{-1} s^{-2} or kg / s^2m
Energy, work, quantity of heat	joule	J	Nm	kg m^2 s^{-2} or kg m^2/s^2
Power, radiant flux	watt	W	J/s or J s^{-1}	kg m^2 s^{-3} or kg m^2/s^3
Quantity of electricity, electric charge	coulomb	C		A s
Electric potential, potential difference, electromotive force	volt	V	W/A or W A^{-1}	kg m^2s^{-3} A^{-1} or kg m^2/s^3 A
Capacitance	farad	F	C / V	A^2 s^4 kg^{-1} m^{-2}
Electric resistance	ohm	Ω	V/A	kg m^2 s^{-3} A^{-2}
Conductance	siemen	S	A / V	m^{-2} kg^{-1} s^3 A^2
Magnetic flux	weber	Wb	V s or J / A	kg m^2 s^{-2} A^{-1}
Magnetic field, magnetic flux density, magnetic induction	tesla	T	Wb / m^2	kg s^{-2} A^{-1}
Inductance	henry	H	Wb / A	kg m^2 s^{-2} A^{-2}
Luminous flux, luminous power	lumen	lm		cd/sr
Illuminance	lux	lx	lm / m^2	m^{-2} cd sr^{-1}

Some Important Conversions

(i)	1 yard	=	0.9144 m ≅ 0.91 m
(ii)	1 foot (1′)	=	0.305 m
(iii)	1 inch (1″)	=	2.54 cm = 0.025 m
(iv)	1 mile	=	1609 m = 1.609 km
(v)	1 ltr.	=	1000 cc = 10^{-3} m^3
(vi)	1 cm^2	=	10^{-4} m^2
(vii)	1 mm	=	10^{-3} m
(viii)	1 atomic mass unit		1 (amu) = 1.67 × 10^{-27} kg
(ix)	1 slug	=	14.57 kg
(x)	1 tonne	=	10 quintal = 1000 kg
(xi)	1 kg/m^3	=	1000 g/cm^3

(xii)	1 km/h	=	$\frac{5}{18}$ m/s and 1 m/s = $\frac{18}{5}$ km/h
(xiii)	1 newton	=	10^5 dyne, 1 kg wt = 9.8 N and 1 g wt = 981 dyne
(xiv)	1 joule	=	10^7 erg, 1eV = 1.6×10^{-19} J
(xv)	1 atm	=	76 cm of Hg = $1.01 \times 10^5 \frac{N}{m^2} = 1.01 \times 10^6 \frac{dy}{cm^2}$
(xvi)	1 h.p.	=	746 watt
(xvii)	1 kw h	=	3.6×10^6 J
(xviii)	1 tesla	=	1 web/m^2 = 10^4 gauss
(ixx)	1 curie	=	3.7×10^{10} disintegration/sec
	1 rutherford	=	10^6 disintegration/sec
(xx)	1 weber	=	10^8 maxwell
(xxi)	1 degree	=	$\frac{\pi}{180}$ radian and 1 radian = $\frac{180}{\pi}$ degree
(xxii)	1 shake	=	10^{-8} sec

Rules for Writing Units and Their Symbols

(i) Units named after scientists are not written with initial letter in capital, e.g. it is incorrect to write 10 Newton, the correct way is 10 newton.
(ii) The symbols of the units named after scientists must be written in capital letter for example, 10 N.
(iii) If the symbols of unit is not from a name then small letters must be used.
(iv) For derived units index notation must be used e.g. 20 ms^{-2} in place of 20 m/s^2.
(v) Units and symbols should not be written in plural form e.g., write 25 centimeter in place of 25 centimeters.

DIMENSIONS OF A PHYSICAL QUANTITY

All physical quantities can be expressed in terms of the fundamental quantities. Consider the physical quantity force.

$$\text{Force} = \text{mass} \times \text{acceleration} = \text{mass} \times \frac{\text{velocity}}{\text{time}}$$

$$= \text{mass} \times \frac{\text{length / time}}{\text{time}} = \text{mass} \times \text{length} \times \text{time}^{-2}$$

$\therefore$ Unit of force = unit of mass × unit of length × (unit of time)$^{-2}$

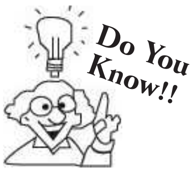
Thus we can express the unit of force as products of different powers of the fundamental units of mass, length and time.

i.e., Force = [MLT^{-2}]

Thus the dimensions of a physical quantity are the powers to which the fundamental quantities mass, length and time must be raised to represent it.

An expression for a physical quantity in terms of fundamental quantities is known as **dimensional formula.**

So the dimensional formula for force is $[MLT^{-2}]$ and that of volume will be $[M^0L^3T^0]$. While writing dimensional formula we will use symbols, *M* for mass, *L* for length, *T* for time, *A* for current, *K* for temperature, *mol* for amount of substance and *cd* for luminous intensity.



A physical quantity may be dimensionless but still may have units. For example, plane angle is dimensionless but has radian as its unit.

A physical quantity that does not have any unit must be dimensionless.

ILLUSTRATION : 1

Derive the dimensional formulae for the following quantities : (i) Pressure (ii) Torque (iii) Angular momentum

SOLUTION: (i)

$$\text{Pressure} = \frac{F}{A}\left(\begin{array}{l} F = ma = m\dfrac{v}{t} \\ A = l \times b \end{array}\right)$$

$$[P] = \frac{[F]}{[A]} = \frac{[MLT^{-2}]}{[L^2]} = [M\,L^{-1}\,T^{-2}]$$

(ii)

$$\text{Torque} = \text{force} \times \text{moment arm}$$

$$[\tau] = [F]\,[l] = [MLT^{-2}]\,[L]$$

$$[\tau] = [ML^2\,T^{-2}]$$

(iii)

$$\text{Angular momentum } (L) = mvr\ [M]\,[v]\,[r] = [M]\,[LT^{-1}]\,[L]$$

$$[L] = [ML^2\,T^{-1}]$$

Dimensions of Some Physical Quantities

The dimensional formulae of various derived quantities can be obtained from their relations with other quantities. In the following table dimensional formulae of various physical quantities are given.

S.No.	Physical quantity	Relation with other quantities	Unit	Dimensions
1.	Length, displacement, distance		m	$[M^0\,L\,T^0]$
2.	Area	length (l) × breadth (b)	m^2	$[M^0\,L^2\,T^0]$
3.	Volume	$V = l \times b \times h$ (height)	m^3	$[M\,L^3\,T^0]$
4.	Mass		kg	$[M\,L^0\,T^0]$
5.	Mass density	$d = \frac{M}{V}$	kg m^{-3}	$[M\,L^{-3}\,T^0]$
6.	Linear mass density	$m = \frac{M}{l}$	kg m^{-1}	$[M\,L^{-1}\,T^0]$
7.	Relative density	$\frac{\text{density of a solid}}{\text{density of water at 4°C}}$	Unitless, dimensionless	$[M^0\,L^0\,T^0]$
8.	Specific gravity	$\frac{\text{density of a liquid}}{\text{density of water at 4°C}}$	Unitless, dimensionless	$[M^0\,L^0\,T^0]$
9.	Speed	$\frac{\text{distance}}{\text{time}}$	ms^{-1}	$[M^0\,L\,T^{-1}]$
10.	Velocity	$\frac{\text{displacement}}{\text{time}}$	ms^{-1}	$[M^0L\,T^{-1}]$
11.	Acceleration	$\frac{\Delta v}{\Delta t}$	ms^{-2}	$[M^0L\,T^{-2}]$
12.	Force	$F = ma$	kg ms^{-2} = N (newton)	$[MLT^{-2}]$
13.	Coefficient of friction	$\mu = \frac{f}{N}$	Unitless, dimensionless	$[M^0L^0T^0]$

14.	Work	$W = F.s$	kg m^{2}s^{-2} = J (joule)	$[ML^2T^{-2}]$
15.	Kinetic energy	$K = \frac{1}{2}mv^2$	kg m^{2}s^{-2} = J	$[M L^2 T^{-2}]$
16.	All forms of energy	—	kg m^{2}s^{-2} = J	$[M L^2 T^{-2}]$
17.	Torque	$\tau = F \times r$	kg m^{2}s^{-2} = J	$[ML^2T^{-2}]$
18.	Linear momentum	$P = mv$	kg ms^{-1}	$[MLT^{-1}]$
19.	Linear impulse	$J = F.\Delta t$	kg ms^{-1}	$[MLT^{-1}]$
20.	Power	$P = \frac{W}{t}$	kg m^{2}s^{-3} = W (watt)	$[ML^2T^{-3}]$
21.	Pressure	$p = \frac{F}{A}$	kg m^{2}s^{-3} = $\frac{N}{m^2}$	$[ML^2T^{-3}]$
22.	Stress	$\sigma = \frac{F}{A}$	kg m^{-1}s^{-2} = $\frac{N}{m^2}$ or pascal	$[ML^{-1}T^{-2}]$
23.	Young's modulus	$Y = \frac{\sigma}{\in} = \frac{stress}{strain}$	kg m^{-1}s^{-2} = $\frac{N}{m^2}$ or pascal	$[ML^{-1}T^{-2}]$
24.	Shear modulus	$G = \frac{F/A}{\theta}$	kg m^{-1}s^{-2} = $\frac{N}{m^2}$ or pascal	$[ML^{-1}T^{-2}]$
25.	Bulk modulus	$B = \frac{P}{\frac{-\Delta v}{v}}$	kg m^{-1}s^{-2} = $\frac{N}{m^2}$ or pascal	$[ML^{-1}T^{-2}]$
26.	Strain	$\in = \frac{\Delta l}{l}$	Unitless, dimensionless	$[M^0L^0T^0]$
27.	Universal constant of gravitation	$G = \frac{Fr^2}{m_1m_2}$	Nm^{2}kg^{-2}	$[M^{-1}L^3T^{-2}]$
28.	Poisson ratio	$\frac{\text{lateral strain}}{\text{longitudinal strain}}$	Unitless, dimensionless	$[M^0L^0T^0]$
29.	Surface tension	$S = \frac{F}{l}$	kg s^{-2} = $\frac{\text{N}}{\text{m}}$	$[ML^0T^{-2}]$
30.	Frequency	$\frac{1}{T}$	s^{-1}	$[M^0L^0T^{-1}]$
31.	Angular velocity	$\omega = \frac{\Delta\theta}{\Delta t}$	s^{-1}	$[M^0L^0T^{-1}]$
32.	Radius of gyration	length	m	$[M^0LT^0]$
33.	Moment of inertia	$I = mr^2$	kg m^2	$[ML^2T^0]$
34.	Angular momentum	$L = I\omega$	kg m^2 s^{-1}	$[ML^2T^{-1}]$
35.	Rotational kinetic energy	$\frac{1}{2}I\omega^2$	J	$[ML^2T^{-2}]$

36.	Wavelength	$\lambda = \frac{v}{f}$	m	$[M^0LT^0]$
37.	Coefficient of visosity	$\eta = \frac{F}{6\pi r v}$	$Nm^{-2}s$ = Deca Poise	$[ML^{-1}T^{-1}]$
38.	Reynold's number	$R_N = \frac{\text{inertial force}}{\text{viscous force}}$	Unitless dimensionless	$[M^0L^0T^0]$
39.	Temperature		K (kelvin)	$[M^0L^0T^0K]$
40.	Heat energy (Q)		J	$[ML^2T^{-2}]$
41.	Specific heat	$S = \frac{Q}{m\Delta\theta}$	$m^2s^{-2}k^{-1} = \frac{J}{kgK}$	$[M^0L^2T^{-2}K^{-1}]$
42.	Heat capacity	$C = ms = \frac{Q}{\Delta\theta}$	$kgm^2\ s^{-1}\ K^{-1} = JK^{-1}$	$[ML^2T^{-2}K^{-1}]$
43.	Latent heat of fusion or vaporisation	$L = \frac{Q}{m}$	$m^2s^{-2} = J\ kg^{-1}$	$[M^0L^2T^{-2}]$
44.	Gas constant	$R = \frac{PV}{nT}$	$kg\ m^2s^{-2}K^{-1} = JK^{-1}$	$[ML^2T^{-2}K^{-1}]$
45.	Boltzmann constant	$K = \frac{R}{N_A}$	$kg\ m^2s^{-2}K^{-1} = JK^{-1}$	$[ML^2T^{-2}K^{-1}]$
46.	Stefan's constant	$\sigma = \frac{P}{AT^4}$	$kg\ s^{-3}K^{-4} = W\ m^{-2}K^4$	$[ML^0T^{-3}K^{-4}]$
47.	Power of a lens	$P = \frac{1}{f}$	m^{-1}	$[M^0L^{-1}T^0]$
48.	Planck constant	$h = \frac{E}{v}$	$kgm^2s^{-1} = Js$	$[ML^{-2}T^{-1}]$
49.	Electric current		A (ampere)	$[M^0L^0T^0\ A]$
50.	Electric charge	$q = It$	As = C (coulomb)	$[M^0L^0TA]$
51.	Electric potential	$V = \frac{W}{q}$	JC^{-1} = volt	$[ML^2T^{-3}\ A^{-1}]$
52.	Absolute permittivity	$\epsilon_0 = \frac{q_1q_2}{4\pi r^2 F}$	$kg^{-1}\ m^{-3}\ s^4A^2 = C^2\ N^{-1}\ m^2$	$[M^{-1}L^{-3}T^4\ A^2]$
53.	Electric field	$E = \frac{V}{l}$ or $\frac{F}{q}$	$kg\ ms^{-3}\ A^{-1} = Vm^{-1} = NC^{-1}\ m^2$	$[M^{-1}LT^{-3}\ A^{-1}]$
54.	Electric dipole moment	$P = q \times 2l$	msA = Cm	$[M^0LTA]$
55.	Electrostatic potential energy	$U = \frac{q_1q_2}{4\pi \epsilon_0\ r}$	$kg^{-1}\ m^2\ s^{-2} = J$	$[ML^2T^{-2}]$
56.	Electric resistance	$R = \frac{V}{I}$	$kg\ m^2\ s^{-3}\ A^{-2}$ = ohm	$[ML^2T^{-3}\ A^{-2}]$

57.	Resistivity	$\rho = \frac{RA}{l}$	kg m^3 $sA^{-2} = \Omega - m$	$[ML^3T^{-3}A^{-2}]$
58.	Capacitance	$C = \frac{q}{V}$	kg^{-1} $m^{-2}s^4A^2$ = farad	$[M^{-1}L^{-2}T^4A^2]$
59.	Magnetic field strength/induction	$B = \frac{F}{qv}$	kg s^{-2} A^{-1} = tesla	$[M^{-1}L^0T^{-2}A^{-1}]$
60.	Magnetic flux	$\phi = BA$	kg m^2 s^{-2} A^{-1}= t – m^2 = weber	$[ML^2T^{-2}A^{-1}]$
61.	Coefficient of self-inductance	$L = \frac{\phi}{I}$	kg m^2 s^{-2} A^{-2} = (H) henry	$[ML^2T^{-2}A^{-2}]$

CHECK Point

- **The division of energy and time is X. Among momentum, power, torque and electric field, which has the same dimensional formula as that of X ?**

Solution

Power is defined as the ratio of work (or energy) to time. Therefore, X has the dimensional formula as that of power.

Uses of Dimensional Analysis

The following are the uses of dimensional analysis.

(i) To convert a unit from one system to other

Dimensions are quite useful for finding the conversion factor for the unit of a physical quantity from one system to another.

We know that $n_1 u_1 = n_2 u_2$

where n_1, n_2 are numerical values and u_1 and u_2 are their respective units. Suppose dimensions of a physical quantity are *a, b, c* in mass, length and time respectively then

$u_1 = [M_1]^a[L_1]^b[T_1]^c$ and $u_2 = [M_2]^a[L_2]^b[T_2]^c$

$$\text{So, } n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

ILLUSTRATION : 2

Convert 1 pascal (N/m^2) into c.g.s units.

SOLUTION :

Pascal is unit of pressure whose dimensional formula is $[ML^{-1}T^{-2}]$

So, $a = 1, b = -1, c = -2$

$n_1 = 1$ Pascal

$n_2 = ?$

Using the formula

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c = \left[\frac{1\,\text{kg}}{1\,\text{g}}\right]^1\left[\frac{1\,\text{m}}{1\,\text{cm}}\right]^{-1}\left[\frac{1\,\text{s}}{1\,\text{s}}\right]^{-2}$$

$$n_2 = \left[\frac{1000\,\text{g}}{1\,\text{g}}\right]\left[\frac{100\,\text{cm}}{1\,\text{cm}}\right]^{-1} \times 1 = 10$$

Therefore, 1 Pa = 10 cgs pressure

ILLUSTRATION : 3

What will be the value of 100 newton in a new system which has 10 g, 1000 cm and 1 minute as fundamental units?

SOLUTION :

Dimensional formula for force is $[M\,L\,T^{-2}]$

So, $a = 1, b = 1, c = -2$

Using $n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$

$$n_2 = 100\left[\frac{1\text{ kg}}{10\text{ g}}\right]\left[\frac{1\text{ m}}{1000\text{ cm}}\right]\left[\frac{1\text{ s}}{1\text{ min}}\right]^{-2}$$

$$n_2 = 100\left[\frac{1000\text{ g}}{10\text{ g}}\right]\left[\frac{100\text{ cm}}{1000\text{ cm}}\right]\left[\frac{\frac{1}{60}\text{ min}}{1\text{ min}}\right]^{-2}$$

$$n_2 = 100\times100\times\frac{1}{10}\times\frac{1}{3600} = 0.27 \text{ in new units.}$$

(ii) To check the correctness of a physical equation

According to the **principle of homogeneity of dimensions**, *only those physical quantities can be added or subtracted which have the same dimensions*. In other words an equation which is containing several terms separated from each other by the equality, plus or minus must be of the same dimensions. An equation will be dimensionally correct only and only if all the terms have the same dimensions. For example in the equation

$$v^2 = u^2 + 2as$$

$$[v^2] = L^2T^{-2}$$

$$[u^2] = L^2T^{-2}$$

$$[2as] = LT^{-2}.\,L = L^2T^{-2}$$

Thus the equation $v^2 = u^2 + 2as$ is dimensionally correct as all the terms are having the same dimensions.

ILLUSTRATION : 4

Verify dimensional accuracy of the formula $Q = \frac{\pi}{8}\frac{Pr^4}{\eta l}$ where Q is volume flow rate, P is pressure, r is radius, η is coefficient of viscosity and l is length.

SOLUTION :

$$\text{LHS } [Q] = \frac{\text{volume}}{\text{time}} = M^0L^3T^{-1}$$

$$\text{RHS } \left[\frac{\pi}{8}\frac{Pr^4}{\eta l}\right] = \frac{\text{pressure}\times(\text{radius})^4}{(\text{coefficient of viscosity})\times(\text{length})}$$

$$= \frac{ML^{-1}T^{-2}L^4}{ML^{-1}T^{-1}L} = M^0L^3T^{-1}$$

As dimensions of both sides of equation are the same, so the formula is dimensionally correct.

(iii) To establish relation among different physical quantities

If the dependence of a physical quantity on other physical quantities is of product type then using dimensional formula for the given physical quantity can be deduced. This method is also based on the principle of homogeneity of dimensions. For example a mass attached with a spring oscillates in the vertical direction, we assume that the time period of oscillation (t) depends upon spring constant (k), gravity (g) and mass (m), then

$$t = \lambda\,(k)^a(g)^b(m)^c$$

where λ is a dimensionless constant and a, b, c are to be evaluated, taking dimensions

$$T = [MT^{-2}]^a[LT^{-2}]^b[M]^c$$

$$M^0L^0T = M^{a+c}.L^b.\ T^{-2a-2b}$$

On comparison $0 = a + c$

$b = 0$

$1 = -2a - 2b$

$\Rightarrow\ a = -½\ \ c = ½$

Hence, time period $t = \lambda\sqrt{\dfrac{m}{k}}$

Thus, by dimensional analysis we have deduced that time period of a spring-mass system is independent of gravity, directly proportional to square root of mass m and is inversely proportional to square root of spring constant k.

ILLUSTRATION : 5

Viscous force (F) on a body depends upon coefficient of viscosity (h) of the medium, velocity (v) of the body and the radius of the spherical body (r), deduce the formula using dimensions.

SOLUTION :

Viscous force, $F = \lambda[\eta]^a[v]^b[r]^c$, λ is dimensionless constant

Taking dimensions on both the sides

$$[MLT^{-2}] = [ML^{-1}T^{-1}]^a[LT^{-1}]^b[L]^c$$

$$[MLT^{-2}] = M^aL^{-a+b+c}\,T^{-a-b}$$

On comparison $a = 1, -a + b + c = 1, -a - b = 2$

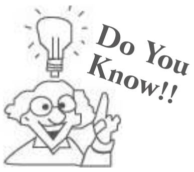
$\Rightarrow$ $b = 1, c = 1$

So, $F = \lambda\eta vr$

The value of λ can be found experimentally, it comes out to be $6\,\pi$.

Limitations of Dimensional Analysis

(i) For deriving a formula, we should know the quantities on which a particular quantity depends.
(ii) The method works only if dependence of physical quantities is of product type only.
(iii) The method does not give any information about the dimensionless constant.
(iv) We cannot derive the formulae which contain exponential, logarithmic and trigonometrical functions.
(v) The method works only if there are as many equations available to us as there are unknowns. For example if in mechanics a physical quantity depends upon more than three other physical quantities then the method fails.



In composite relations, each term has the same dimensions. It is because, only like quantities can be added and subtracted from one another.

Types of Physical Quantities on the Basis of Dimensions

According to dimensions there are four types of physical quantities.

(a) Dimensional constants : Those quantities which have fixed value but possesses dimensions. For example (i) gravitational constant (ii) Planck's constant (iii) velocity of light etc.

(b) Dimensionless constants : Those quantities which have fixed value but does not have dimensions. For example (i) π (ii) e (iii) numerical constants, 1, 3, 6….etc.

(c) Dimensional variables : Those quantities which have dimensions but do not have fixed value. For example velocity, volume, force, *etc*.

(d) Dimensionless variables : Those quantities which do not have dimensions and fixed value. For example (i) strain (ii) relative density (iii) angle *etc*.

ERRORS IN MEASUREMENTS

Measurement is an important and essential part of our life. Generally measured value of a quantity is different from the true value of the physical quantity. *The difference between the true value and measured value is called error*. Before we discuss about errors let us understand two important terms :

(a) **Accuracy :** It is the measure of how close the measured value is to the true value of the physical quantity.

(b) **Precision :** It tells us about the limit or resolution upto which the quantity is measured.

Suppose certain body has true value of its mass as 7.298 kg. In experiment (a) we use an instrument of resolution 0.1 kg and measured value is 7.2 kg and in experiment (b) its measured value is 7.48 kg with a measuring instrument of resolution 0.01 kg. Then the experiment (a) is more accurate (as it is closer to the true value) where as experiment (b) has more precision because of higher resolution.

Types of Errors

(i) **Systematic errors :** Those errors which tend to be in one direction, either positive or negative, generally their cause is known. These errors can be minimised by improving experimental techniques selecting better equipments and removing personal bias. Some of the sources of systematic errors are:

(a) **Instrumental errors :** This type of error arises due to imperfect design or calibration of the measuring instrument, for example zero mark of varnier scale may not coincide with zero mark of main scale in a vernier callipers.

(b) **Imperfection in experimental procedure :** For example, measuring temperature of a human body by placing thermometer under armpit would give lower temperature than the actual body temperature, ignoring force of buoyancy during the measurement of weight of a body etc.

(c) **Personal error :** This type of error arise due to lack of proper setting of the apparatus, individual bias, or due to carelessness while taking observation. For example, if you hold your head too much towards while reading ammeter or voltmeter there will be some error due to parallax.

(d) **Errors due to external factors** like variation in temperature, humidity, pressure, wind etc. may introduce errors. For example wind may introduce error while taking the time period of a simple pendulum.

(ii) **Random errors :** These arise due to unpredictable and random variations in experimental conditions like temperature, voltage supply, personal (unbiased) error by observer etc. These errors are also called 'chance' errors as these occurs irregular and are random with respect to sign (negative or positive) and size.

Random errors can be minimised by taking the observation several times and taking the arithmetic mean of all the observations

(iii) **Least count errors :** *The error associated with the resolution of an instrument is called least count error*. The smallest division on the scale of a measuring instrument is called its **least count**. By using instrument of high precision and improving experimental technique we can minimise least count errors.

(iv) **Gross errors :** These arise entirely due to carelessness of the observer like reading an instrument without proper setting, recording observation incorrectly etc. This type of errors can be minimised if the observer is mentally alert and sincere.

Absolute, Mean Absolute, Relative and Percentage Error

(a) **Absolute error :** *The magnitude of the difference between the true value and the individual measured value is called absolute error of the measurement*.

Suppose individual values obtained in various observations are $a_1, a_2, a_3........a_n$

then the true or mean value is given by $A_{\text{true}} = a_{\text{mean}} = \dfrac{a_1 + a_2 + a_3 + ... + a_n}{n}$

And the absolute errors are :

$$\Delta a_1 = a_{\text{mean}} - a_1$$
$$\Delta a_2 = a_{\text{mean}} - a_2$$
$$.................$$
$$A_n = a_{\text{mean}} - a_n$$

Absolute errors can be negative or positive or zero also.

(b) **Mean absolute error :** It is the arithmetic mean of magnitudes of absolute errors in all measurements.

i.e., Mean absolute error, $\overline{\Delta a} = \Delta a_{\text{mean}} = \dfrac{|\Delta a_1| + |\Delta a_2| + ... + |\Delta a_n|}{n}$

Final measurement in the terms of mean absolute error is expressed as $(a_{\text{mean}} \pm \Delta a_{\text{mean}})$ or $a_{\text{mean}} + \overline{\Delta a}$

Remember the mean absolute error has the same unit as that of the measured quantity.

(c) Relative or fractional error : It is equal to *the ratio of mean absolute error to the mean (true) value of measured quantity*.

$$\text{Relative error} = \frac{\text{mean absolute error}}{\text{mean or true value}}$$

$$\text{Relative error} = \frac{\overline{\Delta a}}{a_{mean}} \text{ ; it is unitless}$$

(d) Percentage error : *If relative error is expressed in terms of percentage then it is called percentage error* (δa).

$$\text{Percentage error } (\delta a) = \text{relative error} \times 100 = \frac{\overline{\Delta a}}{a_{mean}} \times 100$$

Final measurement in terms of the percentage error will be expressed as $(a_{mean} \pm \delta a\%)$.

Combination of Errors (Maximum Permissible Errors)

(a) Error of a sum or a difference : When two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors in the individual quantities.
i.e. if $q = x \pm y$, then the maximum value of the error Δq is given by
$\Delta q = \Delta x + \Delta y$

(b) Error of a product or a quotient : When two or more quantities are multiplied or divided, the fractional error in the result is the sum of the fractional errors in the multipliers.
i.e. if $q = xy/z$, then the maximum permissible error in the measurement of q is given by

$$\frac{\Delta q}{q} = \frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta z}{z}$$

(c) Error due to the power of a measured quantity : The fractional error in a physical quantity raised to the power is the power times the fractional error in the individual quantity.
i.e. if $q = x^a y^b z^{-c}$ then the maximum permissible error in the measurement of q is given by

$$\frac{\Delta q}{q} = a\frac{\Delta x}{x} + b\frac{\Delta y}{y} + c\frac{\Delta z}{z}$$

Remember: The quotient rule is not applicable if the numerator and denominator are dependent on each other.

e.g., if $R = \frac{XY}{X+Y}$ we cannot apply quotient rule to find the error in R. Instead we write the equation as follows

$\frac{1}{R} = \frac{1}{X} + \frac{1}{Y}$. Differentiating both the sides, we get

$$-\frac{dR}{R^2} = \frac{dX}{X^2} - \frac{dY}{Y^2}$$

$$\text{Thus } \frac{r}{R^2} = \frac{x}{X^2} + \frac{y}{Y^2} \quad \text{or} \quad \frac{r}{R^2} = \frac{x}{X^2} + \frac{y}{Y^2}$$

ILLUSTRATION 6 :

Two resistances $R_1 = 10 \pm 1\Omega$ and and $R_2 = 20 \pm 3\Omega$ are connected in series, find equivalent resistance, express your answers in terms of mean absolute errors and percentage error.

SOLUTION :

In series combination of resistances, the resultant is given by

$$R_S = R_1 + R_2 = 10 + 20 = 30\Omega$$

And $\overline{\Delta R_S} = \Delta R_1 + \Delta R_2 = 1 + 3 = 4\Omega$

Resistance in terms of mean absolute error = $30 \pm 4\Omega$

$$\text{Percentage error } \delta a = \frac{\overline{\Delta R_S}}{R_S} = \frac{4}{30} \times 100 = 13.33\%$$

So, resistance in terms of % error = $30\Omega \pm 13.33\%$

ILLUSTRATION : 7

If $L = l_1 - l_2$ where $l_1 \pm \Delta l_1 = 20 \pm 0.2$ cm and $l_2 + \Delta l_2 = 10 \pm 0.5$ cm, find L and express it in terms of mean absolute and percentage error.

SOLUTION :

As, $L = L_1 - L_2 = 20 - 10 = 10$ cm

$$\Delta l = \Delta l_1 + \Delta l_2 = 0.2 + 0.5 = 0.7 \text{ cm}$$

So, the final length 10 cm ± 0.7 cm

Now, percentage error $= \frac{0.7}{10} \times 100 = 7\%$

So, final length in terms of percentage error = 10 cm ± 7%

ILLUSTRATION : 8

Length and breadth of a rectangle are given as $l = 100$ cm ± 2% and $b = 20$ cm ± 4%. Find area in terms of percentage and absolute error.

SOLUTION :

As in multiplication percentage errors are added so,

percentage error in area $\frac{\Delta A}{A} \times 100 = 2 + 4 = 6\%$

where $A = l\,b = 100 \times 20 = 2000 \text{ cm}^2$

Hence, area in terms of percentage error = 2000 cm^2 ± 6%

Now, $\frac{\Delta A}{2000} \times 100 = 6\%$

$$\Delta A = 120 \text{ cm}^2$$

Absolute error = 2000 cm^2 ± 120 cm^2

ILLUSTRATION : 9

Find mean absolute error and percentage error in R if $R = \frac{V}{I}$ where $V = 50 \pm 2$ volt and $I = 10 \pm 1$ ampere.

SOLUTION :

Resistance, $R = \frac{V}{I} = \frac{50}{10} = 5\Omega$

Using, $\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I} = \frac{2}{50} + \frac{1}{10}$

$\Rightarrow \quad \frac{\Delta R}{R} = \frac{2+5}{50}$

$\Delta R = 0.7\Delta$ (Mean absolute error)

Percentage error $= \frac{\Delta R}{R} \times 100 = \frac{0.7}{5} \times 100 = 14\%$

Note : Percentage error in R can also be found by adding percentage error in $V\left(\frac{2}{50} \times 100 = 4\%\right)$ and that in $I\left(\frac{1}{10} \times 100 = 10\%\right)$

ILLUSTRATION : 10

A physical quantity P is related to four observations a, b, c and d as follows : $P = \frac{a^3 b^2}{\sqrt{cd}}$

The percentage errors of measurement in a, b, c and d are 1%, 3%, 4% and 2% respectively. What is the percentage error in the quantity P ?

SOLUTION :

Given $P = \dfrac{a^3b^2}{\sqrt{cd}}$

So, $\dfrac{\Delta P}{P}\times 100 = 3\left(\dfrac{\Delta a}{3}\times 100\right)+2\left(\dfrac{\Delta b}{b}\times 100\right)\times\dfrac{1}{2}\left(\dfrac{\Delta c}{c}\times 100\right)+\dfrac{1}{2}\left(\dfrac{\Delta d}{d}\times 100\right)$

$\dfrac{\Delta P}{P}\times 100 = 3\times 1+2\times 3+\dfrac{1}{2}\times 4+\dfrac{1}{2}\times 2$

Hence, $\dfrac{\Delta P}{P}\times 100 = 13\%$

SIGNIFICANT FIGURES

Significants digits or figures give information about the accuracy of a measurement. It tells us about the number of digits in which we have confidence. Suppose a particular measurement is reported to be 9.28 cm, then the two digits 9 and 2 are reliable and certain while the digit 8 is uncertain. *The reliable and first uncertain digits are known as significant digits or figures*.

There are certain rules for counting significant digits or figure which are as follows :

Rule-1. All the non-zero digits are significant—For example 2134 has four significant digits and 27184 has five significant digits.

Rule-2. All the zeros between two non-zero digits are significant, no matter where the decimal point is, if at all. For example 25089 has five significant digits, 12.0021 has six significant digits.

Rule-3. In a number which is less than one all zeros to the right of decimal point and to the left of a non-zero digit are not significant.

Rule-4. All the zeros on the right of last non-zero digits are significant in a number with a decimal point. For example in 3,500 there are four significant digits and in 0.079000 there are five significant digits.

Rule-5. All the zeros on the right on a non-zero digit are not significant in a number without decimal point. For example 15800 has only three significant digits, 18930000 has only four significant digits.

Rule-6. All the zeros on the right on a non-zero digit are taken to be significant when these come from a measurement. For example some distance is measured to be 7890 m then this number would have four significant digits.

Rule-7. A change of system of units does not change the number of significant digits in a measurement. Also when a number is written in scientific notation ($a \times 10^b$) then the powers of 10 are irrelevant to the determination of significant digits.

ILLUSTRATION : 11

Write down the number of significant figures in the following :

(i) 6729 N (ii) 0.024 (iii) 6.0023 g cm^{-3} (iv) 2.520 × 10^7 m (v) 0.08240 N m^{-2} (vi) 4200 (vii) 91.000 m

SOLUTION :

(i) 6729 N has four significant figures.

(ii) 0.024 cm has two significant figures.

(iii) 6.0023g cm^{-3} has five significant figures.

(iv) 2.520 × 10^7 m has four significant figures.

(v) 0.08240 N m^{-2} has four significant figures.

(vi) 4200 has two significant figures.

(vii) 91.000 has five significant figures.

Rules for Arithmetic Operations with Significant Figures

(i) **Addition and subtraction** : The final result should retain as many decimal places as there are in the number with the least decimal places. For example if we add 1.269,26.57 and 9.1 the sum will be 36.939 but according to the rule the final result must retain only one decimal place i.e. 36.9.

(ii) **Multiplication and division** : In multiplication and division the final result should retain least number of significant digits among all the numbers. Suppose density of a material is 6.921 kg/m^3 and volume is 2.1 m^3 then its mass will be

$m = d \times v = 6.921 \times 2.11 = 14.60331$ kg

But according to the rule of significant digits it may be 14.6 kg.

As the number 6.921 has 4 significant digits and 2.11 has 3 significant digits so the final answer must contain only ‘3’ significant digits.

CHECK Point

Obtain the value of (500.0 m + 600 mm) with due regards to significant figures.

Solution

500.0 m + 600 mm = 500.0 + 0.600 m = 500.600 m ≈ 501m

ILLUSTRATION : 12

Each side of a cube is measured to be 7.293 m. What are the total surface area and volume of the cube to appropriate significant digits ?

SOLUTION :

Here, the side of cube is measured upto ‘4’ significant digits, so the calculated area and volume should be rounded off to 4 significant digits.

Total surface area of the cube = 6 $(\text{side})^2$ = $6(7.293)^2 = 319.12704\ m^2 = 319.1 m^2$

Volume of the cube = $(\text{side})^3 = (7.293)^3 = 387.8989828\ m^3 = 387.9\ m^3$

ROUNDING OFF

Rules for rounding off the uncertain digit

Rule-1. Preceding digit is increased by one if the insignificant (uncertain) digit which is to be rounded off is more than 5.

For example, a number 9.87$\underline{6}$ is to be rounded off to 3 significant digits, then rounded off number will be 9.88 as insignificant digit 6 is more than 5.

Rule-2. Preceding digit remains the same if the insignificant digit which is to be rounded off is less than 5. Suppose the number is 9.87$\underline{3}$ then after rounding off to 3 significant digits it will become 9.87.

Rule-3. If the digit to be rounded off is 5 and preceding digit is even then the preceding digit is left unchanged. For example, the number to be rounded off is 9.86$\underline{5}$ then the rounded off number will be 9.86.

Rule-4. If the digit to be rounded off is 5 and preceding digit is odd then the preceding digit is increased by one.

For example, if the number is 9.83$\underline{5}$ to be rounded off to three significant digits then after rounding off, the new number will be 9.84.

ILLUSTRATION : 13

Rounded off the numbers 978.$\underline{5}$, 12.6$\underline{8}$, 5.73$\underline{5}$, 8.92$\underline{5}$ and 11.2$\underline{2}$

SOLUTION :

Number to be rounded off	Rounded off number	Rule
978.$\underline{5}$	978	3
12.6$\underline{8}$	12.7	1
5.73$\underline{5}$	5.74	4
8.92$\underline{5}$	8.92	3
11.2$\underline{2}$	11.2	2

MISCELLANEOUS SOLVED EXAMPLES

1. Give the S.I. unit for each of the following quantities. (i) Pressure (ii) Universal gravitational constant (iii) Acceleration

Sol. (i) The S.I. unit of pressure is pascal or newton/metre^2

(ii) The S.I. unit of universal gravitational constant is newton metre^2/kg^2

(iii) The S.I. unit of acceleration is metre/second^2

2. Let us consider an equation $\frac{1}{2}mv^2 = mgh$ where m is the mass of the body, v its velocity, g is the acceleration due to gravity and h is the height. Check whether this equation is dimensionally correct or not.

Sol. The dimensions of LHS are $[M]\,[L\,T^{-1}]^2 = [M]\,[L^2\,T^{-2}] = [M\,L^2\,T^{-2}]$

The dimensions of RHS are $[M][L\,T^{-2}]\,[L] = [M][L^2\,T^{-2}] = [M\,L^2\,T^{-2}]$

The dimensions of LHS and RHS are the same and hence the equation is dimensionally correct.

3. Consider a simple pendulum, having a bob attached to a string, that oscillates under the action of the force of gravity. Suppose that the period of oscillation of the simple pendulum depends on its length (l), mass of the bob (m) and acceleration due to gravity (g). Derive the expression for its time period using method of dimensions.

Sol. The dependence of time period T on the quantities l, g and m as a product may be written as :

$$T = k\,l^x\,g^y\,m^z$$

where k is dimensionless constant and x, y and z are the exponents.

By considering dimensions both sides, we have

$$[L^0M^0T] = [L]\,[LT^{-2}]\,y\,[M]\,z = L^{x+y}\,T^{-2y}\,M^z$$

On equating the dimensions both sides, we have

$$x + y = 0;\ -2y = 1;\ \text{and } z = 0$$

So, $$x = \frac{1}{2}, y = -\frac{1}{2}, z = 0$$

Therefore , $T = k l^{½} g^{-½}$ or $T = k\sqrt{\frac{\ell}{g}}$

Note that value of constant k cannot be obtained by the method of dimensions. Here it does not matter if some number multiplies the right side of this formula, because that does not affect its dimensions.

Here, k = 2π therefore, time period of a simple pendulum

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

4. A body travels uniformly a distance of 13.8 ± 0.2 m in a time 4.0 ± 0.3 s. Calculate its velocity with error limits. What is the percentage error in velocity ?

Sol. Velocity, $v = \frac{S}{t} = \frac{13.8}{4} = 3.45\ ms^{-1} \sim 3.4\ ms^{-1}$

(two significant figures)

$$\Delta v = \left[\frac{0.2}{13.8} + \frac{0.3}{4}\right]3.45 = 0.3087 \sim 0.31$$

(two significant figures)

$$v = (3.4 \pm 0.31)ms^{-1}$$

Percentage error $= \frac{0.31}{3.4} \times 100 = 9.11\% \sim 9\%.$

5. Each side of a cube is measured to be 7.203 m. What are the total surface area and the volume of the cube to appropriate significant figures?

Sol. The number of significant figures in the measured length is 4. The calculated area and the volume should therefore be rounded off to 4 significant figures.

Surface area of the cube = 6 (side)^2 = 6(7.203)^2 m^2

= 311.299254 m^2 = 311.3 m^2

Volume of the cube = (side)^3 = (7.203)^3 m^3

= 373.714754 m^3 = 373.7 m^3

ADVANCEDEXERCISE
BASED ON CONNECTING TOPICS

DIRECTIONS : (Qs. 1-44) *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. Which of the following systems of units is not based on units of mass, length and time alone?
(a) SI (b) MKS
(b) CGS (d) FPS

2. Unit of latent heat is
(a) J kg^{-1} (b) J mol^{-1}
(c) N kg^{-1} (d) N mol^{-1}

3. Which of the following is not a unit of time?
(a) Solar year (b) Tropical year
(c) Leap year (d) Light year

4. Dyne–sec is the unit of
(a) momentum (b) force
(c) work (d) angular momentum

5. One shake is equal to
(a) 10^{-8} s (b) 10^{-9} s
(c) 10^{-10} s (d) 10^{9} s

6. One torr is equal to
(a) 1 cm of Hg column (b) Atmosphere
(c) 1 N m^{-2} (d) 1 mm of Hg column

7. What are the units of magnetic permeability?
(a) Wb A^{-1} m^{-1} (b) Wb^{-1} Am
(c) Wb A m^{-1} (d) Wb A^{-1} m

8. The ampere–second is a unit of
(a) current (b) charge
(c) energy (d) power

9. The SI unit of coefficient of mutual inductance of a coil is
(a) henry (b) volt
(c) farad (d) weber

10. The SI unit of magnetic flux is
(a) gauss (b) weber
(c) oersted (d) ampere/metre

11. Unit of specific resistance is
(a) ohm/m^2 (b) ohm m^3
(c) ohm – m (d) ohm/m

12. Light year is
(a) light emitted by the sun in one year
(b) time taken by light to travel from sun to earth
(c) the distance travelled by light in free space in one year
(d) time taken by earth to go once around the sun

13. The SI unit of pressure is
(a) atmosphere (b) bar
(c) pascal (d) mm of Hg

14. Electron volt is a unit of
(a) potential difference (b) charge
(c) energy (d) capacity

15. Dimensions of impulse are
(a) $[MLT^{-1}]$ (b) $[MLT^{2}]$
(c) $[MT^{-2}]$ (d) $[ML^{-1}T^{-3}]$

16. Units of coefficient of viscosity are
(a) Nms^{-1} (b) Nm^2 s^{-1}
(c) Nm^{-2} (d) None of these

17. What are the dimensions of action?
(a) M^2LT^{-3} (b) MLT^{-1}
(c) MLT^{-2} (d) ML^2T^{-1}

18. Which one is dimensionless?
(a) Force/acceleration (b) Velocity/acceleration
(c) Volume/area (d) Energy/work

19. Potential is measured in
(a) joules/coulomb (b) watt/coulomb
(c) newton–second (d) None of these

20. One second is defined to be equal to
(a) 1650763.73 periods of the Krypton clock
(b) 652189.63 periods of the Krypton clock
(c) 1650763.73 periods of the Cesium clock
(d) 9192631770 periods of the Cesium clock

21. If C and L denote the capacitance and inductance, the units of LC are
(a) $M^0 L^0 T^{-1}$ (b) $M^0 L^{-1} T^0$
(c) $M^{-1} L^{-1} T^0$ (d) $M^0 L^0 T^2$

22. The dimensions of electromotive force in terms of current A are
(a) $MT^{-2} A^{-2}$ (b) $ML^2 T^{-2} A^2$
(c) $ML^2 T^{-2} A^{-2}$ (d) $ML^2 T^{-3} A^{-1}$

23. The expression $[ML^{-1} T^{-2}]$ does not represent
(a) pressure (b) power
(c) stress (d) Young's modulus

24. The dimensions of universal gas constant are
(a) $L^2 M T^{-2} K^{-1}$ (b) $L M^2 T^{-2} K^{-1}$
(c) $L M T^{-2} K^{-1}$ (d) $L^2 M^2 T^{-2} K^{-1}$

25. Dimensions of specific heat are
(a) $ML^2 T^{-2} K$ (b) $ML^2 T^{-2} K^{-1}$
(c) $ML^2 T^2 K^{-1}$ (d) $L^2 T^{-2} K^{-1}$

26. Which physical quantities have same dimension?
(a) Moment of couple and work
(b) Force and power
(c) Latent heat and specific heat
(d) Work and power

27. Distance travelled by a particle at any instant '*t*' can be represented as $S = A(t + B) + Ct^2$. The dimensions of *B* are
(a) M^0LT^{-1} (b) M^0L^0T
(c) $M^0L^{-1}T^{-2}$ (d) $M^0L^2T^{-2}$

28. In the eqn. $\left(P+\frac{a}{V^2}\right)(V-b) = \text{constant}$, the units of a are
(a) dyne × cm^5 (b) dyne × cm^4
(c) dyne/cm^3 (d) dyne × cm^2

29. Error in the measurement of radius of a sphere is 1%. Then error in the measurement of volume is
(a) 1% (b) 5%
(c) 3% (d) 8%

30. A quantity is represented by $X = M^a L^b T^c$. The % error in measurement of M, L and T are α%, β% and γ% respectively. The % error in X would be
(a) $(\alpha a+\beta b+\gamma c)\%$ (b) $(\alpha a-\beta b+\gamma c)\%$
(c) $(\alpha a-\beta b-\gamma c)\times 100\%$ (d) None of these

31. Subtract 0.2 J from 7.26 J and express the result with correct number of significant figures
(a) 7.1 J (b) 7.06 J
(c) 7.0 J (d) 7 J

32. Multiply 107.88 by 0.610 and express the result with correct number of significant figures
(a) 65.8068 (b) 65.807
(c) 65.81 (d) 65.8

33. When 97.52 is divided by 2.54, the correct result is
(a) 38.3937 (b) 38.394
(c) 38.39 (d) 38.4

34. The radius of a thin wire is 0.16 mm. The area of cross section of the wire in sq. mm with correct number of significant figures is
(a) 0.08 (b) 0.080
(c) 0.0804 (d) 0.80384

35. S.I. unit of surface tension is
(a) degree/cm (b) N/m
(c) N/m^2 (d) N m

36. Weber/m^2 is equal to
(a) tesla (b) henry
(c) watt (d) None of these

37. ML^2T^{-2} are dimensions of
(a) force (b) moment of force
(c) momentum (d) power

38. Which of the following is not the name of a physical quantity?
(a) Displacement (b) Momentum
(c) Metre (d) Torque

39. Watt–hour meter measures
(a) current (b) voltage
(c) power (d) electric energy

40. If I is regarded as the fourth dimension, then the dimensional formula of charge in terms of current I is
(a) $[IT^2]$ (b) $[I^0T^0]$
(c) $[I^{-1}T^0]$ (d) $[IT]$

41. If time T, acceleration A and force F are regarded as base units, then the dimensional formula of work is
(a) $[FA]$ (b) $[FAT]$
(c) $[FAT^2]$ (d) $[FA^2T]$

42. The number of significant figures in 0.00060 m is
(a) 1 (b) 2
(c) 3 (d) 4

43. In a simple pendulum experiment for the determination of acceleration due to gravity, time period is measured with an accuracy of 0.2% while length was measured with an accuracy of 0.5%. The percentage accuracy in the value of g so obtained is
(a) 0.25% (b) 0.7%
(c) 0.9% (d) 1.0%

44. A force is given by $F = at + bt^2$, where t is time, the dimensions of a and b are
(a) $[M\,L\,T^{-4}]$ and $[M\,L\,T^{-1}]$
(b) $[M\,L\,T^{-1}]$ and $[M\,L\,T^0]$
(c) $[M\,L\,T^{-3}]$ and $[M\,L\,T^{-4}]$
(d) $[M\,L\,T^{-3}]$ and $[M\,L\,T^0]$

DIRECTIONS (Qs. 45-52) : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

45. Which of the following pairs have same dimensions?
(a) Light year and wavelength
(b) Torque and work
(c) Angular momentum and work
(d) Energy and Young's modulus

46. Choose the correct statements :
(a) A dimensionally incorrect equation may be incorrect
(b) A dimensionally correct equation may be incorrect
(c) A dimensionally incorrect equation may be correct
(d) A dimensionally correct equation may be correct

47. Which of the following are not a unit of time?
(a) Light year (b) Second
(c) Parsec (d) Micron

48. When a wave traverses a medium the displacement of a particle located at x, at time t is given by ;
$y = a \sin(bt - cx)$
where a, b and c are constants of the wave.
Which of the following are dimensionless quantities?
(a) cx (b) $\frac{b}{c}$
(c) bt (d) $\frac{y}{a}$

49. Which of the following are the units of mass?

(a) amu (b) Quintal
(c) kg-wt (d) Metric ton

50. Which of the following are true regarding significant figures?

(a) The zeros appearing in the middle of a number are significant, while those at the end of a number without a decimal point are ambiguous.
(b) All non-zero digits are significant
(c) Greater the number of significant figures in a measurement smaller is the percentage error
(d) The power of 10 are counted while counting the number of significant figures.

51. Given $y = a\cos\left(\frac{t}{p} - qx\right)$

where t represents time in second and x represents distance in metre. Which of the following statements are false?

(a) The unit of t is same as that of q
(b) The unit of x is same as that of q
(c) The unit of t is same as that of p
(d) The unit of x is same as that of p

52. Identify the pairs having identical dimensions.

(a) Strain and angle
(b) Planck constant and angular momentum
(c) Linear momentum and moment of force
(d) Pressure and modulus of elasticity

DIRECTIONS (Qs. 53-56): *Following question has four statements (A, B, C and D) given in Column I and four or five statements (p, q, r, s and t) in Column II. Any given statement in Column-I can have correct matching with one or more statement(s) given in Column II. Match the entries in Column-I with entries in Column-II.*

53. Match the quantities having same dimensions :

Column-I	**Column-II**
(A) Stress	(p) Force
(B) Pressure	(q) Strain
(C) Tension	(r) Angle
(D) Refractive index	(s) Young's modulus.
	(t) Energy per unit volume

	A	B	C	D
(a)	p,q	q	r, s	q, r
(b)	s, t	s, t	p	q, r
(c)	r, s	p, q	t	p, r, s
(d)	p,	q, r	r	s

54.

Column-I (Quantity)	**Column-II (Unit)**
(A) Area	(p) $kg\ m^2\ s^{-2}$
(B) Speed	(q) ms^{-1}
(C) Force	(r) m^2
(D) Work	(s) $kg\ ms^{-2}$

	A	B	C	D
(a)	p,q	q	r, s	q, r
(b)	p, q, r, s	q	p, q, r, s	p, q, r, s
(c)	r	q	s	p
(d)	p,	q, r	r	s

55.

Column-I (Quantity)	**Column-II (C.G.S. unit)**
(A) Acceleration due to gravity	(p) erg/g°C
(B) Specific heat capacity	(q) $gc\ ms^{-1}$
(C) Impulse	(r) cms^{-2}
(D) Power	(s) $ergs^{-1}$

	A	B	C	D
(a)	p, r	p,s	q	r
(b)	p, q, r, s	q	p, q, r, s	p, q, r, s
(c)	p, s	q	r, s, t	r
(d)	r	p	q	s

56. Match the quantities (in Column I) with their respective dimensions (in Column II).

Column-I	**Column-II**
(A) Velocity	(p) $[M^0LT^{-1}]$
(B) Force	(q) $[M^0LT^0]$
(C) Acceleration	(r) $[M^0LT^{-2}]$
(D) Displacement	(s) $[MLT^{-2}]$
	(t) $[ML^{-1}T^0]$

	A	B	C	D
(a)	p	s	r	q
(b)	p, r	s, t	r, t	r, s, t
(c)	t	r, t	p, s, t	s, t
(d)	p,	q, r	r	s

DIRECTIONS (Qs. 57-62): *Study the given paragraph(s) and answer the following questions.*

PASSAGE - I

Dimensions are the powers to which fundamental quantities must be raised to represent a given physical quantity. Again the choice of fundamental quantities is not unique. A set of physical quantities which are independent of each other may be taken as fundamental quantities. Based on the above information and knowledge of dimensions, answer the following.

57. Which of the following set of quantities cannot be taken as fundamental?

(a) force, length, time
(b) momentum, length, time
(c) acceleration, length, time
(d) mass, velocity, acceleration.

58. If momentum (P), velocity (V) and Time (T) is taken as fundamental quantities, the dimensional formula for mass will be

(a) $[PV^{-1}T]$ (b) $[PV^{-1}T^0]$
(c) $[P^0VT^{-1}]$ (d) $[PVT^0]$

59. In the previous case, the dimensional formula for force will be

(a) $[PVT]$ (b) $[PV^0T^{-1}]$
(c) $[PV^{-1}T^0]$ (d) $[PV^{-1}T^2]$

PASSAGE - II

The mass of a cube measured with a balance of least count 0.2 g is found to be 5.0 g and its length measured with the help of a vernier calliper of least count 0.01 cm is found to be 1.0 cm. Then.:

60. The percentage error in the measurement of mass of the cube is

(a) 20% (b) 4%
(c) 7% (d) 10%

61. Absolute error in the measurement of volume is

(a) 0.03 cm^3 (b) 0.3 cm^3
(c) 0.1 cm^3 (d) 0.15 cm^3

62. Absolute error in the measurement of density is

(a) 0.05 g/cm^3 (b) 0.15 g/cm^3
(c) 0.25 g/cm^3 (d) 0.35 g/cm^3

DIRECTIONS (Qs. 63-68): *Each of these questions contains an Assertion followed by Reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.
(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.
(c) If **Assertion** is **correct** but **Reason** is **incorrect**.
(d) If **Assertion** is **incorrect** but **Reason** is **correct**.

63. **Assertion :** Light year and wavelength both measure distance.
Reason : Both have dimensions of time.

64. **Assertion :** Force cannot be added to pressure.
Reason : Their dimensions are different.

65. **Assertion :** Density is a derived physical quantity.
Reason : Density cannot be derived from the fundamental physical quantities.

66. **Assertion :** The graph between P and Q is straight line. When P/Q is constant.
Reason : The straight line graph means that P is proportional to Q or P is equal to constant multipled by Q.

67. **Assertion :** Number of significant figures in 0.005 is one and that in 0.500 is three.
Reason : This is because zeros are not significant.

68. **Assertion :** Radian is the unit of distance.
Reason : One radian is the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle.

DIRECTIONS (Qs. 69-72): *Following are integer based/ Numeric based questions. Each question, when worked out will result in one integer or numeric value.*

69. Each side of a cube is measured to be 7.203 m. What are the total surface area and the volume of the cube to appropriate significant figures?

70. 5.74 g of a substance occupies 1.2 cm^3. Express its density by keeping the significant figures in view.

71. The mass of a box measured by a grocer's balance is 2.300 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (a) the total mass of the box, (b) the difference in the masses of the pieces to correct significant figures ?

72. A calorie is a unit of heat or energy and it equals about 4.2 J where $1J = 1\ kg\ m^2\ s^{-2}$. Suppose we employ a system of units in which the unit of mass equals α kg, the unit of length equals β m, the unit of time is γ s. calorie has a magnitude $n\ \alpha^{-1}\ \beta^{-2}\ \gamma^2$ in terms of the new units, find the value of n.

SOLUTIONS

Brief Explanations of Selected Questions

ADVANCED EXERCISE
BASED ON CONNECTING TOPICS

1. (a) SI is based on seven fundamental units.
2. (a) $L = \frac{Q}{m} = \frac{\text{J}}{\text{kg}} = \text{J kg}^{-1}$
3. (b) Tropical year is the year in which there is total solar eclipse. Light year represents distance.
4. (a) As force = change in momentum/time.
$\therefore$ force × time = change in momentum
5. (a)
6. (d)
7. (a) From Biot Savart's law
$$B = \frac{\mu_0}{4\pi}\frac{i\,dl\,\sin\theta}{r^2}$$
$$\mu_0 = \frac{4\pi\,Br^2}{i\,dl\,\sin\theta} = \frac{Wb\;m^{-2}m^2}{A\,m} = Wb\;A^{-1}\,m^{-1}$$
8. (b) charge = current × time.
9. (a)
10. (b) According to Faraday' Law $\in = \frac{d\phi}{dt}$
so dimensionally $[\phi] = [E.M.F][T] = [W/q][T]$
$= [ML^2T^{-2}/AT][T] = [ML^2T^{-2}A^{-1}]$
and the S.I unit of magnetic flux is volt × sec., which is known as weber (Wb).
11. (c) $\rho = \frac{RA}{\ell} = \frac{\text{ohm m}^2}{\text{m}} = \text{ohm} - \text{m}$
12. (c) 1 light year = speed of light in vacuum × no. of seconds in one year = $(3 \times 10^8) \times (365 \times 24 \times 60 \times 60)$
$= 9.467 \times 10^{15}$ m.
13. (c) 1 pascal = 1 N / m^2.
14. (c) Electron volt is a unit of energy &
$1\text{eV} = 1.6 \times 10^{-19}$ joule
15. (a) Impulse = force × time = $MLT^{-2} \times T = [MLT^{-1}]$.
16. (d) Coefficient of viscosity (η)
= Force required per unit area to maintain unit velocity gradient
$= \text{N}/[\text{m}^2 \times (\text{m/s})/\text{m}]$
$= \text{N/m}^2\,\text{s}^{-1} = \text{N m}^{-2}\,\text{s} = [ML^{-1}\,T^{-1}]$
17. (c) Action means force & the dimension of force is MLT^{-2} ($\because F = ma$, where a is acceleration)
18. (d) Both energy and work have same unit.
$\therefore$ energy/work is a pure number.
19. (a) Potential is work done per unit charge.
20. (d)
21. (d) From $\nu = \frac{1}{2\pi\sqrt{LC}}$
$$LC = \frac{1}{(2\pi\nu)^2} = \frac{1}{(T^{-1})^2} = T^2 = [M^0\,L^0\,T^2]$$
22. (d) Electromotive force = potential difference
$$V = \frac{W}{q} = \frac{ML^2\,T^{-2}}{AT} = [ML^2\,T^{-3}\,A^{-1}]$$
23. (b) $\text{Power} = \frac{\text{energy}}{\text{time}} = \frac{ML^2\,T^{-2}}{T} = [ML^2\,T^{-3}]$
24. (a) $R = \frac{PV}{\mu T} = \frac{W}{\mu T} = \frac{ML^2\,T^{-2}}{\text{mol}\,K}$
where μ is number of mole of the gas
$= [ML^2T^{-2}K^{-1}\text{mol}^{-1}]$
25. (d) $s = \frac{Q}{m\theta} = \frac{ML^2T^{-2}}{MK} = [M^0\,L^2\,T^{-2}\,K^{-1}]$
26. (a) Moment of couple = force × distance = $[ML^2T^{-2}]$
work = force × distance = $[ML^2T^{-2}]$.
27. (b) In $S = A(t + B) + Ct^2$; B is added to time t. Therefore, dimensions of B are those of time.
28. (b) As $\frac{a}{V^2} = P$
$$\therefore\; a = PV^2 = \frac{\text{dyne}}{\text{cm}^2}(\text{cm}^3)^2 = \text{dyne} \times \text{cm}^4$$
29. (c) $V = \frac{4}{3}\pi r^3;\; \frac{\Delta V}{V} \times 100 = 3\left(\frac{\Delta r}{r}\right) \times 100 = 3 \times 1\% = 3\%$
30. (a) $X = M^aL^bT^c$;
$$\frac{\Delta X}{X} \times 100 = \left(\frac{a\,\Delta M}{M} + \frac{b\,\Delta L}{L} + \frac{c\,\Delta T}{T}\right) \times 100$$
$= (a\alpha + b\beta + c\gamma)\%$
31. (a) Subtraction is correct upto one place of decimal, corresponding to the least number of decimal places.
7.26 – 0.2 = 7.06 = 7.1 J.
32. (d) Number of significant figures in multiplication is three, corresponding to the minimum number
107.88 × 0.610 = 65.8068 = 65.8
33. (d) $\frac{97.52}{2.54} = 38.393 = 38.4$ (with least number of significant figures, 3).

34. (b) Radius, $r = 0.16$ mm.
Area of cross–section $= \pi r^2$
$= 3.14 \times (0.16)^2$
$= 0.08038 = 0.080$

35. (b) Surface tension $T = \frac{\text{force}}{\text{length}} = \frac{N}{m}$

36. (a) Weber/m^2 = tesla

37. (b) Moment of force $= r \times F = [L]\,[MLT^{-2}] = [ML^2T^{-2}]$

38. (c) The meter is a unit and not a physical quantity.

39. (d) W – hr is a unit of energy

40. (d) $I = \frac{q}{t}$

41. (c) $[A] = [LT^{-2}]$ or $[L] = [AT^2]$ or
[Work] = [Force ×Distance] = $[E] = [FAT^2]$

42. (b) According to rules of significant figures.

43. (c) $T = 2\pi\sqrt{\frac{\ell}{g}},\ g \propto \frac{\ell}{T^2}$

$\therefore \frac{\Delta g}{g} \times 100 = 0.5\% + 2 \times 0.2\% = 0.9\%$

44. (c) [at] = $[F]$ amd $[bt^2] = [F]$
$\Rightarrow [a] = MLT^{-3}$ and $[b] = MLT^{-4}$

45. (a, b)

46. (a, b, d)

47. (a, c, d)

48. (a, c, d)
$[b] = [t^{-1}]$ & $[C] = [x^{-1}]$ & $[y] = [x] = [a]$

$\therefore cx, bt$ & $\frac{y}{a}$ are dimensionless.

49. (a, b, c, d)

50. (a, b, c)

51. (a, b, d)
$[p] = [t]$ & $[q] = [x^{-1}]$

52. (a, b, d)

53. (b) (A) →(s, t) ; (B) →(s,t) ; (C) →(p) ; (D) →(q, r)

54. **(c)** A →(r) ; B →(q) ; C →(s) ; D →(p)

55. (d) A →(r) ; B →(p) ; C →(q) ; D →(s)

56. (a) A →(p) ; B →(s) ; C →(r) ; D →(q)

57. (c) Acceleration can be derived from length and time, i.e. $[a] = [LT^{-2}]$. Hence acceleration, length and time cannot be taken as set of fundamental quantities.

58. (b) We have,
momentum
$P = mV$

or $m = \frac{P}{V}$ or $[m] = [PV^{-1}\,T^0]$

59. (b) We have,

Force $= \frac{\text{Change in momentum}}{\text{Time}}$

or $[F] = \frac{[P]}{t} = [PV^{\circ}T^{-1}]$

60. (c) Here, mass of cube,
$m = (5.0 \pm 0.2)$ g
$l = (1.0 \pm 0.01)$ cm
$\therefore V = l^3 = 1.0$ cm^3

$\frac{\Delta V}{V} = 3\frac{\Delta l}{l} = 0.03\,\text{cm}^3$

$\therefore V = (1.0 \pm 0.03)$ cm^3

$\therefore P = \frac{m}{V} = \frac{5.0}{1.0} = 5.0\,\text{cm}^{-3}$

$\frac{\Delta P}{P} = \frac{\Delta m}{m} + \frac{\Delta V}{V} = \frac{0.2}{5.0} + \frac{0.03}{1.0}$

or $\frac{\Delta P}{P} = 0.07 = 7\%$

61. (a) $\Delta V = 0.03$ cm^3

62. (d) $\Delta P = 0.07 \times 5.0 = 0.35$ g/cm^3

63. (c)

64. (a) Quantities having different dimensions cannot be added or subtracted.

65. (c)

66. (d) Graph between P and Q is straight line if,

$\frac{\Delta P}{\Delta Q}$ = constant

67. (c)

68. (d) Radian is unit of distance.

69. The number of significant figures in the measured length is 4. The calculated area and the volume should therefore be rounded off to 4 significant figures.
Surface area of the cube = $6(7.203)^2$ m^2
$= 311.299254$ m^2 = 311.3 m^2
Volume of the cube = $(7.203)^3$ m^3 = 373.714754 m^3
= 373.7 m^3

70. There are 3 significant figures in the measured mass whereas there are only 2 significant figures in the measured volume. Hence the density should be expressed to only 2 significant figures.

Density $= \frac{5.74}{1.2}\,\text{gcm}^{-3} = 4.8\ \text{g cm}^{-3}$.

71. (a) Total mass = 2.3403 kg = 2.3 kg
(b) Difference = 20.17 g – 20.15 g = 0.02g

72. 1 cal = 4.2 kg m^2 s^{-2}.

SI	New system
$n_1 = 4.2$	$n_2 = ?$
$M_1 = 1$ kg	$M_2 = \alpha$ kg
$L_1 = 1$m	$L_2 = \beta$ metre
$T_1 = 1$s	$T_2 = \gamma$ second

Dimensional formula of energy is [ML^{2}T^{-2}]
Comparing with $[M^aL^bT^c]$, we find that $a = 1$, $b = 2$, $c = -2$

Now, $n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$

$= 4.2\left[\frac{1\,\text{kg}}{\alpha\,\text{kg}}\right]^1\left[\frac{1\text{m}}{\beta\text{m}}\right]^2\left[\frac{1\text{s}}{\gamma\,\text{s}}\right]^{-2}$

$= 4.2\,\alpha^{-1}\beta^{-2}\gamma^2 \quad \therefore$ n = 4.2

Chapter

7

Centre of Mass and Equilibrium of Rigid Bodies

INTRODUCTION

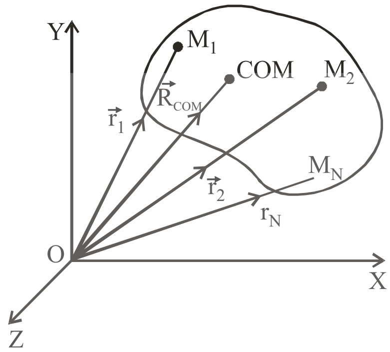
So far we have discussed mechanics of bodies which are having dimensions of a point i.e. particle. But in day to day life we come across bodies which are larger in size and made up of large number of particles. In this chapter we will discuss motion of these bodies under the application of forces and torque. For example when we apply force on the handle of a door which is hinged to the wall, it moves but it retains its shape i.e. it does not get deformed. On the application of force on the handle all parts of the door moves though with different linear velocities.

For describing translatory motion of a body, the centre of mass of the body is the representatory point at which whole mass of the body is suppposed to be concentrated.

In this chapter we will read different variables to describe the motion of a rigid body like moment of inertia, torque, angular velocity, angular acceleration, etc. Actually, these quantities are analogous to some terms used in the motion of a point object. Moment of inertia plays the same role in the rotational motion which the mass plays in the linear motion. Similarly, torque is equivalent to force, angular velocity is equivalent to velocity, angular acceleration to acceleration, angular momentum to momentum and so on. The chapter deals with the centre of mass, coupled with the motion of a rigid body and different aspects of angular variables describing the motion.

CENTRE OF MASS (COM)

For a system of particles **centre of mass,** is that point at which its total mass is supposed to be concentrated.



When we consider the motion of a rigid body then a point inside or outside the body which behaves in such a way that the entire mass of the body is concentrated at that point. This is called centre of mass of the body.

Consider a body made-up of N-particles having masses M_1, M_2,.........M_N with position vectors $\vec{r}_1, \vec{r}_2........\vec{r}_N$ respectively, then the position vector $\vec{R}_{COM}$ of its centre of mass is given by

$$\vec{R}_{COM} = \frac{M_1\vec{r}_1 + M_2\vec{r}_2 + ...+ M_N\vec{r}_N}{M_1 + M_2 + ...+ M_N} = \frac{\Sigma M_i\vec{r}_i}{\Sigma M_i}$$

If particles M_1, M_2.........M_N have co-ordinates (x_1, y_1, z_1), (x_2, y_2, z_2) ....(x_N, y_N, z_N) respectively with respect to origin O, then

$$\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k},\ \vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}\ ....\ \vec{r}_N = x_N\hat{i} + y_N\hat{j} + z_N\hat{k}$$

And $\vec{R}_{COM} = X_{COM}\hat{i} + Y_{COM}\hat{j} + Z_{COM}\hat{k}$

Also $X_{COM} = \dfrac{M_1x_1 + M_2x_2 + ...+ M_Nx_N}{M_1 + M_2 + ...+ M_N} \Rightarrow X_{COM} = \dfrac{\Sigma M_ix_i}{\Sigma M_i}$

$$Y_{COM} = \frac{M_1y_1 + M_2y_2 + ...+ M_Ny_N}{M_1 + M_2 + ...+ M_N} \Rightarrow Y_{COM} = \frac{\Sigma M_iy_i}{\Sigma M_i}$$

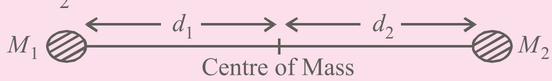
and $Z_{COM} = \dfrac{M_1z_1 + M_2z_2 + ...+ M_Nz_N}{M_1 + M_2 + ...+ M_N} \Rightarrow Z_{COM} = \dfrac{\Sigma M_iz_i}{\Sigma M_i}$

In case of a body made of two particles $\vec{R}_{COM} = \dfrac{M_1\vec{r}_1 + M_2\vec{r}_2}{M_1 + M_2}$

and $X_{COM} = \dfrac{x_1M_1 + M_2x_2}{M_1 + M_2}$, $Y_{COM} = \dfrac{M_1y_1 + M_2y_2}{M_1 + M_2}$ and $Z_{COM} = \dfrac{M_1z_1 + M_2z_2}{M_1 + M_2}$

Knowledge ENHANCER

Consider two particles of mass M_1 and M_2 as shown below :



If d_1 and d_2 are the distances of these particles from their centre of mass then

$$M_1d_1 = M_2d_2$$

Rigid Body

A body which does not deform on the application of whatsoever large force is called a rigid body. Ideally such type of body will not exist but practically, large, extended object can be treated as rigid body. For example, door is a rigid body.

Centre of Mass of Some Symmetrical Regular Objects

When bodies are symmetrical in shape and have uniform densities then their centre of mass would lie on their geometrical centres.

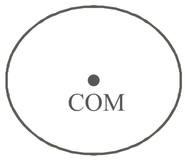
The position of centre of mass depends on following two factors:

(i) The geometrical shape of the body

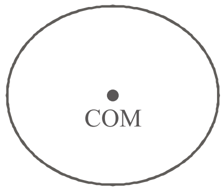
(ii) The distribution of mass in the body

(i) Uniform thin rod

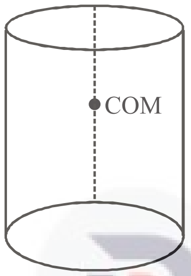
(ii) Ring/ disc



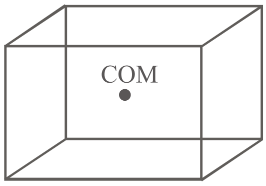
(iii) Hollow / solid sphere



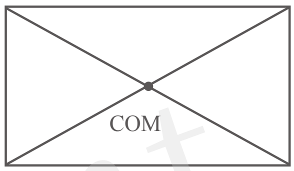
(iv) Cylinder



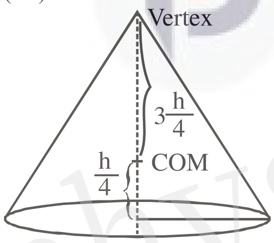
(v) Cube



(vi) Rectangular lamina

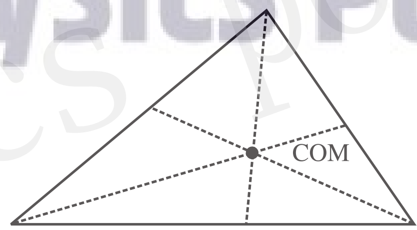


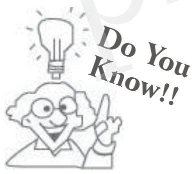
(vii) Cone



where h is its vertical height

(viii) Triangular plate / lamina





The centre of mass of a body is the average position of its mass (rather than its weight). A body's centre of mass is exactly in the same position as its centre of gravity, provided the gravitational field strength does not vary within the body itself. A point where whole weight of a body may be assumed to be act is called centre of gravity of a body. Where there is no gravity or uniform gravity, the centre of gravity of the body coincides with the centre of mass of the body.

CENTRE OF MASS OF CONTINUOUS BODIES

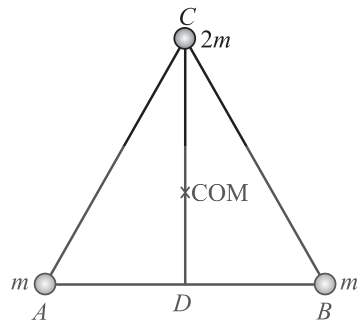
The bodies which are not made of discrete particles but have continuous distribution of matter then the co-ordinates for the centre of mass can be located by following formulae :

$$X_{COM} = \frac{\int x\,dm}{\int dm} \Rightarrow X_{COM} = \frac{\int x\,dm}{M}; \quad Y_{COM} = \frac{\int y\,dm}{\int dm} \Rightarrow Y_{COM} = \frac{\int y\,dm}{M}; \quad Z_{COM} = \frac{\int z\,dm}{\int dm} \Rightarrow Z_{COM} = \frac{\int z\,dm}{M}$$

Where dm is a small element having mass dm and x, y and z are distance of this element from x, y and z-axes respectively.

CHECK Point

- **Two balls each of mass m are placed on two vertices A and B of an equilateral triangle. A ball of mass 2 m is situated at the third vertex figure. Determine the centre of mass of the system.**



Solution

The *COM* of the balls *A* and *B* lies at the mid-point *D* of the side *AB* of the equilateral triangle *ABC*. Now, the system of the given masses is equivalent to a system of two masses, each of mass 2*m* present at the points *C* and *D* on the right bisector *CD* of the side *AB*. Therefore, *COM* of the whole system lies at the mid-point of the right bisector *CD*.

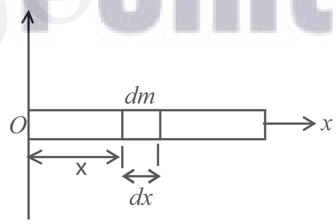
ILLUSTRATION : 1

Show that the centre of mass of a rod of mass *M* and length *L* lies midway between its ends, assuming the rod has a uniform mass per unit length.

SOLUTION :

By symmetry, we see that $y_{COM} = z_{COM} = 0$ if the rod is placed along the *x* axis. Furthermore, if we call the mass per unit length λ (the linear mass density), then $\lambda = M/L$ for a uniform rod.
If we divide the rod into elements of length *dx*, then the mass of each element is $dm = \lambda dx$. Since an arbitrary element of each element is at a distance *x* from the origin, equation gives

$$x_{COM} = \frac{1}{M}\int_0^L x\,dm = \frac{1}{M}\int_0^L x\lambda dx = \frac{\lambda L^2}{2M}$$

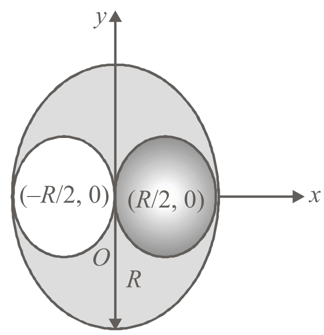


Because $\lambda = M/L$, this reduces to $x_{CM} = \frac{L^2}{2M}\left(\frac{M}{L}\right) = \frac{L}{2}$

One can also argue that by symmetry, $x_{COM} = L/2$.

ILLUSTRATION : 2

Figure shows a uniform disc of radius *R*, from which a hole of radius *R*/2 has been cut out from left of the centre and is placed on the right of the centre of the disc. Find the COM of the resulting disc.



SOLUTION :

Mass of the cut out disc

$$m = \frac{M}{\pi R^2} \times \pi\left(\frac{R}{2}\right)^2 = \frac{M}{4}$$

Let centre of the disc is at the origin of the co-ordinates. Then we can write the COM of the system as

$$x_{COM} = \frac{M\vec{R} - m\vec{r} + m\vec{r}}{M - m + m} = \frac{M \times 0 - \frac{M}{4}\left(\frac{-R}{2}\right) + \frac{M}{4}\left(\frac{R}{2}\right)}{M - \frac{M}{4} + \frac{M}{4}} = \frac{R}{4}$$

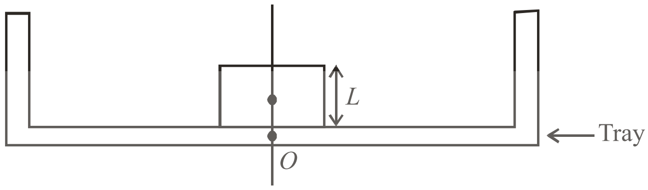
ILLUSTRATION : 3

A cube of ice of mass *m* and side *L* is placed on a very large tray of mass *M*. If the ice melts find the distance by which centre of mass of the system (ice + tray) comes down.

SOLUTION :

Taking O as origin the y-co-ordinates of centre of mass will be

$$Y_{COM} = \frac{m(L/2) + M.0}{m + M}$$



When ice melts it will spread evenly on the surface of tray, as tray is very large, its height will be negligible, hence new centre of mass of 'water + tray' will be at 'O' only. So the centre of mass shifts down by a distance of $= \frac{m(L/2)}{(m+M)}$.

MOTION OF THE CENTRE OF MASS

Assume that the total mass M of the system remains constant with time, then, for our fixed system of particles

$$M\vec{r}_{COM} = m_1\vec{r}_1 + m_2\vec{r}_2 + ... + m_n\vec{r}_n,$$

where r_{COM} is the position vector identifying the centre of mass in a particular reference frame.
Differentiating this equation with respect to time, we obtain

$$\frac{d\vec{r}_{COM}}{dt} = m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt} + ... + m_n\frac{d\vec{r}_n}{dt}$$

or $$M\vec{V}_{COM} = m_1\vec{v}_1 + m_2\vec{v}_2 + ... + m_n\vec{v}_n \quad .... (1)$$

where $\vec{v}_1$ is the velocity of the first particle, etc, and $dr_{COM}/dt \; (= \vec{V}_{COM})$ is the velocity of the centre of mass.
Differentiating equation (1) with respect to time, we obtain

$$M\frac{d\vec{V}_{COM}}{dt} = m_1\frac{d\vec{v}_1}{dt} + m_2\frac{d\vec{v}_2}{dt} + ... + m_n\frac{d\vec{v}_n}{dt} \quad .... (2)$$

$$= m_1\vec{a}_1 + m_2\vec{a}_2 + ... + m_n\vec{a}_n$$

where $\vec{a}_1, \vec{a}_2 .... \vec{a}_n$ are is the acceleration of the first, second and n[th] particle, etc, and $dv_{COM}/dt \; (= \vec{a}_{COM})$ is the acceleration of the centre of mass of the system. Now, from Newton's second law, the force F_1 acting on the first particle is given by $F_1 = m_1 a_1$. Likewise, $F_2 = m_2 a_2$, etc. We can then write equation (2) as $M\vec{a}_{COM}\,\vec{F}_1 + \vec{F}_2 + ... + \vec{F}_n = \vec{F}_{internal} + \vec{F}_{external}$
Internal forces are forces exerted by the particles on each other. However, from Newton's third law, these internal forces will occur in equal and opposite pairs, so that they contribute nothing to the sum.

$\therefore \quad M\vec{a}_{COM} = \vec{F}_{ext}$

This states that the centre of mass of a system of particles moves as though all the mass of the system were concentrated at the centre of mass and all the external forces were applied at that point.

Knowledge ENHANCER

Whatever may be the rearrangement of the bodies in a system, due to internal forces (such as one part moving away from the other or an internal explosion taking place, breaking a body into pieces),

(a) If the body was originally at rest, the COM will continue to be at rest.

(b) If before the change, the body had been moving with a constant velocity, it will continue to move with a constant velocity and in presence of external force if body had been moving with constant acceleration in a particular trajectory, the COM will continue to move in the same trajectory, with the same acceleration as if it had never experienced any explosion only if there is no change in external force.

If $\vec{F}_{ext} = 0$ then $\vec{v}_{COM} = \text{constant}$

If no external force is acting on the system, net momentum of the system must remain constant. (momentum conservation)

MOTION OF CENTRE OF MASS IN A MOVING SYSTEM OF PARTICLES

(1) COM at rest :

If $F_{ext} = 0$ and $v_{cm} = 0$, then COM remains at rest. Individual components of the system may move and have non-zero momentum due to mutual forces (internal), but the net momentum of the system remains zero.

Examples: (i) A bomb at rest suddenly explodes into various smaller fragments, all moving in different directions then, since the explosive forces are internal and there is no external force on the system, for explosion therefore, the COM of the bomb will remain at the original position and the fragment fly such that their net momentum remains zero.

(ii) Two men standing on a frictionless platform, push each other, then also their net momentum remains zero because the push forces are internal for the two men system.

(iii) A boat floating in a lake, also has net momentum zero if the people on it changes their position, because the friction force required to move the people is internal of the boat system.

(iv) Objects initially at rest, if moving under mutual forces (electrostatic or gravitational) also have net momentum zero.

(v) A light spring of spring constant k kept compressed between two blocks of masses m_1 and m_2 on a smooth horizontal surface. When released, the blocks acquire velocities in opposite directions, such that the net momentum is zero.

(vi) In a fan, all particles are moving but COM is at rest.

(2) COM moving with uniform velocity :

If $F_{ext} = 0$, then v_{cm} remains constant therefore, net momentum of the system also remains conserved. Individual components of the system may have variable velocity and momentum due to mutual forces (internal), but the net momentum of the system remains constant and COM continues to move with the initial velocity.

Examples : (i) Internal explosions/breaking does not change the motion of COM and net momentum remains conserved. A bomb moving in a straight line suddenly explodes into various smaller fragments, all moving in different directions then, since the explosive forces are internal and there is no external force on the system for explosion therefore, the COM of the bomb will continue the original motion and the fragments fly such that their net momentum remains conserved.

(ii) Man jumping from cart of buggy also exert internal forces, therefore, net momentum of the system and hence, motion of COM remains conserved.

(iii) Two moving blocks connected by a light spring of spring constant on a smooth horizontal surface. If the acting forces is only due to spring then COM will remain in its motion and momentum will remain conserved.

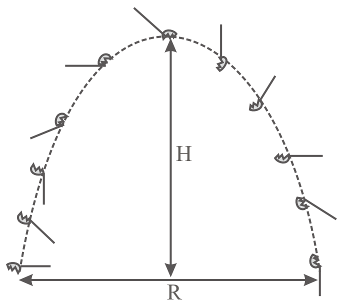
(iv) Particles colliding in absence of external impulsive forces also have their momentum conserved.

(3) COM moving with acceleration :

If an external force is present then COM continues its original motion as if the external force is acting on it, irrespective of internal forces.

Examples :

Projectile motion : An axe thrown in air at an angle θ with the horizontal will perform a complicated motion of rotation as well as parabolic motion under the effect of gravitation.

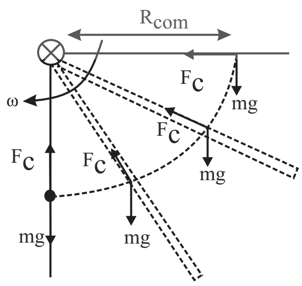


Height of COM, $H_{COM} = \dfrac{u^2 \sin^2 \theta}{2g}$,

Range of COM, $R_{COM} = \dfrac{u^2 \sin^2 \theta}{g}$,

Time of flight of COM, $T_{COM} = \dfrac{2u \sin \theta}{g}$

Circular motion : A rod hinged at an end, rotates then its COM performs circular motion. The centripetal force (F_c) required in the circular motion is assumed to be acting on the COM.



$$F_c = \frac{m\omega^2}{R_{COM}}$$

CHECK Point

(i) Is centre of mass a reality ?

(ii) A child sits stationary at one end of a long trolley moving uniformly with a speed v on a smooth horizontal floor. If the child gets up and runs about on the trolley in any manner, what is the speed of the C.O.M. of the (trolley + child) system?

Solution

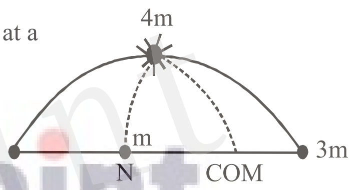
(i) No, it is only a mathematical concept.

(ii) The speed of the centre of mass of the system i.e. trolley and the child remains unchanged, if the child gets up and runs about on the trolley in any manner. It is because, the forces involved are from within the system, whereas the state of the system can change only under the effect of an external force.

ILLUSTRATION : 4

A projectile is fired at a speed of 100 m/s at an angle of 37° above the horizontal. At the highest point, the projectile breaks into two parts of mass ratio 1:3, the smaller coming to rest. Find the distance from the launching point to the point where the heavier piece lands.

SOLUTION :



Internal forces do not affect the motion of the centre of mass, the centre of mass hits the ground at a position where the original projectile would have landed. The range of the original projectile is

$$x_{COM} = \frac{2u^2 \sin\theta\cos\theta}{g} = \frac{2\times 10^4 \times \frac{3}{5}\times\frac{4}{5}}{10}\text{ m} = 960\text{m}$$

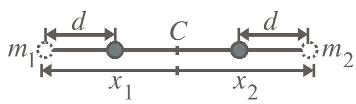
The centre of mass will hit the ground at this position. As the smaller block comes to rest after breaking, it falls down vertically and hits the ground at half of the range, i.e., at $x = 480$ m. If the heavier block hits the ground at x_2, then

$$x_{COM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} \Rightarrow 960 = \frac{(m)(480) + (3m)(x_2)}{(m + 3m)} \qquad \therefore \qquad x_2 = 1120 \text{ m}$$

ILLUSTRATION : 5

Consider a system of two masses m_1 and m_2. If first mass is displaced by distance d towards centre of mass find a displacement of second mass.

SOLUTION :



Assume centre of mass at distances x_1 and x_2 from m_1 and m_2 respectively

From centre of mass definition

$$m_1x_1 = m_2x_2 \qquad \text{........(i)}$$

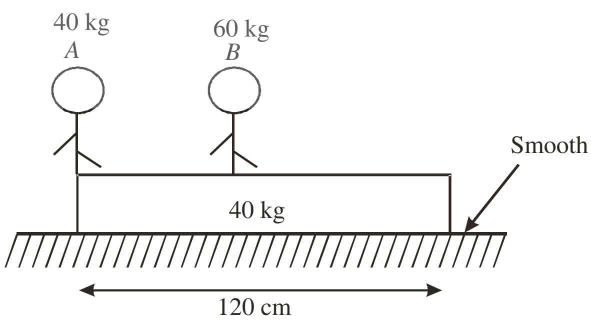
and $$m_1(x_1 - d) = m_2(x_2 - d') \qquad \text{........(ii)}$$

Solving eqs. (i) and (ii)

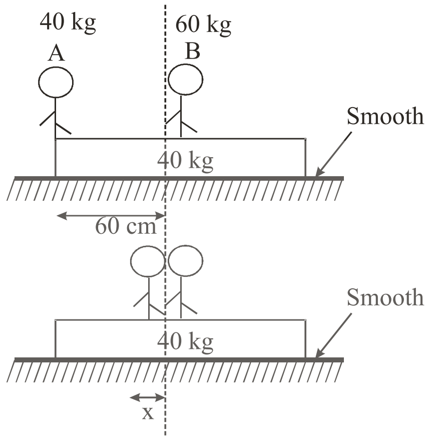
$$m_1 d = m_2 d' \quad \text{or} \quad d' = \frac{m_1}{m_2} d$$

ILLUSTRATION : 6

Two men A and B are standing on a plank. B is at the middle of the plank and A is at the left end of the plank. Surface of the plank is smooth. System is initially at rest and masses are as shown in figure. A and B starts moving such that the position of B remains fixed with respect to ground, then A meets B. Find the point where A meets B.



SOLUTION :



Taking the origin at the centre of the plank.

$m_1\Delta x_1 + m_2\Delta x_2 + m_3\Delta x_3 = 0 \qquad (\because \Delta x_{COM} = 0)$

(Assuming the centres of the two men are exactly at the axis shown)

60 (0) + 40 (60) + 40 (–x) = 0, x is the displacement of the block

$\Rightarrow \quad x = 60$ cm

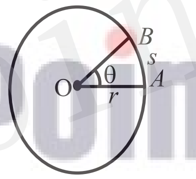
i.e., A and B meet at the right end of the plank.

ROTATIONAL MOTION

If the relative distance between the particles of a system do not change on applying force, then it is called a **rigid body.** A rigid body performs a pure rotational motion, if each particle of the body moves in a circle, and the centre of all the circles lie on a straight line called the axis of rotation.

Angular Displacement ($\Delta\theta$)

The change in position of a particle moving in a circular path with respect to the center is known as it's angular displacement. The counterclockwise rotation is taken as the positive sense of rotation. The angular position is as shown and represented by θ.



Angular displacement, $\theta = \dfrac{\text{arc}}{\text{radius}} = \dfrac{s}{r}$

The angular displacement is $\Delta\theta = \theta_2 - \theta_1$

where θ_1 is the angular position at time t_1 and θ_2 is the angular position at time t_2.

Its **SI unit** is radian.

Angular Velocity (ω)

The rate of change of angular displacement of a body is defined as angular velocity .

Average angular velocity $\omega = \dfrac{\theta_2 - \theta_1}{t_2 - t_1} = \dfrac{\Delta\theta}{\Delta t}$

Instantaneous angular velocity $\omega = \lim\limits_{\Delta t \to 0} \left(\dfrac{\Delta\theta}{\Delta t}\right) = \dfrac{d\theta}{dt}$

Also, angular velocity, $\omega = 2\pi n$ where, n = number of revolution per second.

Its **SI unit** is radian/s and **dimension** $[M^oL^oT^{-1}]$

Relation between angular velocity (ω), linear velocity (v) and radius of circular path (r) $v = r\,\omega$ or, $\omega = \dfrac{v}{r}$

Angular Acceleration (α)

The rate of change of angular velocity of a rotating body is defined as angular acceleration.

Average angular acceleration $\alpha_{avg} = \dfrac{\omega_2 - \omega_1}{t_2 - t_1} = \dfrac{\Delta\omega}{\Delta t}$ ω_1 and ω_2 are the angular velocities at t_1 and t_2

Instantaneous angular acceleration $\alpha = \lim\limits_{\Delta t \to 0} \left(\dfrac{\Delta\omega}{\Delta t}\right) = \dfrac{d\omega}{dt} = \dfrac{d^2\theta}{dt^2}$

Its **SI unit** is radian/s^2 and **dimension** $[M^oL^oT^{-2}]$

Relation between angular acceleration (α) tangential acceleration (a_t) and radius of circular path (r) $a_t = r\alpha$

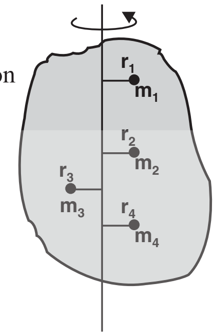
MOMENT OF INERTIA

The property of a body by virtue of which it opposes any change in its state of rest or of rotational motion is defined as its moment of inertia. The moment of inertia of a particle in rotational motion is equal to the product of its mass and square of its distance from the axis.

Moment of inertia, $I = mr^2$

where m is the mass of the particle and r is the distance of the particle from the axis of rotation.

For a system of particles of masses m_1, m_2, m_3, ... at distances r_1, r_2, r_3 ... from the axis of rotation



$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + ... = \sum_{i=1}^{n} m_i r_i^2$$

For a body with uniform mass distribution $I = \int r^2 dm = \int r^2 \rho dV$

If the mass of the i^{th} particle is m_i and its speed is v_i, the KE $_{trans}$ of this particle is

$$K_i = \frac{1}{2} m_i v_i^2$$

Total KE $= \Sigma K_i = \Sigma \frac{1}{2} m_i v_i^2 = \frac{1}{2} \Sigma m_i r_i^2 \omega^2$ $\qquad \omega \to$ common to all particles

$$= \frac{1}{2} \Sigma m_i \cdot r_i^2 \cdot \omega^2 = \frac{1}{2} I \omega^2$$ $\qquad I \to$ moment of inertia.

Thus, quantities *moment of inertia (I) and angular velocity* (ω) *in rotational motion are analogous to mass (m) and linear velocity* (v) *in linear motion, respectively.*

Mathematically, moment of inertia (I) is the sum of the product of mass and square of the perpendicular distance from the axis considered. Moment of inertia depends on axis considered.

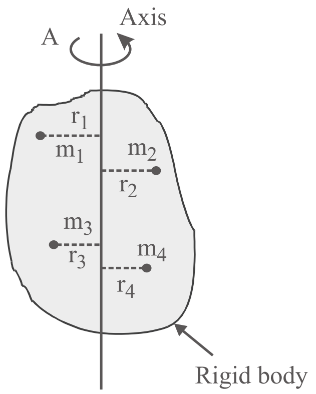
It is neither a scalar nor a vector but it is considered as a **tensor**.

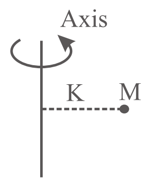
Its **SI unit** is kg m^2 and its **dimensions**$[ML^2T^0]$

RADIUS OF GYRATION (K)

The radius of gyration of a body is the distance from axis of rotation. The square of this distance when multiplied by the mass of body then it gives the moment of inertia of the body.

($I = MK^2$) about same axis of rotation





$I = MK^2$ but $I = \Sigma mr^2$

So, $MK^2 = \Sigma mr^2$

$$\Rightarrow \quad K^2 = \frac{m_1 r_1^2 + m_2 r_2^2 + ...m_n r_n^2}{M}$$

$$\Rightarrow \quad K = \sqrt{\frac{m_1 r_1^2 + m_2 r_2^2 + ...m_n r_n^2}{m_1 + m_2 + ...+ m_n}}$$

If $m_1 = m_2 = m_3 = m$ then, $M = mn$

$$K = \sqrt{\frac{r_1^2 + r_2^2 + ...r_n^2}{n}}$$ [n = total number of particles]

Radius of gyration $K = \sqrt{\frac{I}{M}}$

It has no meaning without axis of rotation, it is a scalar quantity.

Radius of gyration depends on

(i) axis of rotation

(ii) distribution of mass of body

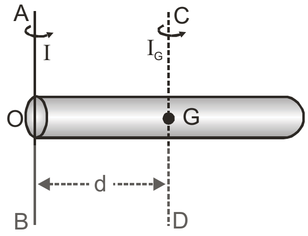
Radius of gyration does not depend on

(i) mass of the body

(ii) angular quantities (angular displacement, angular velocity, etc.)

Parallel and Perpendicular Axes Theorems

Theorem of Parallel Axes

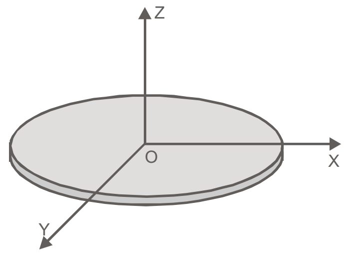


This theorem states that the moment of inertia of a rigid body about any axis is equal to its moment of inertia about a parallel axis through centre of mass plus the product of the mass of the body and the square of the perpendicular distance between the two parallel axes.

Thus $I = I_{CM} + Md^2$

where I_{CM} = M.I. of the body about an axis passing through centre of mass.

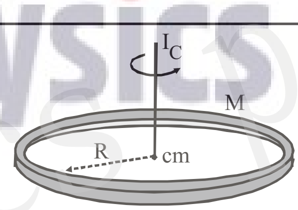
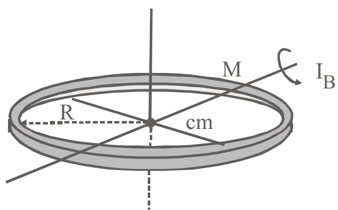
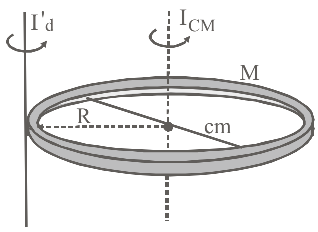
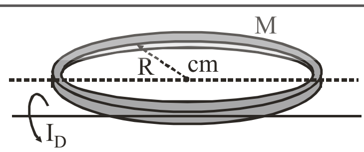
Theorem of Perpendicular Axes

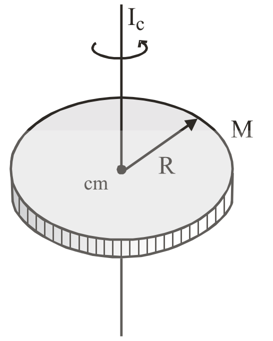
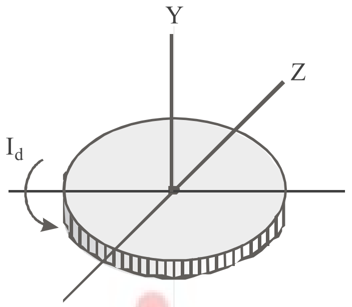
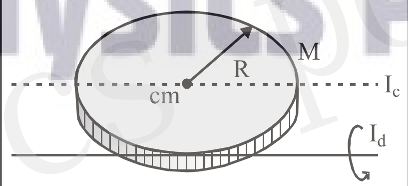
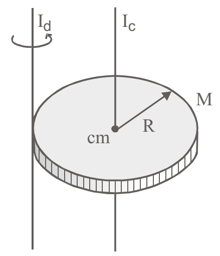
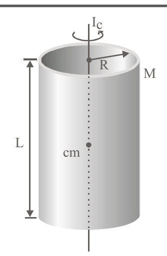
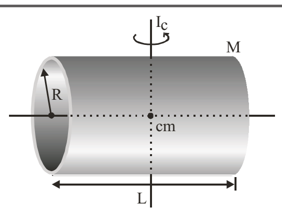


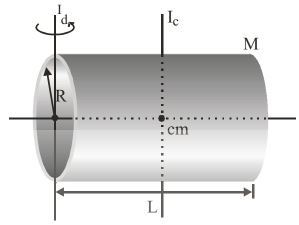
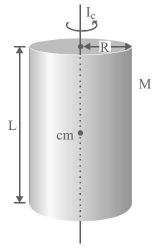
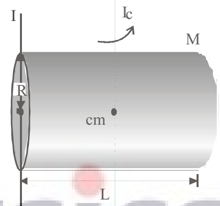
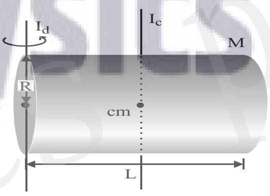
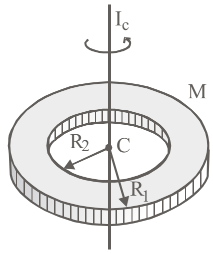
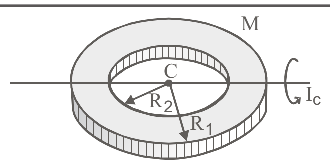
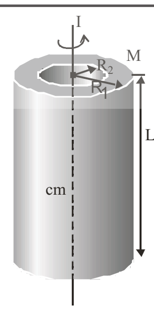
This theorem states that the moment of inertia of an object about an axis perpendicular to its plane is equal to the sum of its moments of inertia about any two mutually perpendicular axes in its plane and intersecting each other at the point where the perpendicular axes pass through it.

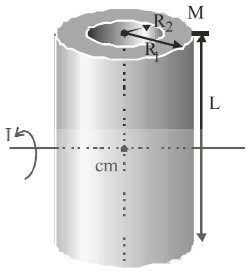
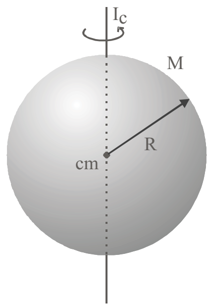
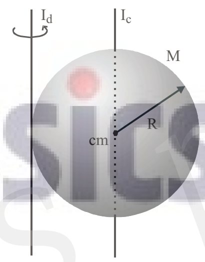
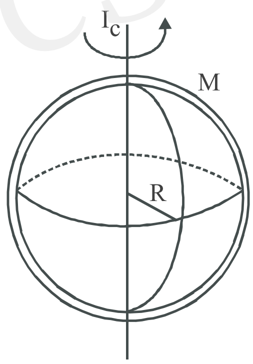
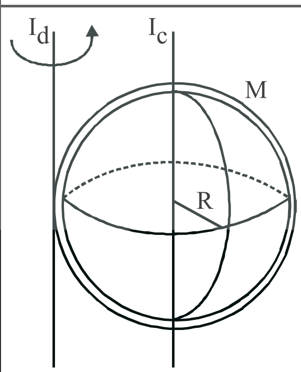
Thus, $I_z = I_x + I_y$

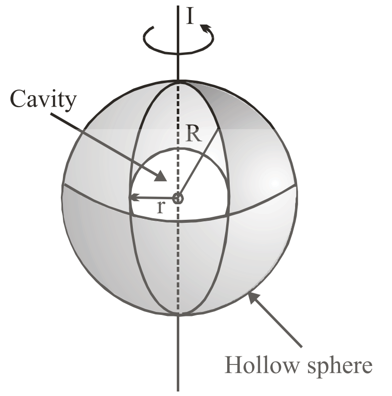
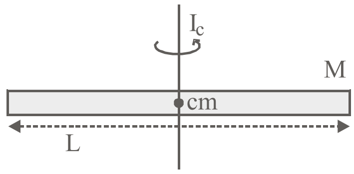
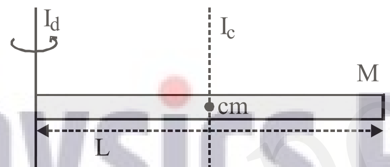
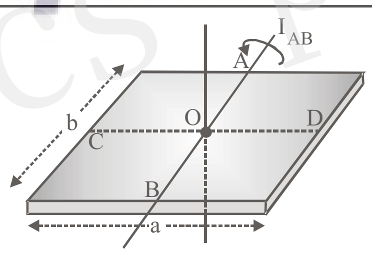
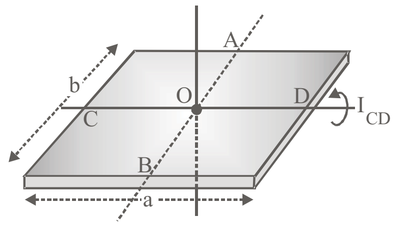
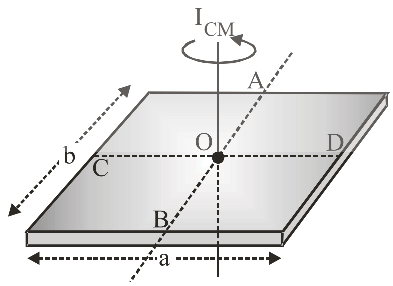
Moment of inertia and radius of gyration of some regular shaped objects

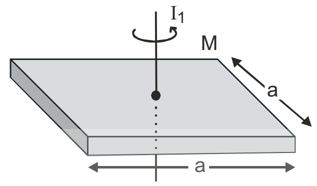
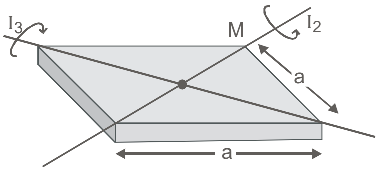
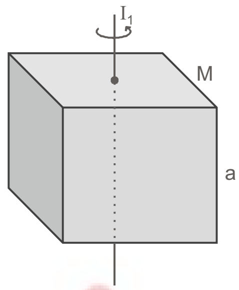
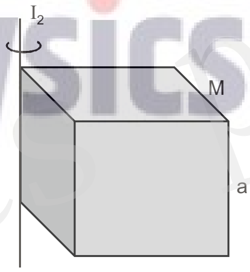
Shape of body	Rotational axis	Figure	Moment of inertia	Radius of gyration
(1) Ring M = mass R = radius	(a) Perpendicular to plane passing through centre of mass		MR^2	R
	(b) Diameter in the plane		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$
	(c) Tangent perpen dicular to plane		$2MR^2$	$\sqrt{2}R$
	(d) Tangent in the plane		$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$

(2) Disc	(a) Perpendicular to plane passing through centre of mass		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$
	(b) Diameter in the plane		$\frac{MR^2}{4}$	$\frac{R}{2}$
	(c) Tangent in the plane		$\frac{5}{4}MR^2$	$\frac{\sqrt{5}}{2}R$
	(d) Tangent perpendicular to plane		$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$
(3) Thin walled cylinder	(a) Geometrical axis		MR^2	R
	(b) Perpendicular to length passing through centre of mass		$M\left(\frac{R^2}{2}+\frac{L^2}{12}\right)$	$\sqrt{\frac{R^2}{2}+\frac{L^2}{12}}$

	(c) Perpendicular to length passing through one end		$M\left(\frac{R^2}{2}+\frac{L^2}{3}\right)$	$\sqrt{\frac{R^2}{2}+\frac{L^2}{3}}$
(4) Solid cylinder	(a) Geometrical axis		$\frac{MR^2}{2}$	$\frac{R}{\sqrt{2}}$
	(b) Perpendicular to length passing through centre of mas		$M\left(\frac{R^2}{4}+\frac{L^2}{12}\right)$	$\sqrt{\frac{R^2}{4}+\frac{L^2}{12}}$
	(c) Perpendicular to length passing through one end		$M\left[\frac{R^2}{4}+\frac{L^2}{3}\right]$	$\sqrt{\frac{R^2}{4}+\frac{L^2}{3}}$
(5) Annular disc	(a) passing through centre of mass		$\frac{M}{2}\left[R_1^2+R_2^2\right]$	$\sqrt{\frac{R_1^2+R_2^2}{2}}$
	(b) Diameter in the plane		$\frac{M[R_1^2+R_2^2]}{4}$	$\sqrt{\frac{R_1^2+R_2^2}{4}}$
(6) Hollow cylinder	(a) Geometrical axis		$M\left[\frac{R_1^2+R_2^2}{2}\right]$	$\sqrt{\frac{R_1^2+R_2^2}{2}}$

	(b) Perpendicular to length passing through centre of mass		$M\left[\frac{L^2}{12}+\frac{(R_1^2+R_2^2)}{4}\right]$	$\sqrt{\frac{L^2}{12}+\frac{R_1^2+R_2^2}{4}}$
(7) Solid sphere	(a) Along the diameter		$\frac{2}{5}MR^2$	$\sqrt{\frac{2}{5}}R$
	(b) Along the tangent		$\frac{7}{5}MR^2$	$\sqrt{\frac{7}{5}}R$
(8) Thin spherical shell	(a) Along the diameter		$\frac{2}{3}MR^2$	$\sqrt{\frac{2}{3}}R$
	(b) Along the tangent		$\frac{5}{3}MR^2$	$\sqrt{\frac{5}{3}}R$

<table>
<tr><td>(9) Hollow sphere</td><td>Along the diameter</td><td>
</td><td>$\frac{2}{5}M\left[\frac{R^5-r^5}{R^3-r^3}\right]$</td><td>$\sqrt{\frac{2}{5}\frac{(R^5-r^5)}{(R^3-r^3)}}$</td></tr>
<tr><td>(10) Thin rod</td><td>(a) Perpendicular to length passing through centre of mass</td><td>
</td><td>$\frac{ML^2}{12}$</td><td>$\frac{L}{2\sqrt{3}}$</td></tr>
<tr><td></td><td>(b) Perpendicular to length passing through one end</td><td>
</td><td>$\frac{ML^2}{3}$</td><td>$\frac{L}{\sqrt{3}}$</td></tr>
<tr><td>(11) Rectangular plate</td><td>(a) Perpendicular to length in the plane passing through centre of mass</td><td>
</td><td>$\frac{Ma^2}{12}$</td><td>$\frac{a}{2\sqrt{3}}$</td></tr>
<tr><td></td><td>(b) Perpendicular to breadth in the plane passing through centre of mass</td><td>
</td><td>$\frac{Mb^2}{12}$</td><td>$\frac{b}{2\sqrt{3}}$</td></tr>
<tr><td></td><td>(c) Perpendicular to plane passing through centre of mass</td><td>
</td><td>$\frac{M(a^2+b^2)}{12}$</td><td>$\frac{\sqrt{a^2+b^2}}{2\sqrt{3}}$</td></tr>
</table>

(12) Square plate	(a) Perpendicular to plane passing through centre of mass		$I_1 = \frac{Ma^2}{6}$	$\frac{a}{\sqrt{6}}$
	(b) Diagonal passing through centre of mass		$I_2 = I_3 \frac{Ma^2}{12}$	$\frac{a}{2\sqrt{3}}$
(13) Cube	(a) Perpendicular to plane passing through centre of mass		$I_1 = \frac{Ma^2}{6}$	$\frac{a}{\sqrt{6}}$
	(b) Perpendicular to plane passing through one end		$I_2 = \frac{2Ma^2}{3}$	$\sqrt{\frac{2}{3}}\, a$

Comparison of Rotational Motion and Linear Motion

	Linear motion	**Rotational motion**
(i)	Internal mass (m)	Moment of inertia (I)
(ii)	Displacement (Δr)	Angular displacement ($\Delta\theta$)
(iii)	Linear velocity $\vec{v} = \frac{d\vec{r}}{dt}$	Angular velocity $\omega = \frac{d\theta}{dt}$
(iv)	Linear acceleration $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$	Angular acceleration $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$
(v)	Linear momentum $\vec{p} = m\vec{v}$	Angular momentum $\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$
(vi)	Force $\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt} = \frac{md\vec{v}}{dt}$	Torque $\vec{\tau} = \vec{r} \times \vec{F} = I\vec{\alpha} = \frac{d\vec{L}}{dt} = I\frac{d\vec{\omega}}{dt}$
(vii)	Impulse $\vec{F}.\Delta t = \Delta\vec{p}$	Angular impulse $\vec{\tau}.\Delta t = \Delta\vec{L}$
(viii)	Kinetic energy $\vec{E} = \frac{1}{2}mv^2$	Rotational kinetic energy $E_{rot} = \frac{1}{2}I\omega^2$
(ix)	Work $W = \vec{F}.\Delta\vec{r}$	Work $W = \vec{\tau}.\Delta\vec{\theta}$
(x)	Power $P = \frac{dW}{dt} = \frac{\vec{F}\cdot\Delta\vec{r}}{\Delta t} = \vec{F}\cdot\vec{v}$	Power $P = \frac{\vec{\tau}\cdot\vec{\theta}}{\Delta t} = \vec{\tau}\cdot\vec{\omega}$
(xi)	Relation between p and E, $p = \sqrt{2mE}$	Relation between E_{rot} I and L, $L = \sqrt{2IE_{rot}}$

(xii) Equations of motion

(a) $v = u + at$

(b) $s = ut + \frac{1}{2} at^2$

(c) $v^2 = u^2 + 2as$

(d) $S_{nth} = u + \frac{1}{2}$ a (2n–1)

Equations of rotational motion

(a) $\omega_2 = \omega_1 + \alpha t$

(b) $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$

(c) $\omega_2^2 = \omega_1^2 + 2\alpha\theta$

(d) $S_{nth} = \omega_1 + \frac{1}{2}$ α (2n–1)

CHECK Point

- **Will two spheres of equal mass, one solid and the other hollow have equal moment of inertia ? Explain.**

Solution

Though the masses of two spheres are equal, the M.I. of the solid and the hollow spheres will not be equal. The M.I. of the hollow-sphere will be more than that of the solid sphere. It is because, the mass is distributed at larger distances from the axis of rotation in case of the hollow sphere.

ILLUSTRATION : 7

Four point masses each *m* are placed at the vertices of a square of side *a,* find moment of inertia of the system about (a) axis as one of the sides (b) diagonal AC (c) axis through any vertex and perpendicular to the plane of the square (d) axis through the centre of the square and perpendicular to the plane of the square.

SOLUTION :

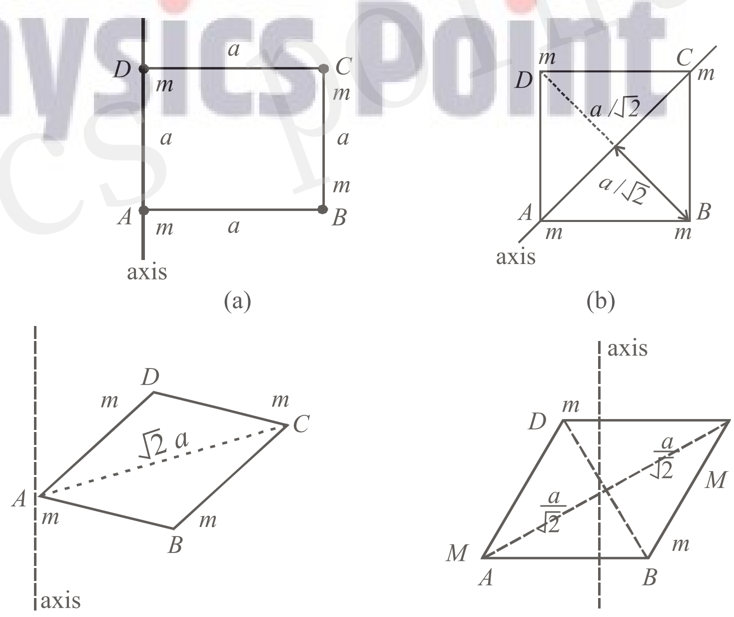
(a) $I = M(0)^2 + M(0)^2 + MA^2 + MA^2 = 2MA^2$

(b) $I = m(0)^2 + m\left(\frac{a}{\sqrt{2}}\right)^2 + m\left(\frac{a}{\sqrt{2}}\right)^2 + m(0)^2$

$I = ma^2$

(c) $I = m(0)^2 + m(a)^2 + m\ m(\sqrt{2}\ a)^2 + m(a)$

$I = 4ma^2$



(a) (b) (c) (d)

(d) $I = m\left(\frac{a}{\sqrt{2}}\right)^2 + m\left(\frac{a}{\sqrt{2}}\right)^2 + m\left(\frac{a}{\sqrt{2}}\right)^2 + m\left(\frac{a}{\sqrt{2}}\right)^2$

$I = 2ma^2$

ILLUSTRATION : 8

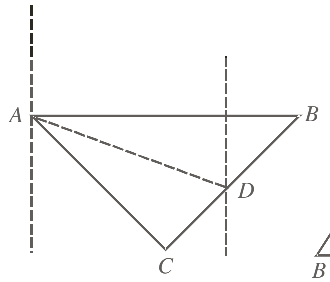
Three thin rods each of mass *m* and length *l* are joined to form an equilateral triangle, find moment of inertia of the system about an axis through (a) any vertex (b) centroid of the triangle. Given axis is perpendicular to the plane of triangle.

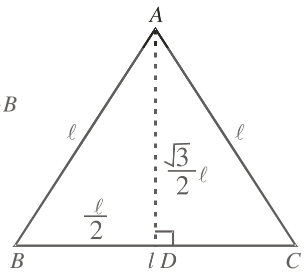
SOLUTION :

(a) M.I. of rods AB and AC about A will be each $\frac{Ml^2}{3}$, for the rod BC we apply parallel axis theorem to find its MI about A.

$$I_A = \frac{M\ell^2}{3} + \frac{M\ell^2}{3} + \left(\frac{M\ell^2}{12} + \left(\frac{\sqrt{3}}{2}\ell\right)^2\right)$$

$$I_A = \frac{3}{2}M\ell^2$$





(b) Distance of centroid from the centre of mass of each rod is $\frac{\ell}{2\sqrt{3}}$ so in order to find moment of inertia about G we can apply parallel axis theorem.

$$I_G = 3\left(\frac{M\ell^2}{12} + M\left(\frac{\ell}{2\sqrt{3}}\right)^2\right)$$

$$I_G = \frac{M\ell^2}{2}$$

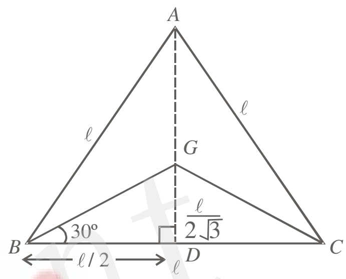
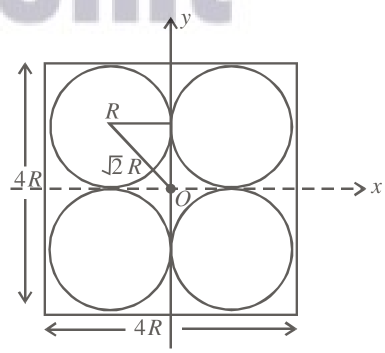


ILLUSTRATION : 9

There is a square plate of side $4R$, four holes each of radius R are cut from it as shown in figure, find moment of inertia of the remaining portion about z-axis, if mass of plate is m.

SOLUTION :



Mass of each hole $= \frac{M}{16R^2} \times \pi R^2 = \frac{\pi M}{16}$

M.I. of remaining portion $= I_{plate} - 4\, I_{\text{hole about (o)}}$

$$= \frac{M}{12}\left((4R)^2 + (4R)^2\right) - 4\left\{\frac{M\pi}{16}\left(\frac{R^2}{2} + 2R^2\right)\right\}$$

$$= \frac{M}{12}(32R^2) - 4\left\{\frac{M\pi}{16} \times \frac{5R^2}{2}\right\} = \left(\frac{8}{3} - \frac{10\pi}{16}\right)MR^2$$

ILLUSTRATION : 10

A flywheel of radius 20 cm starts from rest and has a constant angular acceleration of 60 rad/s^2. Find

(a) the magnitude of the linear acceleration of a point on the rim after 0.15s.

(b) the number of revolutions completed in 0.25s.

SOLUTION :

(a) The tangential acceleration is constant and is given by

$a_t = \alpha r = (60 \text{ rad/s}^2)\ (0.2\text{m}) = 12 \text{ m/s}^2$

In order to calculate the radial acceleration we first need to find the angular velocity at the given time. From equation,

$\omega = \omega_0 + \alpha t = 0 + (60 \text{ rad/s}^2)\ (0.15 \text{ s}) \Rightarrow \omega = 9 \text{ rad/s}$

Then using equation , we have $a_r = \omega^2 r = (81 \text{ rad}^2/\text{s}^2)\ (0.2 \text{ m}) = 16.2 \text{ m/s}^2$

The magnitude of the net linear acceleration is, $a = \sqrt{a_r^2 + a_t^2} = 20.2 \text{ m/s}^2$

(b) From equation, $\theta = 0 + \frac{1}{2}\alpha t^2 = \frac{1}{2}(60 \text{ rad/s}^2)\,(0.25\text{s})^2 = 1.88 \text{ rad}$

This corresponds to (1.88 rad) (1 rev/2π rad) = 0.3 rev.

ILLUSTRATION : 11

A turn table rotates with constant angular acceleration of 2 rad/s^2 about a fixed vertical axis through its centre and perpendicular to its plane. A coin is placed on it at a distance of 1m from the axis of rotation. The coin is always at rest relative to the turntable. If at $t = 0$ the turntable was at rest, then find the total acceleration of the coin after one second.

SOLUTION :

After 1 second angular velocity of the turntable and hence that of the coin about the axis of rotation is

$\omega = 0 + 2(\text{rad/s}^2) \times 1\text{ s} = 2\text{ rad/s}$; $a_T = \alpha r = (2\text{ rad/s}^2) \times 1m = 2\text{ m/s}^2$

$a_R = \omega^2 r = (2\text{ rad/s})^2 \times 1\text{ m} = 4\text{ m/s}^2$ $\therefore$ total acceleration $a = \sqrt{a_T^2 + a_R^2} = 2\sqrt{5}\text{m/s}^2$

ILLUSTRATION : 12

Calculate the torque developed by an airplane engine whose output is 2000 hp at an angular velocity of 2400 rev/min.

SOLUTION :

Here, $\omega = 2\pi n = 2\pi\ (2400/60) = 80\pi$ rad/s

Work done by torque = (torque) × (angular displacement)

Power = work done per sec = $\tau\dfrac{\Delta\theta}{\Delta t}$

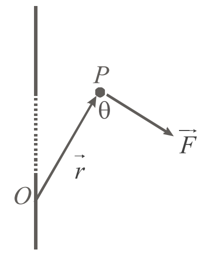
Power = $\tau\omega \Rightarrow \tau = \dfrac{2000 \times 746}{80\pi} = 5937$ Nm

TORQUE

Torque is the turning or twisting action on a body about the axis of rotation due to a force $\vec{F}$.

Only torque can bring a change in the state of rotation of a body.

If force $\vec{F}$ is acting at any point in the body whose position vector relative to any arbitrary point on the axis of rotation is $\vec{r}$, then torque of the force on the body about the axis of rotation is given by the expression



$\vec{\tau} = (\vec{r} \times \vec{F}).\hat{n}$, where $\hat{n}$ is a unit vector along the axis.

Magnitude of the torque is equal to the product of the force and the shortest distance between the force and the axis of rotation.

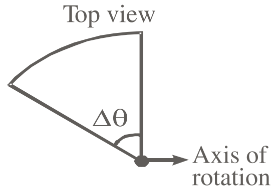
If $\vec{r}$ and $\vec{F}$ are coplanar then

$|\vec{\tau}| = rF\sin\theta$ where θ = angle between $\vec{r}$ and $\vec{F}$.

Relation Between Torque and Angular Acceleration

The angular speed of a body is changed from ω to $(\omega + \Delta\omega)$ in very small time interval Δt.

If angular displacement of the body during this time interval is $\Delta\theta = (\omega\Delta t)$ then



$\tau\Delta\theta = \dfrac{1}{2}I(2\omega\Delta\omega)$, [from work energy theorem]

where $\tau =$ average torque during this interval of time

$\tau\ \theta$ = K.E = ΔK. E. (of rotation) $\Rightarrow \tau\omega\Delta t = I\omega\Delta\omega$ (as $\theta = \omega\Delta t$)

$\Rightarrow \tau = I\dfrac{\Delta\omega}{\Delta t}$ $\Rightarrow \tau = I\bar{\alpha}$

Similarly, instantaneous torque is given by $\tau = I\,\alpha$.

Power delivered by torque: $P = \tau.\omega = \dfrac{\Delta E_{rot}}{\Delta t}$ i.e., the rate of change of rotational kinetic energy of a body.

Rotational kinetic energy, $E_{rot} = \dfrac{1}{2}I\omega^2 = \dfrac{L^2}{2I}$ where, L = angular momentum.

Rigid body in Equilibrium

A rigid body is in equilibrium, if it has zero translational acceleration and zero angular acceleration. In equilibrium, the sum of the externally applied forces is zero, and the sum of the externally applied torque is zero.

i.e., $\Sigma F_x = 0, \Sigma F_y = 0$ and $\Sigma \vec{\tau} = 0$

Principle of Moments for a Lever

Load × load arm = effort × effort arm

Mechanical advantage (M.A.) of lever $\dfrac{\text{load}}{\text{effort}} = \dfrac{\text{effort arm}}{\text{load arm}}$

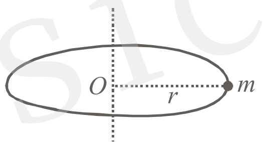
Applications of the conditions of equilibrium to a rigid body:

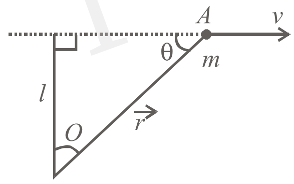
1. Select the object to which the equations for equilibrium are to be applied.
2. Draw a free body diagram (FBD) that shows all the external forces acting on the object, each force with its proper direction.
3. Choose a convenient set of x, y axes and resolve all forces into components that lie along these axes.
4. Apply the equations that specify the balance of forces at equilibrium $\Sigma F_x = 0$ and $\Sigma F_y = 0$.
5. Select a convenient axis of rotation. Identify the point where each external force acts on the object, and calculate the torque produced by each force about the axis of rotation. Set the sum of the torques about this axis equal to zero $\Sigma\tau = 0$.
6. Solve the equations in step 4 and 5 for the desired unknown quantities.

ANGULAR MOMENTUM

In translational motion the measure of quantity of motion possessed by a body is linear momentum and the physical quantity analogous to it in rotational motion is angular momentum, it is represented by L and it is a vector quantity.

1. **Angular momentum of a particle about a point :** Consider a particle of mass m moving with velocity v and there is fixed point O about which we want to find its angular momentum.





Angular momentum $(\vec{L})$ of the particle will be given by moment of its linear momentum

i.e., $\vec{L} = \vec{r} \times \vec{p}$ where $\overrightarrow{OA} = \vec{r}$, $\vec{P} = m\vec{v}$

$\vec{L} = m(\vec{r} \times \vec{v})$

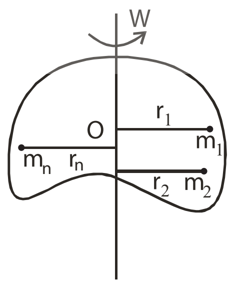
or $L = (r \sin\theta)\, mv$ $\left(\sin\theta = \dfrac{l}{r}\right)$

So, $L = mvl$ (l is perpendicular distance between line of action of linear momentum and point O)

From the above discussion, angular momentum of a particle of mass m moving in a circular path of radius r with speed v is given by :

$L = mvr$ or $L = m\omega r^2$ (as $v = \omega r$)

2. **Angular momentum of a rigid body :** As we know that a rigid body consists of large number of particle, so its angular momentum will be the sum of individual angular momentum about that fixed axis. Consider a rigid body rotating about an axis with angular speed 'ω' then every particle on this rigid body will move in a circular path whose centre will be on this axis, hence



$L = m_1\omega r_1^2 + m_2\omega r_2^2 + ...+ m_n\omega r_n^2$

$L = (m_1 r_1^2 + m_2 r_2^2 + ...)\omega$

$L = I\omega$

Relation Between Torque (τ) and Angular Momentum (L)

We have already seen that

$$L = I\omega$$

Differentiating it w.r.t. time we get

$$\frac{dL}{dt} = I\frac{d\omega}{dt} = I\,\alpha \qquad (\alpha = \text{angular acceleration})$$

$$\frac{dL}{dt} = \tau \qquad (\text{As } \tau = I\,\alpha)$$

CONSERVATION OF ANGULAR MOMENTUM

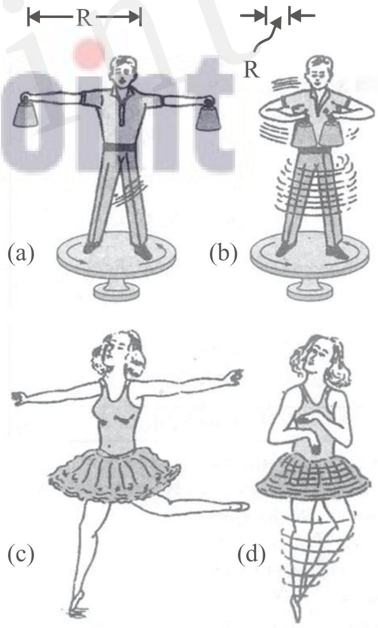
Suppose on a system of particles of a rigid body no external force is acting then its angular momentum remains conserved, this is known as conservation of angular momentum.

We know, $\frac{dL}{dt} = \tau$ if $\tau = 0$

$$dL = 0$$

$$L_{\text{initial}} = L_{\text{final}} \quad \text{or} \quad I_{\omega} = \text{constant.}$$

Applications of Law of Conservation of Angular Momentum



Following are examples of some observed physical phenomena, which can be explained on the basis of the law of conservation of angular momentum:

1. *The angular velocity of a planet revolving in an elliptical orbit around the sun increases, when it comes near the sun and vice-versa.* When the planet moving along its elliptical orbit is near the sun, its moment of inertia about the axis through the sun decreases ande therefore its angular speed increases. On the other hand, when it is far away from the sun, its moment of inertia increases and hence angular speed decreases.
2. *The speed of rotation of a person with some weights in his hands and standing on a rotating turn-table increases, as he draws the weights of his chest.* Suppose that a person is standing on a rotating turn-table with some heavy weights in his hands stretched out as shown in Fig. (a) As he draws his hands to the chest Fig. (b), his angular speed at once increases. It is because, the moment of inertia decreases on drawing the hands to the chest and as a result, the angular speed increases.
3. *An ice-skater or a ballet-dancer can increase her angular velocity by folding her arms and bringing the stretched leg close to the other leg.* When her hands and a leg are stretched outwards Fig. (c) her moment of inertia about the axis of rotation is large and hence the angular speed is quite small. By folding her arms and bringing the stretched by close to the other leg fig (d) she decreases her moment of inertia and thereby her angular speed increases.
4. *The speed of the inner layers of the whirl-wind about its axis in a tarnado is alarmingly high.* When a whirl-wind is formed, the air rotates in the form of concentric layers. Since the moment of inertia of the inner layers is quite small, these layers rotate with alarmingly high speed.
5. *A diver jumping from the spring board sometimes exhibits sommersaults in air before touching the water surface.* After leaving the spring board, he curls his body by rolling the arms and the legs inwards. Due to this, his moment of inertia decreases and he spins in mid air with large angular speed. As he is about to touch the water surface, he stretches out his limbs. This increases his moment of inertia and the diver enters the water at a gentle speed.

CHECK *Point*

- **How will you distinguish between a hard boiled egg and a raw egg by spinning each on a table top?**

Solution

The egg which spins faster will be the hard boiled egg. It is because, a hard boiled egg will spin (rotate) more or less as a rigid body, whereas a raw egg will not do so. In case of a raw egg, its matter in the liquid state moves away from the axis of rotation, thereby increasing its moment of inertia. As M.I. of the raw egg is more, the angular acceleration produced will be lasser, when the same torque is applied in both the cases to set them spinning.

ILLUSTRATION : 13

Two discs of moment of inertia I_1 and I_2 about their respective axes (normal to the disc and passing through the centre) and rotating with angular speeds ω_1 and ω_2 are brought into contact face to face with their axis of rotation coinciding. Calculate (i) angular speed of the two disc system (ii) show that the kinetic energy of the combined system is less than the sum of the initial kinetic energies of the two discs. How do you account for this?

SOLUTION :

As no external force acts on the system we can apply conservation of energy principle

Initial angular momentum, $L_i = I_1\omega_1 + I_2\omega_2$

Final angular momentum, $L_f = (I_1 + I_2)\omega$

As $L_f = L_i$

$$\omega(I_1 + I_2) = (I_1\omega_1 + I_2\omega_2) \Rightarrow \omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$$

Now, Initial KE, $K_i = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2$

Final KE, $K_f = \frac{1}{2}(I_1 + I_2)\omega^2$

So, the change in KE, $\Delta K = K_f - K_i = \frac{1}{2}(I_1 + I_2)\omega^2 - \left(\frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2\right)$

$$\Delta K = \frac{1}{2}(I_1 + I_2)\left(\frac{I_1\omega_1^2 + I_2\omega_2^2}{I_1 + I_2}\right)\left(\frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2\right)$$

$$\Delta K = -\frac{I_1 I_2(\omega_1 - \omega_2)^2}{2\,(I_1 + I_2)}$$

As ΔK is –ve it shows that kinetic energy is not conserved it has decreased. This is due to the work done against friction between the two discs.

ILLUSTRATION : 14

A uniform rod of mass m and length l is initially in the vertical position with its lower end clamped. It is then allowed to fall under gravity, find its angular speed when it is (i) in the horizontal position and (ii) lower most position, also find linear speed of the free end in both the cases. Neglect friction at the clamp.

SOLUTION :

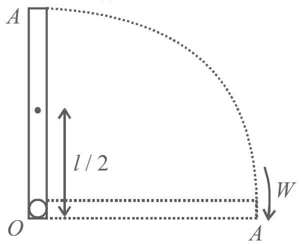
Here we can apply conservation of mechanical energy principle as no energy is lost against friction.

(i) Initially rod has potential energy which converts into rotational kinetic energy

i.e. $\frac{1}{2}I\omega^2 = mg\left(\frac{\ell}{2}\right)$

$\frac{1}{2}\cdot\frac{1}{3}ml^2\omega^2 = mg\frac{\ell}{2} \Rightarrow \omega = \sqrt{\frac{3g}{\ell}}$

Linear velocity of end $A = \omega l = \sqrt{3g\ell}$

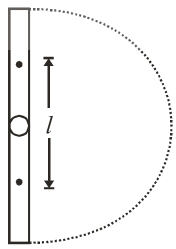


(ii) Using the same principle again

$\frac{1}{2}I\omega^2 = mgl$

$\frac{1}{2}\times\frac{1}{3}ml.\omega^2 = mgl$

$\omega = \frac{6g}{\ell}$ and $V = \omega l = \sqrt{6\ell g}$



ROLLING MOTION

When a body performs translatory as well as rotatory motion, then it is known as rolling motion.

In this type of motion, axis of rotation is not stationary. If a body rotates about an axis with angular velocity ω then with respect to the axis of rotation linear velocity of any particle in the body at a position $\vec{r}$ from the axis of rotation is equal to

$$\vec{v} = \vec{\omega} \times \vec{r}$$

If the axis of rotation also moves with velocity $\vec{v}_0$ then net velocity of the particle relative to stationary frame will be

$$\vec{v} = \vec{\omega} \times \vec{r} + \vec{v}_0 .$$

If a regular rigid body (like sphere, disc or ring) is spinned to a certain angular velocity and placed on a surface such that plane of rotation must be perpendicular to the surface and a velocity $\vec{v}_0$ is given to the centre of mass of the body then net velocity of the particle of the body at a distance r from the centre is equal to $(\vec{\omega} \times \vec{r} + \vec{v}_0)$.

"A body is said to be in pure rolling motion if relative velocity between the point of contacts is zero".

If velocity of the surface on which the body has to roll is $\vec{v}_s$ and R be the radius of the body then, for rolling

$$\vec{\omega} \times \vec{R} + \vec{v}_0 = \vec{v}_s$$

If $\vec{v}_s = 0$ then the condition for rolling is $v_0 = \omega R$.

The **steps for analyzing combined rotation and translation** are as follows:

(i) List the external forces acting on the body.

(ii) The vector sum of external forces divided by the mass of the body gives the acceleration of the centre of mass.

Acceleration of centre of mass, $a_{cm} = \dfrac{\sum F_{ext}}{M}$

(iii) Then find the torque of external forces and the moment of inertia of the body about a line through the centre of mass and perpendicular to the plane of motion of the particles.

If motion of the body is studied from non-inertial frame of reference having an acceleration a in a fixed direction with respect to an inertial frame, we have to apply a pseudo force (-ma) to each particle. These pseudo forces produce a pseudo torque about the axis. In such a case we do not hope $\tau_{ext} = I\alpha$ to hold.

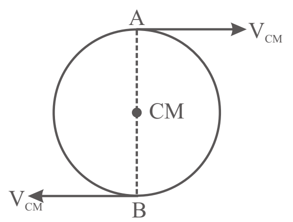
But there exists a very special and very useful case when $\tau_{ext} = I\alpha$ does hold even if the angular acceleration α is measured from a non-inertial frame A. That special case is when axis of rotation passes through the centre of mass.

Take the origin at the centre of mass. The total torque of the pseudo forces is,

$$\sum \vec{r}_i \times (-m_i \vec{a}) = -\left(\sum m_i \vec{r}_i\right) \times \vec{a}$$

where r_i is the position vector of the i^{th} particle as measured from the centre of mass.

But, $\sum m_i \vec{r}_i = 0 \Rightarrow$ Pseudo torque is zero and we get, $\vec{\tau}_{ext} = I_{cm}\vec{\alpha}$



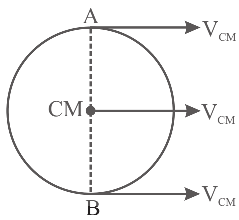
(A) Pure rotational motion

In this case,

Tangential velocity at point A and B

$V_A = V_B = R\omega$

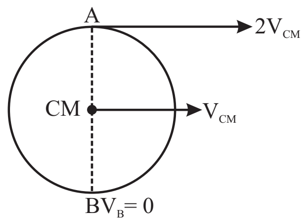
Velocity of CM $V_{CM} = 0$



(B) Pure translational motion

In this case,

$V_A = V_B = V_{CM} = R\omega = V$



(C) Rolling motion = Pure rotational + Pure translational motion

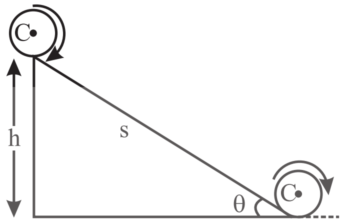
In this case, the tangential velocity at any given point is the vector sum of the velocities in pure rotational and pure translational motion at that point.

Hence, $V_A = 2V_{CM}$, $V_{CM} = V = R\omega$ and $V_B = 0$

In case of rolling motion, because the body is not slipping, the instantaneous velocity at the point of contact B with surface is zero.

MOTION ON AN INCLINED PLANE

(A) For translational motion only (without rotation)



$\frac{1}{2}mv^2 = mgh,$

Velocity, $v = \sqrt{2gh} = \sqrt{2gs\sin\theta}$

Acceleration, $a = g\sin\theta,$ Time $t = \left(\frac{2s}{g\sin\theta}\right)^{1/2}$

(B) For rolling motion (Translational + Rotational)

From law of conservation of mechanical energy, $\frac{1}{2}mv^2 + \frac{1}{2}mk^2\left[\frac{v^2}{R^2}\right]$

or, $\frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right) = mgh$

Velocity $v = \left[\frac{2gh}{\left(1+\frac{K^2}{R^2}\right)}\right]^{1/2} = \left[\frac{2gs\sin\theta}{\left(1+\frac{K^2}{R^2}\right)}\right]^{1/2}$; Acceleration, $a = \frac{g\sin\theta}{\left(1+\frac{K^2}{R^2}\right)}$; Time, $t = \sqrt{\frac{2s}{a}} = \left[\frac{2s\left(1+\frac{K^2}{R^2}\right)}{g\sin\theta}\right]^{\frac{1}{2}}$

Acceleration velocity, time	**Rolling without slipping**	**Sliding without rolling (Translational motion)**
Acceleration	$a = \frac{g\sin\theta}{(1+K^2/R^2)}$	$a = g\sin$
Velocity at bottom	$v = \left\{\frac{2gh}{(1+K^2/R^2)}\right\}^{\frac{1}{2}}$	$v = \sqrt{2gh}$
Time taken	$t = \left\{\frac{2s(1+K^2/R^2)}{g\sin\theta}\right\}^{1/2}$	$t = \sqrt{\frac{2s}{g\sin\theta}}$

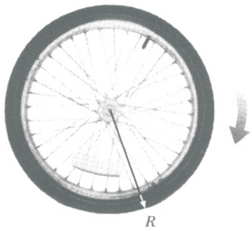
The comparison reveals that : $a_{rolling} < a_{sliding}$, $v_{rolling} < v_{sliding}$, $t_{rolling} > t_{sliding}$

Acceleration, velocity and time of descend for different regular shaped bodies rolling down an inclined plane.

S.No.	**Body**	$a = \frac{g\sin\theta}{\left(1+I/Mr^2\right)}$	$v = \sqrt{\frac{2gh}{1+\left(I/Mr^2\right)}}$	$t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left(1+\frac{I}{Mr^2}\right)}$
1.	Solid sphere	$\frac{5}{7}g\sin\theta$	$\sqrt{\frac{10h}{5g}}$	$\frac{1}{\sin\theta}\sqrt{\frac{14h}{5g}}$
2.	Hollow sphere	$\frac{3}{5}g\sin\theta$	$\sqrt{\frac{6gh}{5}}$	$\frac{1}{\sin\theta}\sqrt{\frac{10h}{5g}}$
3.	Disc	$\frac{2}{3}g\sin\theta$	$\sqrt{\frac{4gh}{3}}$	$\frac{1}{\sin\theta}\sqrt{\frac{3h}{5g}}$
4.	Solid cylinder	$\frac{2}{3}g\sin\theta$	$\sqrt{\frac{4gh}{3}}$	$\frac{1}{\sin\theta}\sqrt{\frac{3h}{g}}$
5.	Hollow cylinder	$g\sin\theta$	$\sqrt{gh}$	$\frac{1}{\sin\theta}\sqrt{\frac{4h}{g}}$
6.	Ring	$g\sin\theta$	$\sqrt{gh}$	$\frac{1}{\sin\theta}\sqrt{\frac{4h}{g}}$

CHECK Point

- **When a wheel of radius R rotates about a fixed axis as in figure (a) do all points on the wheel (including the spokes) have the same angular speed? (b) Do they all have the same tangential speed? (c) If the angular speed is constant and equal to ω, describe the tangential speeds and total translational accelerations of the points located at $r = 0$, $r = R/2$ and $r = R$, where the points are measured from the centre of the wheel.**



Solution

(a) Yes. All points on the wheel make one full rotation in the same time interval.

(b) No. Points farther from the rotation axis have higher tangential speeds.

(c) The point at $r = 0$ has zero tangential speed and zero acceleration; a point at $r = R/2$ has a tangential speed $v = R\omega/2$ and a total translational acceleration equal to the centripetal acceleration $v^2/(R/2) = R\omega^2/2$ (the tangential acceleration is zero at all points because ω is constant). A point on the rim at $r = R$ has a tangential speed $v = R\omega$ and a total translational acceleration $R\omega^2$.

ILLUSTRATION : 15

A force F acts tangentially at the highest point of a disc of mass m kept on a rough horizontal plane. If the disc is in pure rolling motion find acceleration of centre of mass.

SOLUTION :

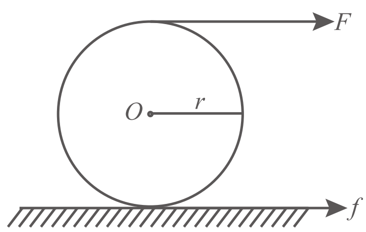
As the point of contact in this case would have a tendency to slip backward 'static friction' will be in forward direction.

Now, $F + f = ma$ ...(i) Using $\Sigma F = ma$)

$Fr - fr = I\alpha$ (Using $\Sigma\tau = I\alpha$)

or $Fr - fr = \frac{1}{2}mr^2\alpha$ $\left(\alpha = \frac{a}{r} \text{ as it is pure rolling}\right)$

$F - f = \frac{1}{2}ma$ ...(ii)



From eqs. (i) and (ii)

$$2F = \frac{3}{2}ma \Rightarrow a = \frac{4F}{3m}$$

ILLUSTRATION : 16

A disc rotating about its axis with angular speed ω is placed lightly (without any translational pull) on a frictionless table. The radius of the disc is R. What are the linear velocities of the points A, B, C and D on disc shown. Will the disc roll in the direction indicated?

SOLUTION :

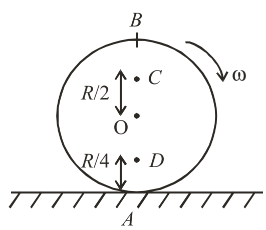
As friction is absent disc will not roll but keep rotating at the same point with angular speed ω.

Using $v = \omega r$

$$v_A = v_B = \omega R$$

$$v_C = \frac{\omega R}{2}$$

$$v_D = \omega(R - R/4) = \frac{3\omega R}{5}$$



MISCELLANEOUS SOLVED EXAMPLES

1. Four particles of masses 1 kg, 2 kg, 3 kg and 4 kg are placed at the four vertices *A*, *B*, *C* and *D* of a square of side 1 m. Find the position of centre of mass of the particles.

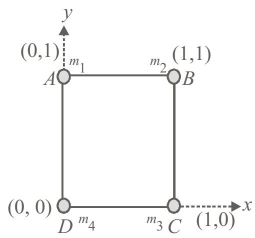
Sol. Assuming *D* as the origin, *DC* as *x*-axis and *DA* as *y*-axis, we have

$m_1 = 1$ kg, $(x_1, y_1) = (0, 1\text{m})$

$m_2 = 2$ kg, $(x_2, y_2) = (1\text{m}, 1\text{m})$

$m_3 = 3$ kg, $(x_3, y_3) = (1\text{m}, 0)$

$m_4 = 4$ kg, $(x_4, y_4) = (0, 0)$



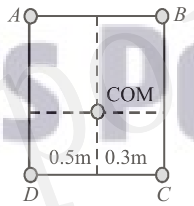
Coordinates of their COM are calculated as follows.

$$X_{COM} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$$

$$= \frac{(1)(0) + 2(1) + 3(1) + 4(0)}{1+2+3+4} = \frac{5}{10} = \frac{1}{2}\text{m} = 0.5\text{m}$$

Similarly, $$Y_{COM} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4}{m_1 + m_2 + m_3 + m_4}$$

$$= \frac{(1)(1) + 2(1) + 3(0) + 4(0)}{1+2+3+4} = \frac{3}{10}\text{m} = 0.3\text{m}$$

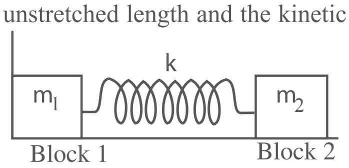


$\therefore \quad (x_{COM}, y_{COM}) = (0.5\text{m}, 0.3\text{ m})$

2. Two blocks of masses m_1 and m_2 connected by a weightless spring of stiffness *k* rest on a smooth horizontal plane. Block 2 is shifted a small distance *x* to the left and released. Find the velocity of the centre of mass of the system after block 1 breaks off the wall.

Sol. We know that the potential energy of compression $= \frac{1}{2}kx^2$

When the block m_1 breaks off from the wall, the spring has its unstretched length and the kinetic energy of the block m_2 is given by



$$\frac{1}{2}m_2v_2^2 = \frac{1}{2}kx^2 \Rightarrow v_2^2 = \frac{kx^2}{m_2} \Rightarrow v_2 = x\sqrt{\frac{k}{m_2}}$$

For centre of mass, $x_{COM} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}$

The distances x_1 and x_2 are measured from the wall.

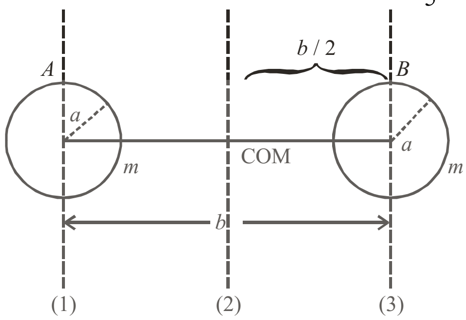
$$\frac{dx_{COM}}{dt} = \frac{m_1}{m_1+m_2}\frac{dx_1}{dt} + \frac{m_2}{m_1+m_2}\frac{dx_2}{dt}$$

At start $\dfrac{dx_1}{dt} = 0 \;\therefore\; \dfrac{dx_{COM}}{dt} = \dfrac{m_2}{m_1+m_2}v_2 = \dfrac{m_2x}{m_1+m_2}\sqrt{\dfrac{k}{m_2}}$

Velocity of centre of mass of system $= \dfrac{x\sqrt{km_2}}{m_1+m_2}$

3. Two solid spheres each of mass *m* and radius *a* are joined by a massless rod such that distance between the spheres is *b*. Find M.I. of the system about axis (a) through any sphere and (b) through the COM of the system.

Sol. (a) We know that M.I. of a solid sphere about its diameter is $\frac{2}{5}Ma^2$ so, MI of sphere B about axis (3) is $\frac{2}{5}Ma^2$ and about (1) will be $\left(\frac{2}{5}Ma^2 + mb^2\right)$.



So the MI of the system about axis (1) will be

$$I_1 = \frac{2}{5}ma^2 + \left(\frac{2}{5}ma^2 + mb^2\right) = \frac{4}{5}ma^2 + mb^2$$

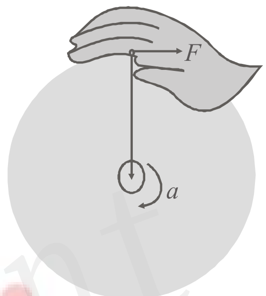
(b) MI about axis (2) is

$$I_2 = 2\left(\frac{2}{5}ma^2 + m\left(\frac{b}{2}\right)^2\right) \quad I_2 = \frac{4}{5}ma^2 + \frac{mb^2}{2}$$

4. A person can spin a grindstone by placing his hand on its edge through a part of revolution.

(a) What is the work done by a force of 200 N through a rotation of 1 rad (57.3°). The force is applied perpendicular to the grindstone at r = 0.320 m.

(b) What is the final angular velocity if the grindstone has a mass of 85.0 kg?



Sol. (a) Net work done :

$W = \tau\Delta\theta = rF\Delta\theta$ = (0.320) (200) (1.00)

= 64.0 J

(b) From kinematic relation, $\omega^2 = \omega_0^2 + 2\alpha\theta$

If the grindstone starts from rest, $\omega_0 = 0$

$\Rightarrow_0 = \sqrt{2\alpha\theta}$

where $\alpha = \frac{\tau}{I}$, $I = MR^2$

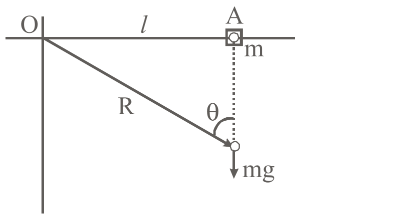
$\vec{\tau} = rF$ = (0.320) (200) = 64.0 Nm

$I = \frac{1}{2}MR^2 = (0.5)(85.0)(0.320)^2$ = 4.352 kgm^2

$\alpha = \frac{T}{I} = \frac{64.0}{4.352}$ = 14.7 rad/s^2

Then, $\omega = (2\alpha\theta)^{1/2} = [2(14.7)(1.00)]^{1/2}$ = 5.42 rad/s

5. A particle of mass m is dropped at point A as shown in figure. Find the torque about O.



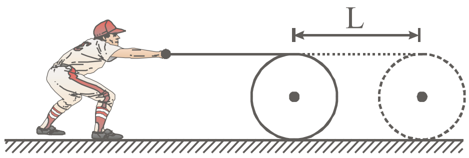
Sol. Torque $\vec{\tau} = \vec{R} \times \vec{F}$

$\Rightarrow \vec{\tau} = RF\sin\theta\,\hat{n} = (R\sin\theta)F\hat{n}$

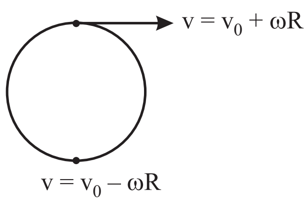
$\Rightarrow \tau = \ell mg$

The direction of torque is inside the paper or in other words, rotation about O is clockwise.

6. A cylindrical drum, pushed along by a board rolls forward on the ground. There is no slipping at any contact. Find the distance moved by the man who is pushing the board, when axis of the cylinder covers a distance L.



Sol. Let v_0 be the linear speed of the axis of the cylinder and ω be its angular speed about the axis. As it does not slip on the ground hence $\omega = \frac{v_0}{R}$. Where R is the radius of the cylinder.



Speed of the topmost point is $v = v_0 + \omega R = 2v_0$

Since time taken by the axis to move a distance L is equal to $t = L/v_0$.

In the same interval of time distance moved by the topmost point is

$$s = 2v_0 \times \frac{L}{v_0} = 2L$$

As there is no slipping between any point of contact hence distance moved by the man is $2L$.

ADVANCED EXERCISE
BASED ON CONNECTING TOPICS

DIRECTIONS (Qs. 1-36): *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

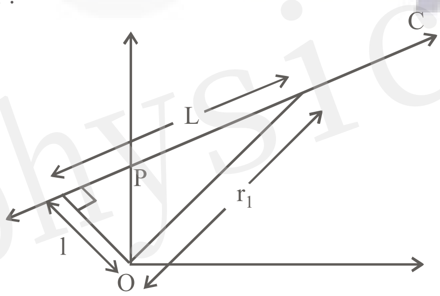
1. Two particles *A* and *B*, initially at rest, moves towards each other under a mutual force of attraction. At the instant when the speed of *A* is *v* and the speed of *B* is 2 *v*, the speed of centre of mass is

(a) zero (b) v
(c) $1.5\ v$ (d) $3\ v$

2. What is the moment of inertia of a solid sphere of density ρ and radius *R* about its diameter?

(a) $\frac{105}{176}R^5\rho$ (b) $\frac{105}{176}R^2\rho$
(c) $\frac{176}{105}R^5\rho$ (d) $\frac{176}{105}R^2\rho$

3. A particle of mass m moves along line PC with velocity v as shown. What is the angular momentum of the particle about P?



(a) mvL (b) mvl
(c) mvr (d) zero

4. A body having moment of inertia about its axis of rotation equal to 3 kg-m^2 is rotating with angular velocity equal to 3 rad/s. Kinetic energy of this rotating body is the same as that of a body of mass 27 kg moving with a speed of

(a) 1.0 m/s (b) 0.5 m/s
(c) 1.5 m/s (d) 2.0 m/s

5. A particle moves in a circle of radius 0.25 m at two revolutions per second. The acceleration of the particle in metre per second^2 is

(a) π^2 (b) $8\pi^2$
(c) $4\pi^2$ (d) $2\pi^2$

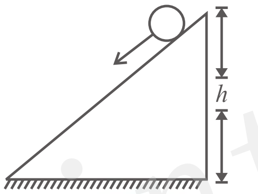
6. A particle of mass *m* is moving in a plane along a circular path of radius *r*. Its angular momentum about the axis of rotation is *L*. The centripetal force acting on the particle is

(a) L^2/mr (b) $L^2 m/r$
(c) $L^2/m r^3$ (d) $L^2/m r^2$

7. Angular momentum of a system of a particles changes, when

(a) force acts on a body
(b) torque acts on a body
(c) direction of velocity changes
(d) None of these

8. A solid cylinder of mass *m* & radius *R* rolls down an inclined plane (as shown in figure)without slipping. The speed of its C.M. when it reaches the bottom is



(a) $\sqrt{2gh}$ (b) $\sqrt{4gh/5}$
(c) $\sqrt{4gh/5}$ (d) $\sqrt{4gh}$

9. Angular momentum is

(a) a polar vector (b) an axial vector
(c) a scalar (d) none of these

10. If a running boy jumps on a rotating table, which of the following is conserved?

(a) Linear momentum
(b) K.E
(c) Angular momentum
(d) None of these

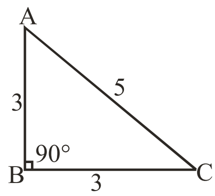
11. A gymnast takes turns with her arms & legs stretched. When she pulls her arms and legs

(a) the angular velocity decreases
(b) the moment of inertia decreases
(c) the angular velocity stays constant
(d) the angular momentum increases

12. Two identical particles move towards each other with velocity 2*v* and *v* respectively. The velocity of centre of mass is

(a) v (b) $v/3$
(c) $v/2$ (d) zero

13. ABC is a triangular plate of uniform thickness. The sides are in the ratio shown in the figure. I_{AB}, I_{BC} and I_{CA} are the moments of inertia of the plate about AB, BC and CA as axes respectively. Which one of the following relations is correct?



(a) $I_{AB} > I_{BC}$
(b) $I_{BC} > I_{AB}$
(c) $I_{AB} + I_{BC} = I_{CA}$
(d) I_{CA} is maximum

14. Moment of inertia of a circular wire of mass M and radius R about its diameter is

(a) $MR^2/2$ (b) MR^2
(c) $2MR^2$ (d) $MR^2/4$

15. A particles performing uniform circular motion. Its angular frequency is doubled and its kinetic energy halved, then the new angular momentum is

(a) $\frac{L}{4}$ (b) $2L$
(c) 4 L (d) $\frac{L}{2}$

16. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same which one of the following will not be affected ?

(a) Angular velocity
(b) Angular momentum
(c) Moment of inertia
(d) Rotational kinetic energy

17. Angular momentum is

(a) moment of momentum
(b) product of mass and angular velocity
(c) product of M.I. and velocity
(d) moment of angular motion

18. The angular momentum of a system of particle is conserved.

(a) when no external force acts upon the system
(b) when no external torque acts upon the system
(c) when no external impulse acts upon the system
(d) when axis of rotation remains same

19. Analogue of mass in rotational motion is

(a) moment of inertia (b) angular momentum
(c) gyration (d) none of these

20. Moment of inertia does not depend upon

(a) angular velocity of body
(b) shape and size
(c) mass
(d) position of axis of rotation

21. A couple is acting on a two particle systems. The resultant motion will be

(a) purely rotational motion
(b) purely linear motion
(c) both a and b
(d) neither of a and b

22. A mass m is moving with a constant velocity along a line parallel to the x-axis, away from the origin. Its angular momentum with respect to the origin

(a) is zero (b) remains constant
(c) goes on increasing (d) goes on decreasing.

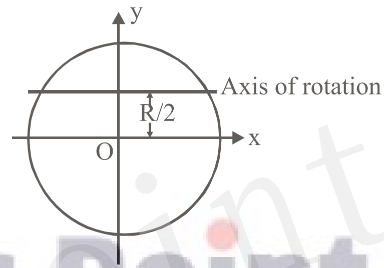
23. Consider a system of two particles having masses m_1 and m_2. If the particle of mass m_1 is pushed towards the mass centre of particles through a distance d, by what distance would the particle of mass m_2 move so as to keep the mass centre of particles at the original position?

(a) $\frac{m_2}{m_1}d$ (b) $\frac{m_1}{m_1+m_2}d$
(c) $\frac{m_1}{m_2}d$ (d) d

24. The moment of inertia of a uniform circular disc of radius 'R' and mass 'M' about an axis passing from the edge of the disc and normal to the disc is

(a) MR^2 (b) $\frac{1}{2}MR^2$
(c) $\frac{3}{2}MR^2$ (d) $\frac{7}{2}MR^2$

25 M.I of a circular loop of radius R about the axis in figure is



(a) MR^2 (b) (3/4) MR^2
(c) $MR^2/2$ (d) $2MR^2$

26. Two bodies have their moments of inertia I and $2I$ respectively about their axis of rotation. If their kinetic energies of rotation are equal, their angular momenta will be in the ratio

(a) 2 : 1 (b) 1 : 2
(c) $\sqrt{2}:1$ (d) $1:\sqrt{2}$

27. A drum of radius R and mass M, rolls down without slipping along an inclined plane of angle θ. The frictional force

(a) dissipates energy as heat.
(b) decreases the rotational motion.
(c) decreases the rotational and translational motion.
(d) converts translational energy to rotational energy

28. In carbon monoxide molecule, the carbon and the oxygen atoms are separated by a distance 1.12×10^{-10} m. The distance of the centre of mass, from the carbon atom is

(a) 0.64×10^{-10} m (b) 0.56×10^{-10} m
(c) 0.51×10^{-10} m (d) 0.48×10^{-10} m

29. The moment of inertia of a disc of mass M and radius R about an axis, which is tangential to the circumference of the disc and parallel to its diameter, is

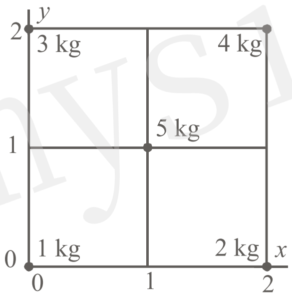
(a) $\frac{3}{2}MR^2$ (b) $\frac{2}{3}MR^2$
(c) $\frac{5}{4}MR^2$ (d) $\frac{4}{5}MR^2$

30. A tube one metre long is filled with liquid of mass 1 kg. The tube is closed at both the ends and is revolved about one end in a horizontal plane at 2 rev/s. The force experienced by the lid at the other end is
(a) $4\pi^2$N (b) $8\pi^2$N
(c) $16\pi^2$N (d) 9.8 N

31. A composite disc is to be made using equal masses of aluminium and iron so that it has as high a moment of inertia as possible. This is possible when
(a) the surfaces of the disc are made of iron with aluminium inside
(b) the whole of aluminium is kept in the core and the iron at the outer rim of the disc
(c) the whole of the iron is kept in the core and the aluminium at the outer rim of the disc
(d) the whole disc is made with thin alternate sheets of iron and aluminium

32. A ball rolls without slipping. The radius of gyration of the ball about an axis passing through its centre of mass is K. If radius of the ball be R, then the fraction of total energy associated with its rotational energy will be
(a) $\frac{R^2}{K^2+R^2}$ (b) $\frac{K^2+R^2}{R^2}$
(c) $\frac{K^2}{R^2}$ (d) $\frac{K^2}{K^2+R^2}$

33. Five masses are placed in a plane as shown in figure. The coordinates of the centre of mass are nearest to



(a) 1.2, 1.4 (b) 1.3, 1.1
(c) 1.1, 1.3 (d) 1.0, 1.0

34. A solid sphere of mass 1 kg rolls on a table with linear speed 1 ms^{-1}. Its total kinetic energy is
(a) 1 J (b) 0.5 J
(c) 0.7 J (d) 1.4 J

35. The moment of inertia of a body about a given axis is 1.2 kg m^2. Initially, the body is at rest . In order to produce a rotational kinetic energy of 1500 J, an angular acceleration of 25 rad s^{-2} must be applied about that axis for a duration of
(a) 4 s (b) 2 s
(c) 8 s (d) 10 s

36. A sphere rolls down on an inclined plane of inclination θ. What is the acceleration as the sphere reaches bottom?
(a) $\frac{5}{7}g\sin\theta$ (b) $\frac{3}{5}g\sin\theta$
(c) $\frac{2}{7}g\sin\theta$ (d) $\frac{2}{5}g\sin\theta$

DIRECTIONS (Qs. 37-49): *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

37. Some particles are placed along x-axis, some at positive x-axis and other at negative x-axis. Which of the following statements are correct ?
(a) COM of the system may lie at the origin
(b) COM of the system may lie at a point on the positive x-axis.
(c) COM of the system may lie at a point of the negative x-axis.
(d) COM of the system must lie at the origin.

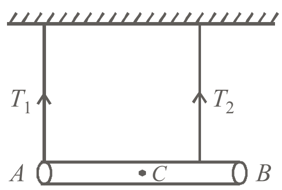
38. A constant force acts on a system of two blocks of masses m_1 and m_2 ($m_1 > m_2$) then
(a) Acceleration of COM is constant
(b) Velocity of COM is constant
(c) Velocity of m_1 may be constant
(d) Velocity of m_2 may be constant

39. Which of the following is/are true about COM of a system?
(a) COM may lie at a point where no mass is present.
(b) COM of a two particle system may lie at a point outside the lie joining the two particles.
(c) Moment of weights about COM is always zero.
(d) COM of a two particle system lies near the lighter particle.

40. Two non-zero forces F_1 and F_2 act on a rigid body in opposite direction at different points. Then
(a) Net force on the body is non-zero if $F_1 = F_2$
(b) Net force on the body is non-zero if $F_1 \neq F_2$
(c) Net torque on the body is non-zero if $F_1 = F_2$
(d) Net torque on the body is non-zero if $F_1 \neq F_2$

41. A body is said to be in equilibrium if
(a) net force on the body is zero
(b) net torque on the body is zero
(c) either (a) or (b)
(d) neither (a) nor (b).

42. A uniform rod of mass m and length l is hanging with the help of two strings as shown in the figure.

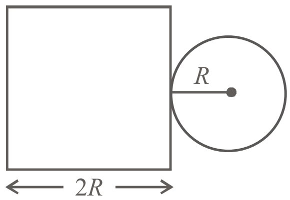


If T_1 and T_2 be tensions in the strings, then
(a) $T_1 > T_2$ (b) $T_1 < T_2$
(c) $T_1 + T_2 = mg$ (d) $T_1 = T_2$

43. When a bomb explodes in mid-air, its
(a) momentum increases
(b) momentum remains constant
(c) kinetic energy increases
(d) kinetic energy remains constant

44. Two particles of same mass move towards each other under their mutual gravitational intersection. If the particles are initially at rest, then
(a) the speed of particles at any instant is equal.
(b) the speed of particles after some time become constant.
(c) the speed of COM of the system is always zero.
(d) the speed of COM of the system is increasing.

45. A square sheet and a disc of same thickness are placed in contact as shown.



(a) If densities of disc and square are equal, COM lies inside the square.
(b) If densities of disc and square are equal, COM lies at the point of contact.
(c) If masses of disc and square are equal, COM lies inside the disc.
(d) If masses of disc and square are equal, COM lies at the point of contact.

46. A constant force F acts on a system of particles. If the force acts towards positive x-axis :
(a) the acceleration of COM must be along positive x-axis.
(b) the acceleration of some of the particles may be along negative x-axis.
(c) the acceleration of all the particles must be along positive x-axis.
(d) the acceleration of some of the particles may be zero.

47. A body is said to be in translatory equilibrium if
(a) net force acting on it is zero
(b) it is at rest
(c) it is in uniform motion
(d) it is in acceleration motion

48. Two unequal masses are tied together with a compressed spring. When the chord is burnt with a match-stick releasing the spring, the two masses fly apart with
(a) equal momentum
(b) equal speeds
(c) equal kinetic energy
(d) different kinetic energy

49. Consider a system of two particles of masses m_1 and m_2 ($m_1 > m_2$) moving in opposite directions with equal kinetic energy. Which of the following statements is/are correct ?
(a) Velocity of COM is zero
(b) Velocity of COM is in direction of motion of m_1
(c) Velocity of COM is in direction of motion of m_2
(d) Kinetic energy of system is non-zero.

DIRECTIONS (Q. 50): *Following question has four statements (A, B, C and D) given in Column I and five statements (p, q, r, s, t ....) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in Column I with entries in Column II.*

50. A man is standing on a plank which is resting on a rough horizontal surface. If the man starts walking on the plank towards right, match all the possibilities.

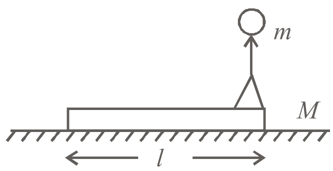
Column - I	**Column - II**
(A) Displacement of COM	(p) must be towards right
(B) Frictional force on plank due to ground	(q) must be towards left
(C) Frictional force on plank due to feet of the boy	(r) may be towards right
(D) Displacement of the plank	(s) may be towards left
	(t) may be zero

	A	B	C	D
(a)	p	p	q	s, t
(b)	p, q, r, s	q	p, q, r, s	p, q, r, s
(c)	p, s	q	r, s, t	r
(d)	r, t	r, s, t	p	q, r

DIRECTIONS (Qs. 51-53): *Study the given paragraph(s) and answer the following questions.*

PASSAGE

A boy of mass m is standing at one end of a platform of mass M and length ℓ placed on a smooth horizontal surface as shown in figure. Now the boy walks on the platform towards left to reach other end of the platform.



51. If the boy walks with uniform velocity, the platform will
(a) move towards right with uniform velocity
(b) move towards right with variable velocity
(c) move towards left with uniform velocity
(d) not move

52. The velocity of COM of the boy-platform system is
(a) always towards left
(b) always towards right
(c) towards left if $m > M$ and towards right if $M > m$
(d) always zero

53. If $M > m$, the resultant displacement of the boy *w.r.t.* ground will be
(a) greater than that of the platform
(b) less than that of the platform.
(c) equal to that of the platform
(d) either (b) or (c).

DIRECTIONS (Qs. 54-59): *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.
(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.
(c) If **Assertion** is **correct** but **Reason** is **incorrect**.
(d) If **Assertion** is **incorrect** but **Reason** is **correct**.

54. **Assertion :** Two blocks of mass m_1 and m_2 are at rest. If they move towards each other due to their mutual interaction, the velocity of COM is zero.
Reason : If no external force acts on the system, then velocity of COM is always zero.

55. **Assertion :** Centre of mass of a three particle system which are not collinear, may lie on line joining any two particles.
Reason : Centre of mass of a three particle system lies within the triangle with the particles at its vertices.

56. **Assertion :** The centre of mass of a body may lie where, there is no mass.
Reason : Centre of mass of a body is a point, where the whole mass of the body is supposed to be concentrated.

57. **Assertion :** When a very small horizontal force is applied on the top of a heavy almirah placed on a rough horizontal surface, it remains in equilibrium.
Reason : Normal reaction and friction balance torque produced by the applied force.

58. **Assertion :** If two bodies of equal mass move in opposite directions, velocity of COM is zero.
Reason : Velocity of COM of a system of two bodies is given by :

$$v_c = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

59. **Assertion :** If no external force acts on a system, velocity of COM is constant.
Reason : $F_c = Ma_c$, hence if $F_c = 0$, $a_c = 0$

DIRECTIONS (Qs. 60-65): *Following are integer based/ Numeric based questions. Each question, when worked out will result in one integer or numeric value.*

60. A wheel having moment of inertia 2 kg-m^2 about its vertical axis, rotates at the rate of 60 rpm about this axis. Find the torque which can stop the wheel's rotation in one minute.

61. A sphere of mass 0.5 kg and diameter 1m rolls without sliding with a constant velocity of 5 m/s, calculate the ratio of the rotational K.E. to the total kinetic energy of the sphere.

62. A body of moment of inertia 3 kg-m^2 rotating with an angular velocity of 2 rad/sec has the same kinetic energy as that of a body of mass 12 kg moving with a velocity of v. Find the value of v.

63. A flywheel rotating about a fixed axis has a kinetic energy of 360 joule when its angular speed is 30 rad/sec. Find the moment of inertia of the wheel about the axis of rotation.

64. An automobile crankshaft transfers energy from the engine to the axle at the rate of 100 hp (=74.6 kW) when rotating at a speed of 1800 rev/min. What torque (in newton-meters) does the crankshaft deliver?

65. Two point masses 4 kg and 6 kg are moving along the same straight line with speed 3 m/s and 2 m/s respectively. Find the velocity of their centre of mass if both the masses are moving in the same direction.

SOLUTIONS

Brief Explanations of Selected Questions

ADVANCED EXERCISE
BASED ON CONNECTING TOPICS

1. (a) If we consider the two masses in a system then no external force is acting on the system. Mutual forces are internal forces. Since the centre of mass is initially at rest, it well remain at rest

2. (c) For solid sphere

$$I = \frac{2}{5} M R^2 = \frac{2}{5}\left(\frac{4}{3}\pi R^3 \rho\right) R^2$$

$$= \frac{8}{15} \times \frac{22}{7} R^5 \rho = \frac{176}{105} R^5 \rho$$

3. (d)

4. (a) $E_r = \frac{1}{2} I \omega^2 = \frac{1}{2} \times 3 \times (3)^3 = 13.5\, J$

$$K.E. = \frac{1}{2} m v^2 = \frac{1}{2} \times 27 \times v^2 = 13.5$$

$v = 1$ m/s

5. (c) Centripetal acceleration

$a_c = 4\pi^2 \nu^2 r = 4\pi^2 \times 2 \times 2 \times 0.25 = 4\pi^2\ \text{ms}^{-2}$

6. (c) $L = m v r$ or $v = L/mr$

Centripetal force $\frac{m v^2}{r} = \frac{m(L/m r)^2}{r} = \frac{L^2}{m r^3}$

7. (b) If we apply a torque on a body, then angular momentum of the body changes according to the relation

$$\vec{\tau} = \frac{d\vec{L}}{dt} \Rightarrow \text{if } \vec{\tau} = 0 \text{ then, } \vec{L} = \text{constant}$$

8. (b) By energy conservation

$$(K.E)_i + (P.E)_i = (K.E)_f + (P.E)_f$$

$$(K.E)_i = 0, (P.E)_i = mgh, (P.E)_f = 0$$

$$(K.E)_f = ½ I \omega^2 + ½ m v_{cm}^2$$

Where I (moment of inertia) = ½ mR^2 (for solid cylinder)

$$\text{so } mgh = ½(½ mR^2)\left(\frac{v_{cm}^2}{R^2}\right) + ½ m v^2 cm$$

$$\Rightarrow v_{cm} = \sqrt{4gh/3}$$

9. (b) Angular momentum $\vec{L}$ is defined as $\vec{L} = \vec{r} \times m(\vec{v})$

so $\vec{L}$ is, an axial vector.

10. (c) The boy does not exert a torque to rotating table by jumping, so angular momentum is conserved i.e.,

$$\frac{d\vec{L}}{dt} = 0 \Rightarrow \vec{L} = \text{constant}$$

11. (b) Since no external torque act on gymnast, so angular momentum ($L = I\omega$) is conserved. After pulling her arms & legs, the angular velocity increases but moment of inertia of gymnast, decreases in, such a way that angular momentum remains constant.

12. (c) Conserving Linear Momentum

$2Mv_c = 2Mv - Mv \Rightarrow v_c = v/2.$

13. (b) The intersection of medians is the centre of mass of the triangle. Since distances of centre of mass from the sides are related as : $x_{BC} < x_{AB} < x_{AC}$ therefore $I_{BC} > I_{AB} > I_{AC}$ or $I_{BC} > I_{AB}$.

14. (a)

15. (a) Angular momentum $\propto \frac{1}{\text{Angular frequency}}$

$\propto$ Kinetic energy

$$\Rightarrow \vec{L} = \frac{K.E.}{\omega}$$

$$\frac{L_1}{L_2} = \left(\frac{K.E_1}{\omega_1}\right) \times \frac{\omega_2}{K.E_2} = 4 \Rightarrow L_2 = \frac{L}{4}$$

16. (b) Angular momentum will remain the same since external torque is zero.

17. (a) Angular momentum = × linear momentum

18. (b) We know that $\tau_{ext} = \frac{dL}{dt}$

if angular momentum is conserved, it means change in angular momentum = 0

or $dL = 0$

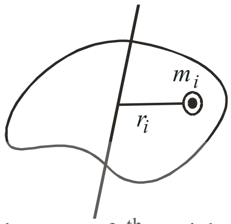
$$\frac{dL}{dt} = 0 \Rightarrow \tau_{ext} = 0$$

Thus total external torque = 0.

19. (a) Analogue of mass in rotational motion is moment of inertia. It plays the same role as mass plays in translational motion.

20. (a) Basic equation of moment of inertia is given

by $I = \sum_{i=1}^{n} m_i r_i^2$



where m_i is the mass of i^{th} particle at a distance of r_i from axis of rotation.

Thus it does not depend on angular velocity.

21. (a) A couple consists of two equal and opposite forces whose lines of action are parallel and laterally separated by same distance. Therefore, net force (or resultant) of a couple is null vector, hence no translatory motion will be produced and only rotational motion will be produced.

22. (b) Angular momentum of mass m moving with a constant velocity about origin is

L = momentum × perpendicular distance of line of action of momentum from origin

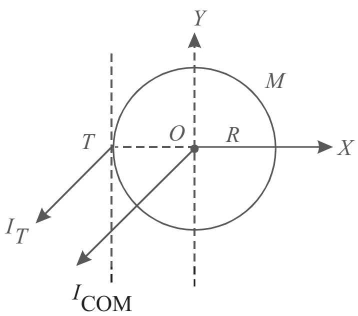
$L = mv \times y$

In the given condition mvy is a constant. Therefore angular momentum is constant.

23. (c) $m_1 d = m_2 d_2 \Rightarrow d_2 = \dfrac{m_1 d}{m_2}$

24. (c) *M.I.* of a uniform circular disc of radius '*R*' and mass '*M*' about an axis passing through COM and normal to the disc is

$$I_{com} = \frac{1}{2} MR^2$$



From parallel axis theorem

$$I_T = I_{COM} + MR^2 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

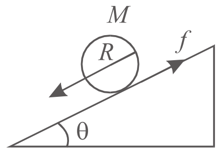
25. (b) Use theorem of parallel axes

26. (d) $K = \dfrac{L^2}{2I} \Rightarrow L^2 = 2KI \Rightarrow L = \sqrt{2KI}$

$$\frac{L_1}{L_2} = \sqrt{\frac{K_1}{K_2}.\frac{I_1}{I_2}} = \sqrt{\frac{K}{K}.\frac{I}{2I}} = \frac{I}{\sqrt{2}}$$

$L_1 : L_2 = 1 : \sqrt{2}$

27. (d) Net work done by frictional force when drum rolls down without slipping is zero.



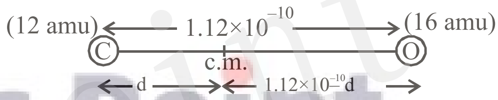
$W_{net} = 0,\ W_{trans.} + W_{rot.} = 0$

$\Delta K_{trans.} + \Delta K_{rot} = 0$

$\Delta K_{trans} = -\Delta K_{rot}$

i.e., converts translation energy to rotational energy.

28. (a)



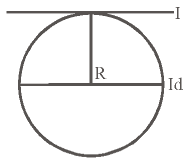
From definition of centre of mass.

$$d = \frac{16\times1.12\times10^{-10} + 12\times0}{16+12} = \frac{16\times1.12\times10^{-10}}{28}$$

$= 0.64 \times 10^{-10}$ m.

29. (c) Moment of inertia of disc about its diameter is

$$I_d = \frac{1}{4}MR^2$$



MI of disc about a tangent passing through rim and in the plane of disc is

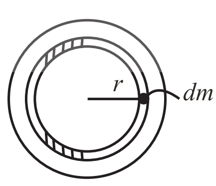
$$I = I_G + MR^2 = \frac{1}{4}MR^2 + MR^2 = \frac{5}{4}MR^2$$

30. (b) $F = mr\omega^2 = 1\times\frac{1}{2}\times4\pi^2\times2\times2 = 8\pi^2$N

31. (b) Density of iron > density of aluminium

moment of inertia = $\int r^2 dm$.

$\therefore$ Since $\rho_{iron} > \rho_{aluminium}$ so whole of aluminium is kept in the core and the iron at the outer rim of the disc.



32. **(d)** Rotational energy = $\frac{1}{2}I(\omega)^2 = \frac{1}{2}(mK^2)\omega^2$

Linear kinetic energy = $\frac{1}{2}m\omega^2R^2$

$\therefore$ Required fraction

$$= \frac{\frac{1}{2}(mK^2)\omega^2}{\frac{1}{2}(mK^2)\omega^2 + \frac{1}{2}m\omega^2R^2} = \frac{K^2}{K^2+R^2}$$

33. **(c)** $X_{COM} = \frac{1\times0+2\times2+3\times0+4\times2+5\times1}{1+2+3+4+5}$

$$= \frac{4+8+5}{15} = \frac{17}{15} = 1.1$$

$$Y_{COM} = \frac{1\times0+2\times0+3\times2+4\times2+5\times1}{1+2+3+4+5}$$

$$= \frac{6+8+5}{15} = 1.3$$

34. **(c)** $E = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$

$$= \frac{1}{2}Mv^2 + \frac{1}{2}\times\frac{2}{5}MR^2\times\frac{v^2}{R^2} = \frac{7}{10}Mv^2 = 0.7\ J$$

35. **(b)** $E = 1500 = 1/2 \times 1.2\ \omega^2$

$$\omega^2 = \frac{3000}{1.2} = 2500$$

$\omega = 50$ rad/sec

$$t = \frac{\omega}{\alpha} = \frac{50}{25} = 2\,\text{sec}$$

36. **(a)** Acceleration of a body rolling down an inclined plane is given by, $a = \frac{g\sin\theta}{1+\frac{K^2}{R^2}}$

In case of a solid sphere , we have

$$\frac{K^2}{R^2} = \frac{[(I/M)]}{R^2} = \frac{I}{MR^2} = \frac{(2/5)MR^2}{MR^2} = \frac{2}{5}$$

Substituting $\frac{K^2}{R^2} = \frac{2}{5}$ in equation (1) we get

$$a = \frac{5}{7}g\sin\theta$$

37. **(a, b, c)**

COM may lie anywhere on the line joining the particles i.e. positive, negative x-axis or origin depending on the masses of particles and their positions.

38. **(a, c, d)**

As F_c = constant, $a_e = \frac{F_c}{m}$ is also constant

Also, as the COM is accelerated, velocity of COM cannot be constant but velocity of any of the particles may be constant.

39. **(a, c)**

40. **(b, c, d)**

Net force on the body will be zero if

$F_1 = F_2$ and non-zero if $F_1 \neq F_2$.

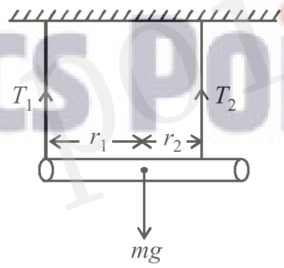
Also as point of application of forces are different and forces are in opposite direction, not torque cannot be zero for any value of F_1 and F_2.

41. **(a, b)**

A rigid body is said to be in equilibrium when net force as well as net torque acting on the body are zero i.e. it is in rotatory as well as translatory equilibrium.

42. **(b, c)**

Condition is shown in the figure.



For translatory equilibrium

$$T_1 + T_2 = mg$$

Hence (c) is correct.

For rotatory equilibrium about COM of the rod,

$$T_1 r_1 = T_2 r_2$$

or $\frac{T_2}{T_1} = \frac{r_1}{r_2}$ but $r_1 > r_2$

$\therefore T_2 > T_1$

Hence (b) is correct and (a) and (d) are wrong.

43. **(b, c)**

As the bomb explodes in mid-air, no external force acts on the bomb hence its momentum remains conserved, but its kinetic energy increases because its chemical energy gets converted into kinetic energy.

44. **(a, c)**

As no external force is acting on the system velocity of COM remains constant i.e. zero, also as mass of both the particles is equal, speed of each particle must be equal at any instant.

45. **(a, d)**
Centre of mass of a two body system lies near the heavier mass.
Hence if masses of disc and square are equal, COM will lie at point of contact, but if their densities are equal, mass of square will be more than that of the disc, hence COM will lie inside the square.
Hence (a) and (d) are correct.

46. **(a, b, d)**
The acceleration of COM must be in the direction of applied force, i.e. positive x-axis but acceleration of a randomly selected particle of system may be it any direction or even zero.

47. **(a, b, c)**
For translatory equilibrium,
$\vec{F}_{net} = 0 \qquad \therefore \ \vec{a} = 0$
or $\vec{v}$ = constant or zero

48. **(a, d)**
As no external force is acting on the system momentum of COM must remain zero. Hence the blocks fly a part with equal and opposite momenta but as their masses are different, their kinetic energy will be different.
Hence (a) and (d) are correct.

49. **(b, d)**
Momenta of particles are given by
$P_1 = \sqrt{2m_1k}$ and $P_2 = \sqrt{2m_2k}$
where k is kinetic energy of each particle,
As $m_1 > m_2$, $P_1 > P_2$
Hence COM will move in the direction of motion of m_1.
Also as kinetic energy is never subtracted, K.E. of the system may not be zero.
Hence (b) and (d) are correct.

50. **(a)** (A) → (p); (B) → (p); (C) → (q); (D) → (s, t)
As the boy moves towards right, he pushes the plank towards left and hence ground exerts friction on the plank towards right.
As the net external force on the system is only friction between the plank and the ground, COM must displace towards right.
Again, plank may move towards left or remain at rest depending of magnitude of both the frictional forces.

51. **(a)** As no external force is acting on the system, velocity of COM must remain zero, hence the platform will move in a direction opposite to the direction of motion of the boy with uniform velocity.

52. **(d)** As no external force is acting on the system, velocity of COM will not change *i.e.* will remain zero.

53. **(a)** If S_1 and S_2 be magnitudes of displacement of the boy and the platform respectively, then
$mS_1 = MS_2$
$$S_1 = \frac{M}{m}S_2$$
as $M > m$
$S_1 > S_2$

54. **(c)** If no external force acts on the system, velocity of COM remains constant which may or may not be zero, hence reason is false. Here assertion is true, as initial velocity of COM is zero and no force acts on the system, final velocity of COM must be zero.

55. **(d)** COM of three non-collinear particles can never lie on the line joining any two particles but lies somewhere inside the triangle with the particles at its vertices.

56. **(b)** Both assertion and reason are true but reason does not explain assertion.

57. **(a)** When we apply a small force on the almirah, it remains at rest, hence it is in rotatory as well as translatory equilibrium.

58. **(d)** In this case velocity of centre of mass is zero only if speeds of two particles are equal.

59. **(a)** Both assertion and reason are true and reason is correct explanation of assertion.

60. $\tau \times \Delta t = L_0 \qquad \{\because \text{ since } L_f = 0\}$
$\Rightarrow \tau \times \Delta t = I\omega$
or $\tau \times 60 = 2 \times 2 \times 60\pi/60$
$(\because \ f = 60\text{rpm} \ \therefore \ \omega = 2\pi f = 2\pi \times \frac{60}{60})$
$$\tau = \frac{\pi}{15}\text{N} - \text{m}$$

61. $$\frac{K_R}{K_T} = \frac{K^2/R^2}{1+K^2/R^2} = \frac{2/5}{1+2/5} = 2/7$$

62. $$\frac{1}{2}I\omega^2 = \frac{1}{2}mv^2$$
$$\frac{1}{2}\times 3\times(2)^2 = \frac{1}{2}\times 12\times v^2 \Rightarrow v = 1m/s$$

63. $$\frac{1}{2}I\omega^2 = 360 \Rightarrow I = \frac{2\times 360}{(30)^2} = \frac{2\times 360}{30\times 30} = 0.8\ \text{kg}\times m^2$$

64. Given, power $P = 74.6$ kW

$= 74.6 \times 1000$ W

Angular speed, $\omega = 1800$ rev/min

$$= \frac{1800}{60} \times 2\pi = 60\,\pi \text{ rad/s}$$

Using the formula $P = \tau\omega$

$$\tau = \frac{P}{\omega} = \frac{74.6 \times 1000}{60 \times 3.14} = 396 \text{ N-m}$$

65. Given

$m_1 = 4$ kg, $v_1 = 3$ m/s, $m_2 = 6$ kg, $v_2 = 2$ m/s

$$\text{Using, ; } v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{4 \times 3 + 6 \times 2}{4 + 6}$$

$$= \frac{24}{10} = 2.4 \text{ m/s}$$

Chapter

8

Mechanical Properties of Solids and Fluids

INTRODUCTION

Although collapsing buildings and falling bridges are a rarity, these things happen, especially with some provocation like an earthquake. Boats still snap in half of the high seas and aeroplanes occasionally come apart in awkward ways. This chapter deals with an introduction to the mechanical properties of materials : how they stretch and compress, fatigue, break and shear. But its real focus is on one of the most interesting properties, elasticity– the ability of a system once distorted to spontaneously return to its original configuration, and in so doing, more often than not, to oscillate. Opposite to elasticity we have a property called plasticity in which material (plastic) have no gross tendency to regain their previous shape, and they get permanently deformed. Putty and mud are close to ideal plastics.

Liquids and gases flow, and we call both fluids. Their atom and/or molecules can move around fairly freely, and that contributes to a range of shared properties. The mobility inherent in fluids makes them crucial to all known life forms. The human body itself is a fluid dynamical system, we breathe, drink, bleed and excrete fluids.

A moment doesn't go by without each of us somehow interacting with fluids in motion. We walk, drive, and fly through the air, all the while breathing at least 6 quarts of it per minute.

The ability to flow, the hallmark of the fluid, varies with the cohesive force from one substance to the next from acetone to water to motor oil, to molasses, to tar. This notion was first treated analytically by Newton. Viscosity is the internal resistance or friction, offered to an object moving through a fluid. Usually small molecules, such as water and benzene, move around easily and manifest little viscosity compared to large complex molecules, such as tar. Much of our discussion will be simplified by treating ideal liquids that are incompressible and nonviscous-characteristics well approximated by water and many other real liquids under ordinary conditions.

ELASTICITY

The property of the body by virtue of which it tends to regain its original shape and size after removing the deforming force is called **elasticity**. If the body regains its original shape and size completely, after the removal of deforming forces, then the body is said to be **perfectly elastic**.

The property of the body by virtue of which it tends to retain its deformed state after removing the deforming force is called **plasticity**. If the body does not have any tendency to recover its original shape and size, it is called **perfectly plastic**.

STRESS AND STRAIN

Stress

When a deforming force is applied on a body, it changes the configuration of the body by changing the normal positions of the molecules or atoms of the body. As a result, an internal restoring force comes into play, which tends to bring the body back to its original configuration. This *internal restoring force acting per unit area of a body is called stress*.

i.e., *Stress = Restoring force/ Area*

Its **SI unit** is N/m^2.

Types of Stress

(i) **Normal or longitudinal stress :** If area of cross-section of a rod is A and a deforming force F is applied along the length of the rod and perpendicular to its cross-section, then in this case stress produced in the rod is known as normal or longitudinal stress.

$$\text{Longitudinal stress} = \frac{F_n}{A}$$

Longitudinal stress is of two types

(a) **Tensile stress :** When length of the rod is increased on application of deforming force over it, then stress produced in rod is called tensile stress.

(b) **Compressive stress :** When length of the rod is decreased on application of deforming force, then the stress produced is called compressive stress.

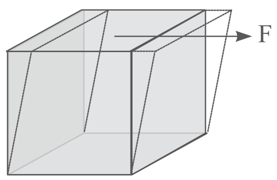
(ii) **Volumetric stress :** When a force is applied on a body such that it produces a change in volume and density and shape remaining same

1. at any point, the force is perpendicular to its surface.
2. at any small area the magnitude of force is directly proportional to its area.

Then force per unit area is called volumetric stress.

$$\text{Volumetric stress} = \frac{F_v}{A}$$

(iii) **Shearing or tangential stress :** When the force is applied tangentially to a surface, then it is called tangential or shearing stress.



$$\text{Tangential stress} = \frac{F_t}{A}$$

It produces change in shape, volume remaining same.

Strain

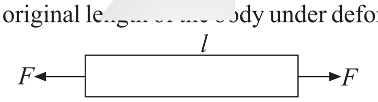
When a deforming force is applied on a body, there is a change in the configuration of the body. The body is said to be strained or deformed. The ratio of change in configuration to the original configuration is called strain.

i.e., $$\text{Strain} = \frac{\text{Change in configuration}}{\text{Original configuration}}$$

Strain being the ratio of two like quantities has **no units** and **dimensions**.

Types of Strain

(i) **Longitudinal strain :** It is defined as the change in length per unit original length of the body under deformation by the external force. Thus,

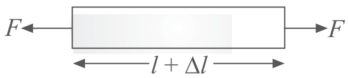


$$\text{Longitudinal strain} = \frac{\text{Change in length}}{\text{Original length}}$$

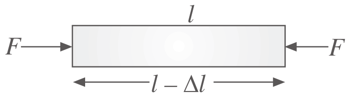
or $\varepsilon = \dfrac{\Delta l}{l}$

It is of two types :

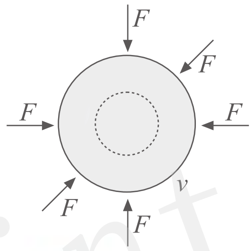
(a) **Tensile strain :** If on applying a deforming force, there is an increase of Δl in length of a rod, then strain produced in the rod is called tensile strain.



(b) **Compressive strain :** If on applying a deforming force there is decrease of Δl in length of a rod, then strain produced in the rod is called compressive strain.



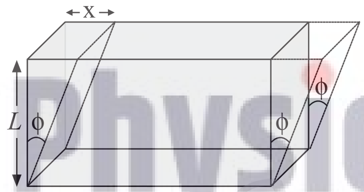
(ii) Volumetric strain : It is defined as the change in volume per unit original volume of the body under deformation by the external force.



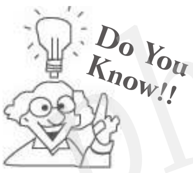
$$\textit{Volumetric strain} = \frac{\textit{Change in volume}}{\textit{Original volume}}$$

or $\varepsilon_V = \dfrac{\Delta V}{V}$

(iii) Shearing strain : This type of strain is produced when a shearing stress is present.



It is defined as the angle in radians through which a plane perpendicular to the fixed surface of the cubical body is turned under the effect of tangential force, $\phi = \dfrac{x}{L}$



Materials behave differently under stress. When dropped, a glass tumbler shatters into pieces, a rubber ball deforms then bounces back and a metal suffers dents.

ILLUSTRATION : 1

A wire is made of a material of density 10 g/cm^3 and breaking stress 5×10^9 N/m^2. What length of the wire will break under its own weight when suspended vertically?

SOLUTION :

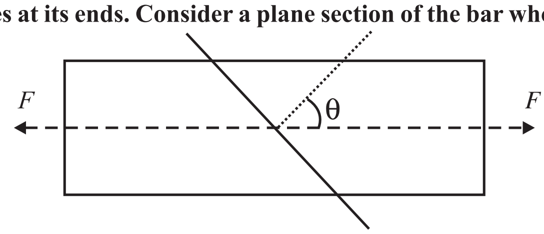
Let L be the required length. Then weight of wire = $LA\rho g$

where ρ is the density of the material.

Stress = $L\rho g = 5 \times 10^9$ N/m^2 $\quad \therefore \; L = \dfrac{5\times10^9}{10\times10^3\times10} = 5 \times 10^4$ m

ILLUSTRATION : 2

A bar of cross section A is subjected to equal and opposite tensile forces at its ends. Consider a plane section of the bar whose normal makes an angle θ with the axis of the bar.



(a) What is the tensile stress on the plane ?

(b) What is the shearing stress on this plane ?

(c) For what value of θ is the tensile stress maximum ?

(d) For what value of θ is the shearing stress maximum ?

SOLUTION :

(a) The resolved part of F along the normal is the tensile force on this plane and the resolved part parallel to the plane is the shearing force on this plane.

$\therefore$ Tensile stress $= \dfrac{\text{force}}{\text{area}} = \dfrac{F\cos\theta}{A\sec\theta} = \dfrac{F}{A}\cos^2\theta$

($\because$ area of cross section = A sec θ)

(b) Shearing stress $= \dfrac{\text{force}}{\text{area}} = \dfrac{F\cos\theta}{A\sec\theta} = \dfrac{F}{2A}\sin 2\theta$

(c) Obviously tensile stress on the plane is maximum when $\cos^2\theta$ is maximum, that is, $\cos\theta = 1$ or $\theta = 0°$.

(d) Obviously shearing stress is maximum when $\sin 2\theta$ is maximum, that is $\sin 2\theta = 1$ or $2\theta = 90°$ or $\theta = 45°$

HOOKE'S LAW

Elastic limit is the upper limit of deforming force up to which, if deforming force is removed, the body regains its original form completely and beyond which if deforming force is increased, the body loses its property of elasticity and gets permanently deformed. *Within the elastic limit, the extension of an elastic body is directly proportional to the force or stress is proportional to strain, within the elastic limit.*

i.e., Stress $\propto$ strain or, stress = E × strain

or, $\dfrac{\text{stress}}{\text{strain}} = \text{constant } E$

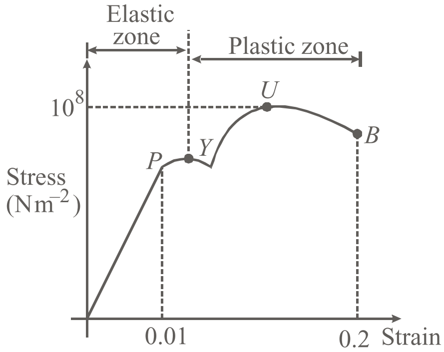
This constant E is known as **modulus of elasticity** or **coefficient of elasticity**. It depends upon the nature of the materials.

On the removal of the deforming force even within the elastic limit, only very few solids regain their original condition. They take some time to regain the original state. This delay in recovering back the original condition, on the removal of the deforming force is called *elastic after-effect*. Quartz, silver, gold and phosphor bronze are exceptions. Glass has a large elastic after-effect.

Elastic materials get tired or fatigued when subjected to repeated strain. Lord Kelvin observed that the vibrations of a wire which was kept vibrated continuously for a longer time died away much faster than that of a fresh wire. The continuously vibrated wire gets tired or fatigued and this is called *elastic fatigue.* A knowledge of elastic fatigue is essential in estimating the stresses that the various parts of machinery can bear, from the point of view of safety.

STRESS-STRAIN CURVE

The stress-strain graph of a ductile metal is shown in figure. Initially, the stress-strain graph is linear and it obeys the Hooke's law upto the point P called the proportional limit. After the proportional limit the graph is non-linear but it still remains elastic upto the yield point Y where the slope of the curve is zero. At the yield point the material starts deforming under constant stress and it behaves like a viscous liquid.



The yield point is the beginning of the plastic zone. After the yield point, the material starts gaining strength due to excessive deformation and this phenomenon is called ***strain hardening***. The point shows the ultimate strength of the material. It is the maximum stress that the material can sustain without failure. After the point the curve goes down towards the breaking point because the calculation of the stress is based on the original cross-sectional area whereas the cross-sectional areas of the sample actually decrease.

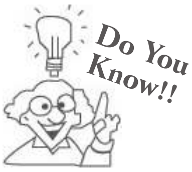
Brittle, Ductile and Malleable solids

There are some materials which break as soon as the stress is increased beyond the elastic limit. They are called ***brittle***, e.g. glass, ceramics etc.

Materials which have large plastic range of extension are called ***ductile***. Using this property, materials can be drawn into thin wires, e.g. copper, aluminium etc.

Materials which can be hammered into thin sheets are called ***malleable*** e.g. gold, silver, lead, etc.

Elastomers : Rubber has a large elastic region. It can be stretched several times its original length. On the removal of stress it returns to its original state. But the stress-strain graph is not a straight line. This means, it does not obey Hooke's law e.g. rubber, elastic tissue of aorta etc.



Metals are polycrystalline materials. They are elastic for small strains and for large strains, metals become plastic.

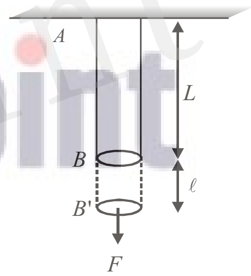
ELASTIC MODULI

The modulus of elasticity or coefficient of elasticity of a body is defined as the ratio of the stress to the corresponding strain produced, within the elastic limit.

Types of Modulus of Elasticity

Corresponding to three types of strain, there are three types of modulus of elasticity as described below:

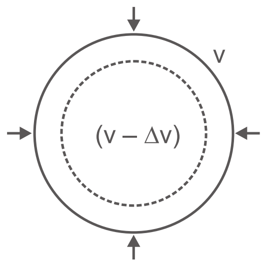
(i) Young's Modulus of Elasticity (Y): It is defined as the ratio of normal stress to the longitudinal strain within the elastic limit.



Thus, $Y = \dfrac{\text{normal stress}}{\text{longitudinal strain}}$

$\therefore \quad Y = \dfrac{F/\pi r^2}{\ell / L} = \dfrac{MgL}{\pi r^2 \ell}$

(ii) Bulk or Volume Modulus of Elesticity (K): It is defined as the ratio of normal stress to the volumetric strain, within the elastic limit.



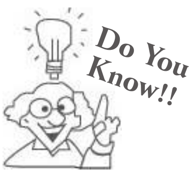
Thus, $K = \dfrac{\text{Normal stress}}{\text{Volumetric strain}}$

$K = \dfrac{F/A}{\Delta v / v} = \dfrac{Fv}{A\,\Delta v} = \dfrac{PV}{\Delta V}$

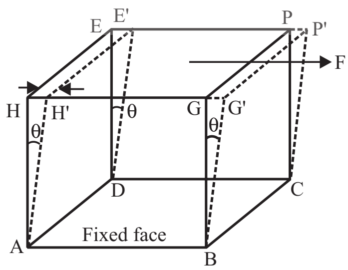
If p is the increase in pressure applied on the spherical body then, $F/A = P$

The reciprocal of bulk modulus of elasticity of a material is called its **Compressibility.**

$\text{Compressibility} = \dfrac{1}{K}$



Liquids and gases have only volume elasticity .



(iii) Shear Modulus or Rigidity Modulus of Elasticity (η): It is defined as the ratio of tangential stress to the shearing strain, within the elastic limit.

Thus, $\eta = \dfrac{\text{Tangential stress}}{\text{Shearing strain}}$

or, $\eta = \dfrac{F/A}{\theta} \Rightarrow \dfrac{F}{A\theta} = \dfrac{FL}{A\,\Delta L}$

Relation Between Y, K, η and σ

(i) $Y = 3K(1 - 2\sigma)$ (ii) $Y = 2\eta(1 + \sigma)$

(iii) $\sigma = \dfrac{3K - 2\eta}{2\eta + 6K}$ (iv) $\dfrac{9}{Y} = \dfrac{1}{K} + \dfrac{3}{\eta}$

(Here, σ is the poisson's ratio)

Relation between angle of shear and angle of twist:

In case of a rod of length ℓ and radius r fixed at one end, angle of shear ϕ is related to the angle of twist θ by the relation, $r\theta = \ell\phi$

ILLUSTRATION : 3

A litre of glycerine contracts 0.21 cm^3/Nm^2, what is the bulk modulus of glycerine ?

SOLUTION :

Here, $-\Delta V = 0.21 \text{ cm}^3 = 0.21 \times 10^{-6} \text{ m}^3$ and $V = 1$ litre $= 10^{-3} \text{ m}^3$

$$\text{Bulk modulus } K = \frac{p}{\left(\frac{-\Delta V}{V}\right)} = \frac{9.8 \times 10^5}{0.21 \times 10^{-6}} \times 10^{-3} = 4.7 \times 10^9 \text{ N/m}^2$$

ILLUSTRATION : 4

The Young's modulus of brass and steel are respectively 10^{10} N/m^2 and 2×10^{10} N/m^2. A brass wire and a steel wire of the same length are extended by 1 mm under the same force, the radii of brass and steel wires are R_B and R_S respectively. Find a relation between R_B and R_S.

SOLUTION :

We know that $Y = FL/\pi r^2 \ell$ or $r^2 = FL/(Y\pi\ell)$

$\therefore \quad R_B^2 = FL/(Y_B \pi \ell)$ and $R_S^2 = FL/(Y_S \pi \ell)$

or $\quad \dfrac{R_B^2}{R_S^2} = \dfrac{Y_S}{Y_B} = \dfrac{2 \times 10^{10}}{10^{10}} = 2$

or $\quad R_B^2 = 2R_S^2$ or $R_B = \sqrt{2}\, R_S$

WORK DONE IN A STRETCHED WIRE

Elastic potential energy U stored in the wire is given by $U = \frac{1}{2} F \times \ell = \frac{1}{2}$ (stress) × (strain) × volume of the wire

$\therefore$ Elastic potential energy per unit volume of the wire (energy density)

$$U = \frac{1}{2} \text{(stress)} \times \text{(strain)} = \frac{1}{2} \text{(Young's modulus} \times \text{strain)} \times \text{strain} \qquad (\because \text{ Young's modulus = stress/strain})$$

$$\therefore \quad U = \frac{1}{2} \text{(Young's modulus)} \times \text{(strain)}^2$$

IDEA BOX

idea box

The energy stored by an elastic material is the area under the force-extension graph. The area under the stress-strain graph gives the energy stored per unit volume.

ILLUSTRATION : 5

Calculate the elastic potential energy per unit volume of water at a depth of 1 km. Compressibility (β) of water = 5 × 10^{-10} SI units. Density of water = 10^3 kg/m^3.

SOLUTION :

Energy per unit volume = (1/2) stress × strain = (1/2) stress × $\frac{\text{stress}}{K}$

= (1/2)β(stress)^2 = (1/2)β($h\rho g$)^2

= (1/2) × (5 × 10^{-10}) × (1000 × 1000 × 9.8)^2

= 2.4 × 10^4 J/m^3

ILLUSTRATION : 6

A catapult consists of two parallel rubber strings, each of lengths 10 cm and cross sectional area 10 mm^2. When stretched by 5 cm, it can throw a stone of mass 100 g to a vertical height of 25 m. Determine Young's modulus of elasticity of rubber.

SOLUTION :

A stretched catapult has elastic potential energy stored in it

$$U = \left(\frac{1}{2}\frac{YAl^2}{L}\right) \times 2$$

This energy, when imparted to the stone, it flies off a height 20 m.

Energy possessed by the stone = $mg(h + l)$.

Now, $U = mgh \Rightarrow \frac{YAl^2}{L} = mg(h + l) \approx mgh$

$$\therefore \quad Y = \frac{mghL}{Al^2} = \frac{1\times10^{-1}\times9.8\times25\times10^{-1}}{10\times10^{-6}\times(5\times10^{-2})^2} = 9.8 \times 10^7 \text{ N/m}^2$$

FLUIDS

Fluids include liquids any gases. They begins to flow when a shearing stress is applied. Fluids have no definite shape. They assume the shape of containing vessel.

Density (ρ)

Mass per unit volume is defined as density. So density at a point of a fluid is represented as

$$\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} = \frac{dm}{dV}$$

where m is the mass and v is the volume of the fluid.

Density is a positive scalar quantity.

SI unit : kg/m^3

- For a solid body volume and density will be same as that of its constituent substance of equal mass.

  i.e. $M_{body} = M_{sub}$ then $V_{body} = V_{sub}$ and $\rho_{body} = \rho_{sub}$.

  But for a hollow body or body with air gaps

  $M_{body} = M_{sub}$ and $V_{body} > V_{sub}$ then $\rho_{body} < \rho_{sub}$

- If m_1 mass of liquid of density ρ_1 and m_2 mass of liquid of density ρ_2 are mixed then,

$$M_{mix} = m_1 + m_2 \text{ and } V_{mix} = V_1 + V_2 = \frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}$$

$$\therefore \quad \rho_{mix} = \frac{M_{mix}}{V_{mix}} = \frac{m_1 + m_2}{\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}}$$

If *same masses are mixed*, i.e. $m_1 = m_2 = m$ then

$$\rho_{mix} = \frac{2\rho_1\rho_2}{\rho_1+\rho_2}$$ (harmonic mean of individual densities)

- If V_1 volume of liquid of density ρ_1 and V_2 volume of liquid of density ρ_2 are mixed then

  $V_{mix} = V_1 + V_2$ and,

  $M_{mix} = m_1 + m_2 = \rho_1 V_1 + \rho_2 V_2$

$$\therefore \quad \rho_{mix} = \frac{M_{mix}}{V_{mix}} = \frac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2}$$

- If same volumes are mixed, i.e. $V_1 = V_2 = V$ then ,

$$\rho_{mix} = \frac{\rho_1+\rho_2}{2}$$ (arithmetic mean of individual densities)

Specific Weight or Weight Density (W)

It is defined as *the ratio of the weight of the fluid to its volume or the weight acting per unit volume of the fluid.*

$$\text{Specific weight}, \ W = \frac{\text{Weight}}{\text{Volume}}$$

$$W = \frac{mg}{V} = \left[\frac{m}{V}\right] g = \rho g$$

S I Unit: N/m^3

Specific weight of pure water at 4°C is 9.81 kN/m^3

Relative Density

It is defined as the ratio of the density of the given fluid to the density of pure water at 4°C.

$$\text{Relative density (R.D).} = \frac{\text{Density of given liquid}}{\text{Density of pure water at } 4°C}$$

The density of water is maximum at 4°C and is equal to $1.0 \times 10^3 \ kgm^{-3}$

Relative density or specific gravity is a **unitless** and **dimensionless** positive scalar physical quantity.

Being a dimensionless/unitless quantity R.D. of a substance is same in SI and CGS system.

Specific Gravity

It is defined as *the ratio of the specific weight of the given fluid to the specific weight of pure water at 4°C.*

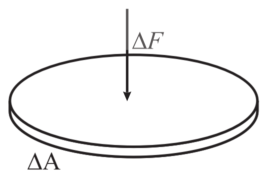
$$\text{Specific gravity} = \frac{\text{Specific weight of given liquid}}{\text{Specific weight of pure water at } 4°C \ (9.81 \ kN/m^3)}$$

$$= \frac{\rho_\ell \times g}{\rho_w \times g} = \frac{\rho_\ell}{\rho_w} = \text{R.D. of the liquid}$$

Thus specific gravity of a liquid is numerically equal to the relative density of that liquid and for calculation purposes they are used interchangeably.

PRESSURE IN A FLUID

The *pressure exerted by a fluid is defined as the force per unit area at a point within the fluid*. Consider an element of area ΔA as shown in figure and an external force ΔF acting normal to the surface. The average pressure in the fluid at the position of the element is given by $P_{av} = \frac{\Delta F}{\Delta A}$



As $\Delta A \to 0$, the element reduces to a point, and thus, pressure at a point is defined as

$$P = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} = \frac{dF}{dA}$$

When the force is constant over the surface , the above equation reduces to $P = F/A$

The **SI unit** of pressure is Nm^{-2} and is also called pascal (*Pa*). The other common pressure units are atmosphere and bar.

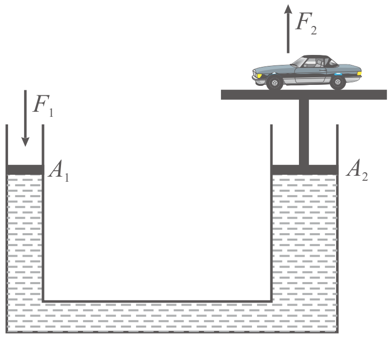
1atm = 1.01325×10^5 Pa, 1 bar = 1.00000×10^5 Pa,

1atm = 1.01325 bar

Expression for liquid pressure and total pressure

(i) The liquid pressure at a depth h is given by $P = \rho g h$ where ρ is the density of the liquid.
(ii) The total pressure at the same depth h $P_{total} = P_{atm} + \rho g h$, where P_{atm} is atmospheric pressure.

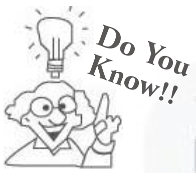
Pascal's Law



According to equation $p = p_0 + \rho g h$, pressure at any depth h in a fluid may be increased by increasing the pressure p_0 at the surface. Pascal recognized a consequence of this fact that we now call Pascal's Law.
According to Pascal's law– *A pressure applied to a confined fluid at rest is transmitted equally undiminished to every part of the fluid and the walls of the container.* This principle is used in a hydraulic presses, brakes, jack or lift, as shown in the figure.
The pressure due to a small force F_1 applied to a piston of area A_1 is transmitted to the larger piston of area A_2. The pressure at the two pistons is the same because they are at the same level.

$$P = \frac{F_1}{A_1} = \frac{F_2}{A_2} \quad \text{or} \quad F_2 = \left(\frac{A_2}{A_1}\right)F_1$$

Consequently, the force on the larger piston is large.
Thus, a small force F_1 acting on a small area A_1 results in a larger force F_2 acting on a larger area A_2.



The total (or absolute) pressure at a depth h below the free liquid surface is more than the outside atmospheric pressure by an amount ρhg.
The pressure at the bottom of container will not depend upon the shape or size of the container.

ATMOSPHERIC PRESSURE, ABSOLUTE PRESSURE AND GAUGE PRESSURE

Atmospheric Pressure

Force exerted by air column on unit cross-section area of sea level is called atmospheric pressure (P_0)

$$P_0 = \frac{F}{A} = 101.3 kN/m^2$$

Barometer is used to measure atmospheric pressure which was **discovered by Torricelli**.
Atmospheric pressure varies from place to place and at a particular place from time to time.

Absolute Pressure

Sum of atmospheric and gauge pressure is called absolute pressure.

$$P_{abs} = P_{atm} + P_{gauge}$$
$$\Rightarrow \quad P_{abs} = P_0 + h\rho g$$

The pressure which we measure in our automobile tyres is gauge pressure.

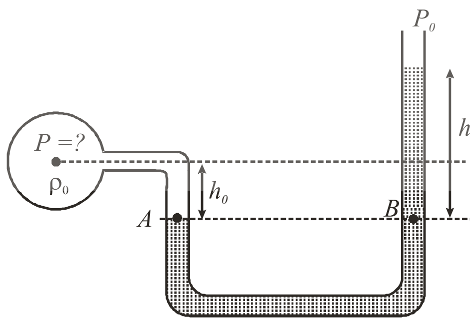
Gauge Pressure

Excess pressure ($P - P_a$) measured with the help of pressure measuring instrument is called Gauge pressure.

$$P_{gauge} = h\rho g \quad \text{or} \quad P_{gauge} \propto h.$$

Gauge pressure is always measured by "**manometer**".

Manometer :



Manometer

A manometer is a tube open at both the ends and bent into the shape of a "*U*" and is partially filled with mercury. When one end of the tube is subjected to an unknown pressure P, the mercury level drops on that side of the tube and rises on the other so that the difference in mercury level is h as shown in the figure.
According to Pascal's law, when we move down in a fluid, pressure increases with depth and when we move up, the pressure decreases with height. When we move horizontally in a fluid, pressure remains constant. Therefore,

$$P + \rho_0 g h_0 - \rho_m g h = P_0$$

where P_0 is the atmospheric pressure, and ρ_0 is the density of the fluid inside the vessel.

Do You Know!!

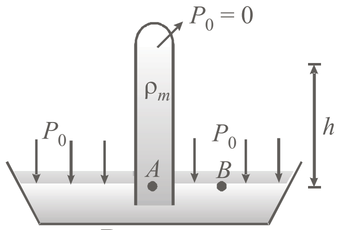
Limitation of a simple manometer are as follows.
- *It cannot measure pressure in a gas.*
- *It can measure positive gauge pressures in liquids only.*
- *It cannot measure very large pressures as it may require very long tube.*

Mercury barometer :

It is a straight glass tube (closed at one end) completely filled with mercury and inserted into a dish which is also filled with mercury as shown in the figure. Atmospheric pressure supports the column of mercury in the tube to a height h. The pressure between the closed end of the tube and the column of mercury is zero, $P = 0$.

Therefore, pressure at points A and B are equal and thus

$$P_0 = 0 + \rho_m gh$$



Barometer

At the sea level, P_0 can support a column of mercury about 76 cm in height

Hence, $P_0 - (13.6 \times 10^3)\,(9.81)\,(0.76) = 1.01 \times 10^5\ \text{Nm}^{-2}$ or Pa

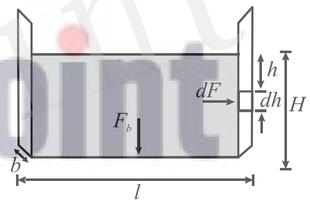
FORCES ON FLUID BOUNDARIES

Whenever a fluid comes in contact with solid boundaries, it exerts a force on it.
Consider a rectangular vessel filled with water to a height H as shown in the figure.
The force acting at the base of the container is given by

$$F_b = P \times (\text{area of the base})$$

Because pressure is same everywhere at the base and is equal to ρgH therefore,

$$F_b = \rho gH(\ell b) = \rho g \ell bH$$



Since, $\ell bH = V$ (volume of the liquid)

Thus, $F_b = \rho gV$ = weight of the liquid inside the vessel

A fluid contained in a vessel exerts forces on the boundaries.

Unlike the base, the pressure on the vertical wall of the vessel is not uniform but increases linearly with depth from the free surface. Therefore, we have to perform the integration to calculate the total force on the wall. Consider a small rectangular element of width b and thickness dh at a depth h from the free surface. The liquid pressure at this position is given by

$$P = \rho gh$$

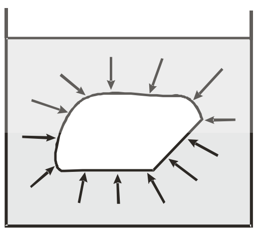
The force at the element, $dF = P\,(b\,dh) = \rho gbh\,dh$

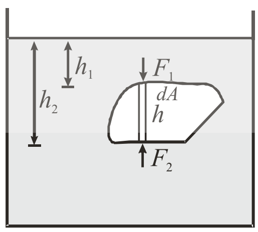
The total force, $F = \rho gb \int_0^H h\,dh = \frac{1}{2}\rho gbH^2$

ARCHIMEDES' PRINCIPLE

A body immersed in a fluid partly or wholly experiences an upward buoyant force equivalent to the weight of the fluid displaced by it. The buoyant force acts through the centre of gravity of the displaced fluid.

Imagine a body of arbitrary shape completely immersed in a liquid of density ρ as shown in the figure (a). A body is being acted upon by the forces from all directions. Let us consider a vertical element of height h and cross-sectional area dA as shown in the figure (b).


(a)



(b)

The force acting on the upper surface of the element is F_1 (downward) and that on the lower surface is F_2 (upward). Since $F_2 > F_1$, therefore, the net upward force acting on the element is $dF = F_2 - F_1$

It can be easily seen from the figure(b), that

$F_1 = (\rho g h_1)dA$ and $F_2 = (\rho g h_2)dA$ so $dF = \rho g(h)dA$

Also, $h_2 - h_1 = h$ and $d(hA) = dV$

$\therefore$ The net upward force $F = \int \rho g dV = \rho V g$

Hence, for the entire body, the buoyant force is the weight of the volume of the fluid displaced.

Buoyancy

If a body is partially or wholly immersed in a fluid, it experiences an upward force due to the fluid surrounding it. The phenomenon of force exerted by fluid on the body is called buoyancy and the force is called buoyant force.

A body experiences buoyant force whether it floats or sinks, under its own weight or due to other forces applied on it.

Do You Know!!

The buoyant force acts through the centre of gravity of the displaced fluid.

Knowledge ENHANCER

- When a body is in air, the net downward force on it is due to earth's gravity only. This net force = $mg = Vdg$. This force is known as weight of the body.
- When the same body is immersed in a liquid of density σ, the net downward force on it is = $mg - B$.
- Buoyant force = $Vdg - v\rho g$.
- This downward force is also known as apparent weight in the liquid.
- When a body is immersed in a liquid, it feels lighter also known as loss in weight (= $v\rho g$.)

CHECK Point

A boat containing some pieces of material is floating in a pond. What will happen to the level of water in the pond if on unloading the pieces in the pond, the pieces (a) float, (b) sink?

Solution

If M is the mass of boat and m of pieces in it, then initially as the system is floating,

$$M + m = V_D \sigma_W$$

i.e., the system displaces water $V_D = \frac{M}{\sigma_W} + \frac{m}{\sigma_W}$

When the pieces are dropped in the pond, the boat will still float, so it displaces water $M = V_1\sigma_W$ i.e., $V_1 = (M/\sigma_W)$

(a) Now if the unloaded pieces float in the pond, the water displaced by them

$$m = V_2 \sigma_W$$

$$\Rightarrow \quad V_2 = (m/\sigma_W) \quad ...(i)$$

So the total water displaced by the boat and the floating pieces,

$$V_1 + V_2 = \frac{M}{\sigma_W} + \frac{m}{\sigma_W} \quad ...(ii)$$

which is same as the water displaced by the floating system initially (Eq.(i)) so the level of water in the pond will remain unchanged.

(b) Now if the unloaded pieces sink, the water displaced by them will be equal to their own volume, i.e.,

$$V_2 = \frac{m}{\rho}\left[\text{as } \rho = \frac{m}{V}\right]$$

and so in this situation the total volume of water displaced by the boat and sinking pieces will be

$$V_1 + V_2 = \left(\frac{M}{\sigma_W} + \frac{m}{\rho}\right) \qquad ...(iii)$$

Now as the pieces are sinking $\rho > \sigma_W$, so that volume will be lesser than initial water displaced by the floating system (Eq. i); so the level of water in the pond will go down (or fall).

ILLUSTRATION : 7

A hydraulic automobile lift is designed to lift car with a maximum mass of 3000 kg. The area of cross-section of the piston carrying the load is 425 cm^2. What maximum pressure would the smaller piston have to bear ?

SOLUTION :

Here mass of car = 3000 kg.

Area of cross section of larger piston = 425 cm^2 = 425 × 10^{-4} m^2.

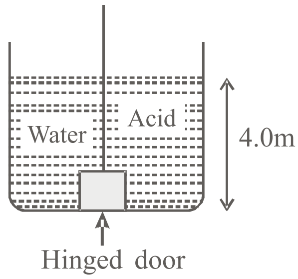
∴ The maximum pressure that the smaller piston would have to bear

$$= \frac{\text{weight of car}}{\text{area of cross-section}} = \frac{3000 \times 9.8}{425 \times 10^{-4}} = 6.92 \times 10^5 \,\text{Nm}^{-2}$$

ILLUSTRATION : 8

A tank with a square base of area 1.0 m^2 is divided by a vertical partition in the middle. The bottom of the partition has a small-hinged door of area 20 cm^2. The tank is filled with water in one compartment, and an acid (of relative density 1.7) in the other, both to a height of 4.0 m. Compute the force necessary to keep the door close.

SOLUTION :



Here $h_w = 4.0$ m, $\rho_w = 1.0 \times 10^3$ kg m^{-3}, $h_a = 4.0$ m, $\rho_a = 1.7 \times 10^3$ kg m^{-3}.

$A = 20$ cm^2 = 20 × 10^{-4} m^2.

Pressure due to water column $P_w = h_w \rho_w g$

Pressure due to acid column $P_a = h_a \rho_a g$

∴ $\Delta P = P_a - P_w = (h_a\rho_a - h_w \rho_w)\, g$

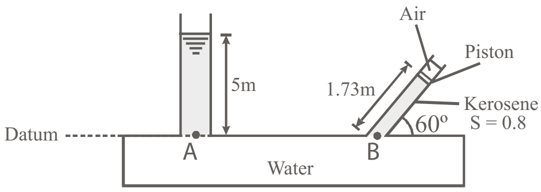
∴ $\Delta P = (4.0 \times 1.7 \times 10^3 - 4.0 \times 1 \times 10^3) \times 9.8 \,\text{Nm}^{-2}$.

∴ Force required to keep the door close = $A.\Delta P = 20 \times 10^{-4} \times 4.0 \times 0.7 \times 10^3 \times 9.8 = 55$ N

ILLUSTRATION 9 :

Find the pressure in the air column at which the piston remains in equilibrium. Assume the piston to be massless and frictionless.

SOLUTION :



Let p_a be the air pressure above the piston.

Applying Pascal's law at point A and B.

$$P_{atm} + \rho_w g(5) = p_a + \rho_k g\,(1.73)\,\frac{\sqrt{3}}{2}$$

⇒ $P_a = 138\ kPa$

STREAMLINE AND TURBULENT FLOW

Streamline Flow

When a liquid (fluid) flows, such that each particle of the liquid passing a point moves along the same path and has the same velocity as its predecessor then the flow is called stream line flow. It is also called laminar flow.

The path of a particle is called a streamline. Streamlines represent the trajectories of fluid particles. It is a curve (or a straight line) drawn such that the tangent to it at a point gives the direction of flow at that point. Two streamlines will never cross each other. If they cross then this means that the fluid particle flowing through the point of intersection can flow in two different directions at the same time, which is impossible. The closer the streamline, the greater the liquid velocity and vice versa.

Remember: In a steady flow,

(i) Velocity of a particle at any point is a constant and is independent of time.
(ii) The velocity will be different for particles in different locations.
(iii) The liquid layer in contact with the solid surface will be at rest.
(iv) The motion of the liquid follows Newton's law of viscous force.
(v) The energy dissipated varies as the first power of velocity.
(vi) The different layers are parallel one above the other, with a gradually varying velocity with distance, perpendicular to the directions of flow.

Turbulent Flow

When the velocity at a point in the liquid changes with time the flow is called unsteady flow. The unsteady flow is called turbulent when there are bends in the path of a fast moving liquid. The velocity of liquid changes continuously and haphazardly both in magnitude and direction. The turbulent flow is characterized by small, erratic whirlpool-like circles called eddy current or eddies which absorb a large amount of energy.

Equation of Continuity

Let us consider the steady flow of an incompressible liquid. Then

$m_1 = m_2$

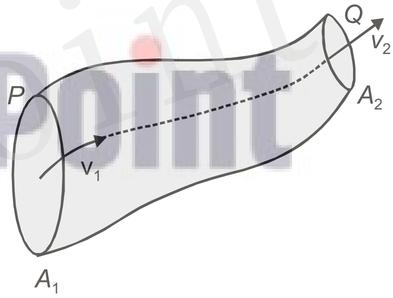
i.e. $v_1 A_1 \rho_1 = v_2 A_2 \rho_2$ .... (1)

As we have considered the fluid incompressible thus,

$\rho_1 = \rho_2$

$\therefore$ $v_1 A_1 = v_2 A_2$ .... (2)

Equations (1) and (2) are said to be as **equation of continuity**.
It is statement of the conservation of mass.

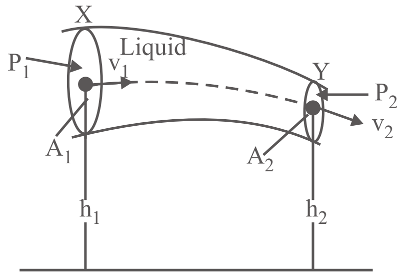


Bernoulli's theorem

When incompressible, non-viscous, irrotational liquid i.e., ideal liquid flow from one position to other in streamline path then in its path at every point, the sum of pressure energy, kinetic energy and potential energy per unit volume remains constant.

i.e., $P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + -\rho v_2$

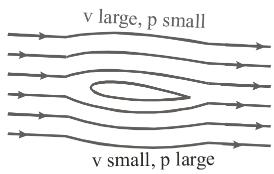
or $P + \rho g h + \frac{\rho v^2}{2}$ = *constant*



Applications of Bernoulli's principle

(1) Dynamic lift :

(i) Wings of aeroplane : The wings of the aeroplane are having tapering as shown in figure. Due to this specific shape of wings when the aeroplane runs, air passes at higher speed over it as compared to its lower surface. This difference of air speeds above and below the wings, in accordance with Bernoulli's principle, creates a pressure difference, due to which an upward force called 'dynamic lift' acts on the plane. If this force becomes greater than the weight of the plane, the plane will rise up.

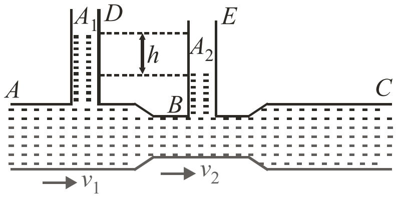


(ii) Ball moving without spin: The velocity of fluid (air) above and below the ball at corresponding points is the same resulting in zero pressure difference. The air therefore, exerts no upward or downward force on the ball.

(iii) Ball moving with spin: A ball which is spinning drags air along with it. If the surface is rough more air will be dragged. The ball is moving forward and relative to it the air is moving backwards. Therefore, the velocity of air above the ball relative to it is larger and below it is smaller. The streamlines thus get crowded above and rarified below.
This difference in the velocities of air results in the pressure difference between the lower and upper faces and there is a net upward force on the ball. This dynamic lift due to spinning is called *Magnus effect*.

(2) Venturimeter :

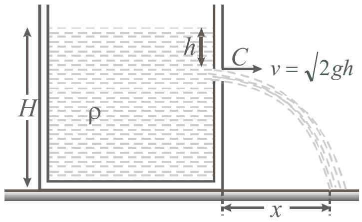
It is used for the measurement of rate of flow of fluid through a tube. The working of venturimeter is based on Bernoulli's principle. Rapidly moving fluids sustain less pressure than slowly moving fluids.



$$v = A_1 A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}$$

(3) Velocity of Efflux (Torricelli's theorem):

If a liquid is filled in a vessel up to height H and a hole is made at a depth h below the fr ee surface of the liquid as shown in figure, then taki ng the level of hole as reference level (i.e., zero point of potential energy) and applying Bernoulli's principle to the liquid just inside and outside the hole (assuming the liquid to be at rest inside) we get



$$(P_0 + h\rho g) + 0 = P_0 + \frac{1}{2}\rho v^2 \quad \text{or} \quad v = \sqrt{2gh}$$

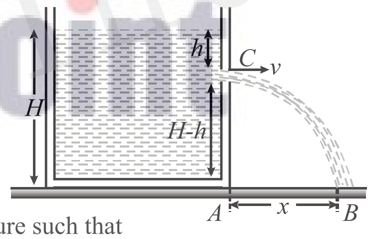
which is the same speed that an object would acquire in falling from rest through a distance h and is called 'velocity of efflux' or velocity of flux (Torricelli's theorem). From this expression it is clear that:

(i) The speed of the liquid coming out of the orifice is independent of the nature and quantity of liquid in the container or the area of the orifice, it depends on shape of orifice.

(ii) Greater is the distance of the hole from the free surface of liquid greater will be the velocity of efflux (i.e., $v \propto \sqrt{h}$). This is why liquid gush-out with maximum velocity from the orifice which is at maximum vertical distance from the free surface of the liquid.

(iii) As the vertical velocity of liquid at the orifice is zero and it is at a height $(H-h)$ from the base, the time taken by the liquid to reach the base-level

$$t = \sqrt{\frac{2(H-h)}{g}}$$



Now during this time liquid is moving horizontally with constant velocity , so it will hit the base level at a horizontal distance x (called range) as shown in figure such that

$$x = vt = \sqrt{2gh} \times \sqrt{[2(H-h)/g} = 2\sqrt{h(H-h)}$$

From this expression it is clear that range will be maximum when x^2 is maximum

i.e., $\frac{d}{dh}(x^2) = 0$ or $4\frac{d}{dh}(Hh - h^2) = 0$ or $H - 2h = 0$, i.e., $h = H/2$

So that $x_{max} = 2\sqrt{\frac{H}{2}\left[H - \frac{H}{2}\right]} = H$ i.e., range x will be maximum $(= H)$ when $h = H/2$.

(iv) If A_0 is the area of orifice at a depth y below the free surface and A that of container, the volume of liquid coming out of the orifice per second will be

$(dV/dt) = vA_0 = \sqrt{2gy}A_0$ [as $v = \sqrt{2gy}$]

Due to this, the level of liquid in the container will decrease and so if the level of liquid in the container above the hole changes from y to $y - dy$ in time t to $t + dt$ then

$-dV = A\,dy$

$-A\frac{dy}{dt} = A\sqrt{gy}$ i.e., $\int dt = -\frac{A}{A_0}\frac{1}{\sqrt{2g}}\int y^{-1/2}dy$

So the time taken for the level to fall from H to H'

$$t = -\frac{A}{A_0}\frac{1}{\sqrt{2g}}\int_H^{H'} y^{-1/2}dy = \frac{A}{A_0}\sqrt{\frac{2}{g}}[\sqrt{H} - \sqrt{H'}]$$

If the hole is at the bottom of the tank, time taken to empty the tank

$$t = \frac{A}{A_0}\sqrt{\frac{2H}{g}} \qquad \text{[as here } H' = 0]$$

ILLUSTRATION : 10

A liquid is flowing through a non-sectional tube with its axis horizontally. If two points *X* and *Y* on the axis of tube has a sectional area 2.0 cm^3 and 25 mm^2 respectively then find the flow velocity at Y when the flow velocity at *X* is 10 m/s.

SOLUTION :

According to principle of continuity

$$v_x A_x = v_y A_y$$

therefore $v_y = \dfrac{v_x A_x}{A_y} = \dfrac{10(\text{m/s}) \times 2(\text{cm}^2)}{25 \times 10^{-2}(\text{cm}^2)} = 80$ m/s

Therefore, the flow velocity at y is 80 m/s.

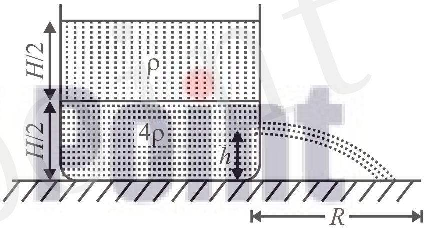
ILLUSTRATION : 11

A container of large uniform cross sectional area holds two immiscible, non-viscous and incompressible liquids of densities ρ and 4ρ, each of height H/2. Determine the position where small hole be punched, as that the heavier liquid comes out with a maximum range *R* initially.

SOLUTION :

Let the hole be punched at a height h above the base level, so that the heavier liquid gushes out of it (as shown in Fig.)

First of all, let us find out the velocity of efflux. Applying Bernoulli's theorem between two points close to the hole one inside and the other just outside the hole,



$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

(reduced form of Bernoulli's equation for horizontal flow)

$$\Rightarrow P_0 + \frac{H}{2}\rho g + \left(\frac{H}{2} - h\right)4\rho g = P_0 + \frac{1}{2}(4\rho)v_2$$

($v_1 = 0$ and the vessel is open to the atmosphere)

$$\Rightarrow g[(H/2) + 2H - 4h] = 2v^2 \;\Rightarrow\; v = \frac{1}{2}\sqrt{g(5H - 8h)}$$

Now, since the vertical component of velocity of liquid is zero initially, so the time taken by it to reach the ground is $t = \sqrt{2h/g}$

ILLUSTRATION : 12

Water flows through a tunnel a reservoir of dam towards the turbine installed in the power plant. The is situated h m below the reservoir. If the ratio of cross-sectional areas of the tunnel at the reservoir and power station is η, find the speed of the water entering into the turbine.

SOLUTION :

Applying Bernoulli's theorem at reservoir and power plant for the flowing water, we obtain

$$P_0 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_0 + \rho g h_2 + \frac{1}{2}\rho v_2^2 \;\Rightarrow\; v_2^2 = v_1^2 + 2g(h_1 - h_2).$$

Putting $(h_1 - h_2) = h_1$ we obtain $\quad v_2 = \sqrt{h_1 + h_2}$ ....(1)

Equation of continuity yields $A_1 v_1 = A_2 v_2$ ....(2)

Eliminating v_1 from equation (1) and (2), we obtain

$$v_2 = \sqrt{\left(\frac{A_2}{A_1}v_2\right)^2 + 2gh} \;\Rightarrow\; v_2 = \sqrt{\frac{2gh}{1 - \left(\frac{A_2}{A_1}\right)^2}}$$

Putting $A_1/A_2 = \eta$, we obtain $\quad v_2 = \eta\sqrt{\dfrac{2gh}{\eta^2 - 1}}$

ILLUSTRATION : 13

The ratio of the radius of the tube of a venturimeter is η (η > 1). The ratio of the densities of the liquid in the manometer and the moving fluid is η_1. If the difference in heights of the liquid column in the manometer is *h*, find the minimum speed of flow of the fluid.

SOLUTION :

The speed of flow is minimum when the cross-sectional area of the tube is maximum. The equation for minimum speed flow of the fluid in a venturimeter is given as

$$v_1 = A_2\sqrt{\frac{2\rho_0 gh}{\rho(A_1^2 - A_2^2)}} = \sqrt{\frac{2\rho_0 / gh}{\left(\frac{A_1}{A^2}\right)^2 - 1}}$$

Putting $\rho_0/\rho = \eta_1$, $\frac{A_1}{A_2} = \frac{\pi r_1^2}{\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \eta^2$, we obtain $Q = \sqrt{\frac{2\eta_1 gh}{\eta^4 - 1}}$.

VISCOSITY, STOKE'S LAW, TERMINAL VELOCITY AND REYNOLD'S NUMBER

Viscosity

If water in a tube is whirled and then left to itself, the motion of the water stops after some time. This is a very common observation. What stops the motion? There is no external force to stop it. A natural conclusion is, that whenever there is relative motion between parts of a fluid, internal forces are set up in the fluid, which oppose the relative motion between the parts in the same way as forces of friction operate when a block of wood is dragged along the ground. To maintain relative motion between layers of a fluid an external force is needed.

"This property of a fluid by virtue of which it opposes the relative motion between its different layers is known as viscosity and the force that is into play is called the viscous force".

Consider the slow and steady flow of a fluid over a fixed horizontal surface. Let v be the velocity of a thin layer of the fluid at a distance x from the fixed solid surface. Then according to Newton, the viscous force acting tangentially to the layer is proportional to the area of the layer and the velocity gradient at the layer. If F is the viscous force on the layer, then

$F \propto A$, where A is the area of the layer

Also, $F \propto -\frac{dv}{dx}$

The negative sign is put to account for the fact that the viscous force is opposite to the direction of motion.

$\Rightarrow \quad F = -\eta A\frac{dv}{dx}$

where η is a constant depending upon the nature of the liquid and is called the **coefficient of viscosity**.

$\frac{dv}{dx}$ is called the **velocity gradient**.

i.e., velocity gradient $= \frac{dv}{dx}$

If $A = 1$ and $\frac{dv}{dx} = 1$, we have $F = -\eta$

Thus the *coefficient of viscosity of a liquid may be defined as the viscous force per unit area of the layer where velocity gradient is unity.*

The coefficient of viscosity has the **dimensions** $[ML^{-1}T^{-1}]$ and its **S.I.unit** is newton second per square metre (Nsm^{-2}) or kilogram per meter per second ($kgm^{-1}s^{-1}$). In ***CGS*** the unit of viscosity is poise.

Effect of Temperature and Pressure on Viscosity

Effect of temperature : On increasing temperature viscosity of a liquid decreases.

Effect of pressure : On increasing pressure viscosity of a liquid increases except water whose viscosity decreases with pressure ries.

CHECK Point

Why should the lubricant oils be of high viscosity?

Solution

An oil, when used as lubricant in a machine, forms a thin layer of the oil over the metallic parts of the machinery. During working of the machinery, the metallic parts do not come in direct contact with each other. The friction between solid-solid surfaces gets converted into friction between solid-liquid surfaces. So that the oil layer is effective as lubricant for a long time, the oil should be of high viscosity.

Stoke's Law

When a solid moves through a viscous medium, its motion is opposed by a viscous force depending on the velocity and shape and size of the body. The energy of the body continuously decreases in overcoming the viscous resistance of the medium. This is why cars, aeroplanes etc. are shaped streamline to minimize the viscous resistance on them.
The viscous drag on a spherical body of radius r, moving with velocity v, in a viscous medium of viscosity η is given by

$$F_{viscous} = 6\pi\eta rv$$

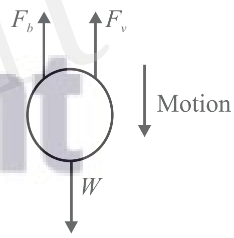
This relation is called Stokes' law.

Importance of Stoke's law:

(i) It is used in the determination of electronic charge with the help of Milikan's experiment.
(ii) It accounts the formation of clouds.
(iii) It accounts why the speed of rain drops is less than that of a body falling freely with a constant velocity from the height of clouds.
(iv) It helps a man coming down with the help of a parachute.

Terminal velocity

It is the maximum constant velocity acquired by the body while falling freely in a viscous medium.
Let a body be falling in a viscous medium.
Three types of force acts on it.
(i) F_b, the buoyant force, vertically upward
(ii) F_V, the viscous force, vertically upward
(iii) W, the weight, vertically downward



$$F_b + F_V = W \qquad \text{.......... (1)}$$

Where F_b = upward buoyant force = $\frac{4}{3}\pi r^3 \rho_0 g$, F_V = upward viscous darg = $6\pi\eta r v$
W = weight of the body acting vertically downward = $\frac{4}{3}\pi r^3 \rho g$. Here ρ = density of the body

and ρ_0 = density of the liquid.
By putting these values in equation (1) and on solving it we get

Terminal velocity $v = \dfrac{2r^2(\rho - \rho_0)g}{9\eta}$

Critical Velocity

The critical velocity is that velocity of liquid flow, upto which its flow is streamlined and above which its flow becomes turbulent.

It is given by $v_C = \dfrac{K\eta}{\rho r}$

Reynold's Number

The stability of laminar flow is maintained by viscous forces. It is observed, however that laminar or steady flow is disrupted if the rate of flow is large. Irregular, unsteady motion, turbulence, sets in at high flow rates.
Reynolds defined a dimensionless number whose value gives an approximate idea, whether the flow rate would be turbulent. This number, called the Reynolds number R_e is defined as, $R_e = \dfrac{\rho v D}{\eta}$

where ρ is the density of the fluid flowing with a speed v. The parameter D stands for the typical dimension of the obstacle or boundary to fluid flow.
It is found that *flow is streamline or laminar for R_e less than 1000. The flow is turbulent for R_e > 2000. The flow becomes unsteady for R_e between 1000 and 2000*. The critical value of R_e (known as critical Reynolds number), at which turbulence sets, is found to be the same for the geometrically similar flows. For example when oil and water with their different densities and viscosities, flow in pipes of same shapes and sizes, turbulence sets in at almost the same value of R_e.

Knowledge ENHANCER

Knowledge of viscosity of various liquids and gases have been put to use in daily life. Some applications of its knowledge are discussed as follows.

1. As the viscosity of liquids vary with temperature, proper choice of lubricant is made depending upon season.
2. Liquids of high viscosity are used in shock absorbers and buffers at railway stations.
3. The phenomenon of viscosity of air and liquid is used to damp the motion of some instruments.
4. The knowledge of the coefficient of viscosity of organic liquids is used in determining the molecular weight and shape of the organic molecules.
5. It finds an important use in the circulation of blood through arteries and veins of human body.

ILLUSTRATION : 14

A metal plate 100 cm^2 in area rests on a layer of castor oil (η = 15.5 poise) 0.2 cm thick. Calculate the horizontal force required to move the plate with a speed of 3 cm/s.

SOLUTION :

As we know, $F = -\eta A\dfrac{dv}{dx}$ where $\eta = 15.5$ poise, $A = 100\text{ cm}^2$

$$\frac{dv}{dx} = \frac{3}{0.2} = 15\text{ s}^{-1}$$

$$F = 15.5\times100\times15 = -23250\ dyne = -0.233\text{ N}$$

ILLUSTRATION : 15

Spherical particles of pollen are shaken up in water and allowed to settle. The depth of water is 2 × 10^{-2} m. What is the diameter of the largest particles remaining in suspension one hour later? Density of pollen = 1.8 × 10^3 kg m^{-3}, viscosity of water = 1 × 10^{-2} poise and density of water, = 1 × 10^{-5} kg m^{-3}.

SOLUTION :

Terminal velocity, $v = \dfrac{2r^2}{9}\dfrac{(\rho-\sigma)g}{\eta}$ ...(i)

But we know $v = \dfrac{s}{t}$

$$\therefore\ \frac{s}{t} = \frac{2}{9}r^2\frac{(\rho-\sigma)g}{\eta} \Rightarrow r^2 = \frac{9s}{2t}\frac{\eta}{(\rho-\sigma)g}$$

Given $s = 2\times10^{-2}$ m, $t = 1$ $h = 3600$ s

$\therefore\ \eta = 1\times10^{-2}$ poise $= 1\times10^{-3}\text{ kg m}^{-1}\text{s}^{-1}$

Substituting given values, we get

$$r^2 = \frac{9\times2\times10^{-2}}{2\times3600}\times\frac{1\times10^{-3}}{(1.8\times10^3-1\times10^3)\times10} = \frac{9}{36}\times\frac{1}{8}\times10^{-10} = \frac{1}{32}\times10^{-10}$$

$$\therefore\ r = \sqrt{\frac{100}{32}}\times10^{-6}\text{m} = 1.77\times10^{-6}\text{m}$$

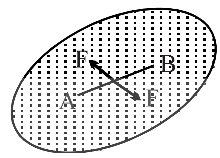
Diameter $D = 2r = 2\times1.77\ \mu\text{m} = 3.54$ m

SURFACE TENSION

Surface tension is basically a property of liquid. The liquid surface behaves like a stretched elastic membrane which has a natural tendency to contract and tends to have a minimum possible surface area. This property of liquid is called surface tension.

Definition of Surface Tension

Surface tension can be defined in the form of an imaginary line on the liquid surface or by relating it to the work done. *The force acting per unit length of an imaginary line drawn on the free liquid surface at right angles to the line and in the plane of liquid surface, is known as surface tension*. Let an imaginary line AB be drawn in any direction on a liquid surface. The surface on either side of this line exerts a pulling force, which is perpendicular to line AB. If force is F and length of AB is L then surface tension,



$$T = \frac{F}{L}$$

Its **SI unit :** N/m or J/m^2; **Dimensions :** [ML^0T^{-2}]

Examples of surface tension

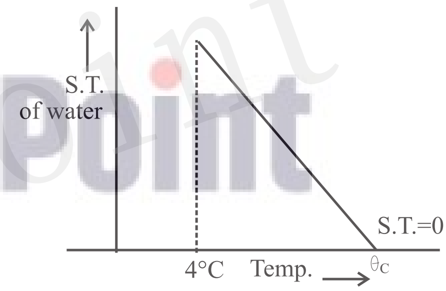
(i) Raindrops are spherical in shape.
(ii) The hair of a shaving brush cling together when taken out of water.
(iii) Oil spread on cold water but remains as a drop on hot water etc.

Factors Affecting Surface Tension

1. **Cohesive force:** The larger the value of cohesive force the larger the value of surface tension and its vice-versa.
2. **Impurities:** If the impurity is completely soluble then on mixing it in the liquid, its surface tension increases e.g., on dissolving ionic salts in small quantities in a liquid, its surface tension increases. If the impurity is partially soluble in a liquid then its surface tension decreases because adhesive force between insoluble impurity molecules and liquid molecules decreases cohesive force effectively.

Ex. (i) On mixing detergent in water its surface tension decreases.
(ii) Surface tension of water is more than alcohol – water mixture.

3. **Temperature:** On increasing temperature surface tension decreases. At critical temperature and boiling point it becomes zero. Surface tension of water is maximum at 4°C.
4. **Electrification :** The surface tension of a liquid decreases due to electrification because a force starts acting due to it in the outward direction normal to the free surface of liquid.



SURFACE ENERGY

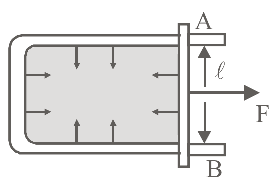
According to molecular theory of surface tension the molecules in the surface have some additional energy due to their position. This additional energy per unit area of the surface is called surface energy.

i.e., $$\text{Surface energy} = \frac{\text{Work done}}{\text{Increase in surface area}}$$

Its **SI unit** is Jm^{-2}

Surface Energy and Surface Tension

Let a liquid film be formed on a wire frame and a straight wire of length ℓ can slide on this wire frame as shown in figure. The film has two surfaces and both the surfaces are in contact with the sliding wire and hence, exert force of surface tension on it. If T be the surface tension of the solution, each surface will pull the wire parallel to itself with a force Tℓ. Thus, net force on the wire due to both the surface is 2Tℓ. Apply an external force F equal and opposite, to keep the wire in equilibrium. Thus, $F = 2T\ell$



Now, suppose the wire is moved through a small distance dx, the work done by the force is, dW = F dx = (2Tℓ) dx
But (2ℓ) (dx) is the total increase in area of both the surface of the film. Let it be dA.

Then, $dW = T\,dA$

$$\Rightarrow \quad T = \frac{dW}{dA}$$

Thus, *the surface tension T can also be defined as the work done in increasing the surface area by unity*. Further, since there is no change in kinetic energy, the work done by the external force is stored as the potential energy of the new surface.

$$T = \frac{dU}{dA} \quad [\text{as } dW = dU]$$

DROPS AND BUBBLES

- Work done (surface energy) in the formation of a water drop of radius r = work done against surface tension

  W = surface tension × change in area

  or $W = T \times \Delta A = T \times 4\pi r^2 = 4\pi r^2 T$
- Work done (surface energy) in the formation of a soap bubble of radius r

  $W = T \times \Delta A = T \times 2 \times 4\pi r^2 = 8\pi r^2 T$

  [$\because$ soap bubble has two surfaces]

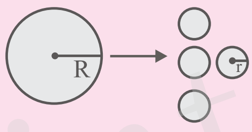
Knowledge ENHANCER

Dividing One Big Drop into Small Droplets

If a big drop is divided into small droplets then volume always remains conserved.

So, we divide a drop of radius R into n small droplets. Then

$$\frac{4}{3}\pi R^3 = n\frac{4}{3}\pi r^3$$



Big drop Small droplets

or $R^3 = nr^3 \Rightarrow n = \frac{R^3}{r^3} \Rightarrow n = \left(\frac{R}{r}\right)^3$

Initial surface area of drop, $4\pi R^2$

Final surface area of drop, $n(4\pi r^2)$

Change in area $\Delta A = n4\pi r^2 - 4\pi R^2 = 4\pi(nr^2 - R^2)$

So, work done against surface tension

$W = 4\pi T\,(nr^2 - R^2)$

or $W = 4\pi R^2 T\,(n^{1/3} - 1) = 4\pi R^2 T\left(\frac{R}{r} - 1\right)$ or $W = 4\pi R^3 T\left(\frac{1}{r} - \frac{1}{R}\right)$

In this process, there is increasing surface of the liquid. So energy is absorbed and decrease in temperature takes place. Due to this reason we feel cool near fountain.

Decrease in temperature $\Delta\theta = \frac{3T}{Jsd}\left(\frac{1}{r} - \frac{1}{R}\right)$

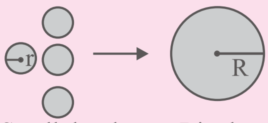
s = specific heat of liquid, d = density of liquid, J = mechanical equivalent of heat.

Small Droplets Coalesce to form a Big Drop

In this process, there is decrease in surface area so energy is liberated and the temperature of big drop increases.

Volume of big drop = Volume of n small drops.

$$n = \left(\frac{R}{r}\right)^3 \Rightarrow r = \frac{R}{n^{1/3}}$$



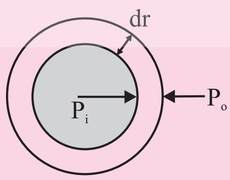
Small droplets Big drop

Thus work done

$W = 4\pi T\,(nr^2 - R^2)$ joules.

Increase in temperature of big drop :

$\Delta\theta = \frac{3T}{Jsd}\left(\frac{1}{r} - \frac{1}{R}\right)$; s = specific heat of liquid, d = density of liquid

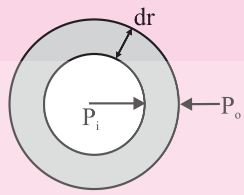


Excess pressure in liquid drop :

$$P = \frac{2T}{R}$$

Excess pressure in the bubble formed in a liquid :

$$P = \frac{2T}{R}$$



Excess pressure in soap bubble :

There is two free surfaces in a soap bubble, so $P = \frac{4T}{R}$

ANGLE OF CONTACT

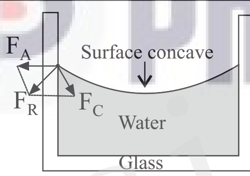
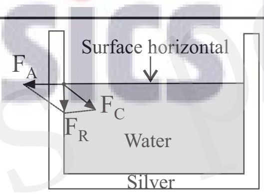
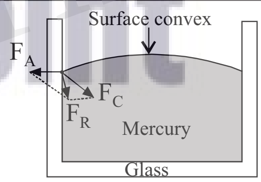
The angle enclosed between the tangent plane at the liquid surface and the tangent plane at the solid surface at the point of contact inside the liquid is termed as the ***angle of contact****.*

The angle of contact depends on the nature of the solid and liquid in contact.

Shape of liquid surface : When a liquid is brought in contact with a solid surface, the surface of the liquid becomes curved near the place of contact. The shape of the surface (concave or convex) depends upon the relative magnitudes of the cohesive force between the liquid molecules and the adhesive force between the molecules of the liquid and the solid.

The free surface of a liquid which is near the walls of a vessel and which is curved because of surface tension is known as **meniscus**. The cohesive force acts at a n angle 45° from liquid surface whereas the adhesive force acts at right angles to the solid surface.

The relation between cohesive and adhesive force, angle of contact and meniscus of the surface liquid :

Relation between cohensive and adhesive force	 Here, $F_A > \frac{F_C}{\sqrt{2}}$	 Here, $F_A = \frac{F_C}{\sqrt{2}}$	 Here, $F_A < \frac{F_C}{\sqrt{2}}$
Shape of meniscus	Concave	Plane	Convex
Angle of contact	$\theta_C < 90°$ (Acute angle)	$\theta_C = 90°$ (Right angle)	$\theta_C > 90°$ (Obtuse angle)
Level of liquid	Liquid rises up	Liquid neither	Liquid falls rises nor falls
Wetting property	Liquids wet	Liquid does not wet the solid surface	Liquid does not wet the solid surface
Example	Glass-water	Silver-water	Glass-mercury

Factors Affecting Angle of Contact

(i) **Temperature :** On increasing temperature surface tension decreases, thus cos θ_C increases $\left[\because \cos\theta_C \propto \frac{1}{T}\right]$ and θ_C decreases. So, on increasing temperature, θ_C decreases.

(ii) **Impurities :**

(a) Soluble impurities increase surface tension, so cos θ_C decreases and angle of contact increases.

(b) Partially soluble impurities decrease surface tension, so angle of contact θ_C decreases.

(iii) **Water proofing agent :** Angle of contact increases due to water proofing agent. It gets converted from acute to obtuse angle.

(iv) **Surfaces in contact :** Angle of contact depends upon the surfaces in contact.

Angle of contact of various solid-liquid pairs

Solid - liquid pair	**Angle of Contact θ_C**	
Glass -normal water	8°	Acute angle
Glass -distilled water	0°	
Glass - alcohol	0°	

Glass - mercury	135°	Obtuse angle
Paraffin wax - water	108°	Obtuse angle
Silver - water	90°	Right angle

ILLUSTRATION : 16

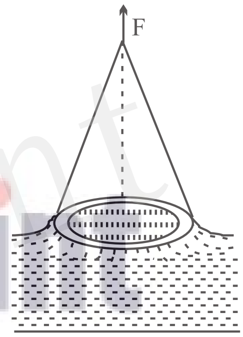
A film of water is formed between two straight parallel wires each 10 cm long and at separation 0.5 cm. Calculate the work required to increase 1 mm distance between the wires. Surface tension of water = 72×10^{-3} N/m.

SOLUTION :

Initial surface area = 2 × length × separation = 2×10 cm $\times 0.5$ cm = 10 cm^2 = 10×10^{-4} m^2
Final surface area = 2×10 cm $\times (0.5 + 0.1)$ cm = $2 \times 10 \times 0.6$ cm^2 = 12×10^{-4} m^2
Required work, $W = T \Delta A = 72 \times 10^{-3} \times (12 \times 10^{-4} - 10 \times 10^{-4})$ J $= 72 \times 10^{-3} \times 2 \times 10^{-4} = 144 \times 10^{-7}$ J

ILLUSTRATION : 17

A ring is cut from a platinum tube 8.5 cm internal and 8.7 cm external diameter. It is supported horizontally from a pan horizontally from a pan of balance so that it comes in contact with the water in a glass vessel. What is the surface tension of water if an extra 3.97 g weight is required to pull it away from water? (g = 980 cm/s^2)



SOLUTION :

The ring is in contact with water along its inner and outer circumference.
So when pulled out, the total force on it due to surface tension will be

$$F = T(2\pi r_1 + 2\pi r_2)$$

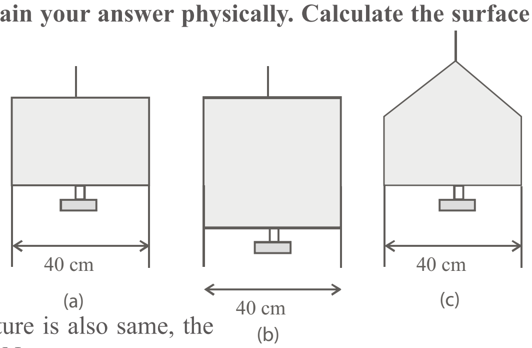
$$\Rightarrow \quad T = \frac{mg}{2\pi(r_1 + r_2)} \quad [\text{as } F = mg]$$

i.e., $T = \dfrac{3.97 \times 980}{3.14 \times (8.5 + 8.7)} = 72.13$ dyne/cm

ILLUSTRATION : 18

Figure (a) shows a thin liquid film supporting a small weight = 4.5×10^{-2} N. What is the weight supported by a film of the same liquid at the same temperature in Fig. (b) and (c) ? Explain your answer physically. Calculate the surface tension of liquid.

SOLUTION :



Length of the film = 40.0 cm. = 0.4 m

$\therefore$ Total weight supported = 4.5×10^{-2} N

$F = S \times 2L$

$$\therefore \quad S = \frac{F}{2L} = \frac{4.5 \times 10^{-2}}{2 \times 0.4} = 5.625 \times 10^{-2}\ \text{Nm}^{-1}$$

As length of the film supporting the weight is same and temperature is also same, the weight supported by the film will also remain same i.e., 4.5×10^{-2} N.

ILLUSTRATION : 19

A mercury drop of radius 1 cm is sprayed into 10^6 droplets of equal size. Calculate the energy expanded if surface tension of mercury is 35×10^{-3} N/m

SOLUTION :

If a drop of radius R is sprayed into n droplets of equal radius r, then as drop has only one surface, initial surface area will be $4\pi R^2$ while final area $n(4\pi r^2)$. So, the increase in area

$$\Delta S = n(4\pi r^2) - 4\pi R^2$$

So, energy expanded in the process

$$W = T\Delta A = 4\pi T(nr^2 - R^2) \quad \text{....(1)}$$

Now, since the total volume of n droplets is the same as that of initial drop, i.e.,

$$\frac{4}{3}\pi R^3 = n\left[\frac{4}{3}\pi r^3\right] \text{ or } r = R/n^{1/3} \quad \text{....(2)}$$

So, substituting the value of r from eq^n (2) in (1)

$W = 4\pi R^2 T\,[(n)^{1/3} - 1]$

So, here $W = 4 \times 3.14 \times (1 \times 10^{-2})^2 \times 35 \times 10^{-3}\,[10^2 - 1] = 4.356 \times 10^{-3}$ J

CAPILLARY RISE

A glass tube with fine bore and open at both ends is known as **capillary tube**. *The property by virtue of which a liquid rise or fall in a capillary tube is known as* ***capillary rise*** *or* ***fall*** *or* ***capillarity***. Rise or fall of liquid in tubes of narrow bore (capillary tube) is called capillary action. Rise of kerosene in lanterns, rise of ink in fountain pen etc. are due to capillary action.

Let the radius of the meniscus is R and the radius of the capillary tube is r. The angle of contact is θ, surface tension is T, density of liquid is ρ and the liquid rises to a height h.

Now let us consider two points A and B at the same horizontal level as shown. By Pascal's law

$$P_A = P_B \Rightarrow P_A = P_C + \rho gh$$

$$\Rightarrow P_A - P_C = \rho\, gh \qquad (\because P_B = P_C + \rho gh)$$

Now, the point C is on the curved meniscus which has P_A and P_C as the two pressures on its concave and convex sides respectively.

$$\Rightarrow P_A - P_C = \frac{2T}{R} = \frac{2T}{r/\cos\theta}$$

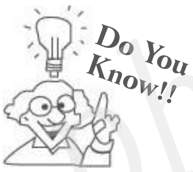
$$\Rightarrow \frac{2T}{r/\cos\theta} = \rho gh \Rightarrow 2T\cos\theta = r\rho\, gh \qquad ....(i)$$

$$\Rightarrow h = \frac{2T\cos\theta}{r\rho g} \qquad \text{Height of liquid rise in capillary tube}$$

Thus the height of rise of liquid in a capillary tube is inversely proportional to the radius of the capillary tube.

If T, θ, ρ and g are constant, $h \propto \frac{1}{r}$ or $rh = constant.$

This is **Zurin's law.** It implies that liquid will rise more in capillary tube of less radius and vice versa.



When the capillary tube is of insufficient length, the liquid will not overflow. It rises upto the top end of the tube and then adjusts the radius of curvature of its meniscus.

ILLUSTRATION : 20

The end of a capillary tube with a radius *r* is immersed in water. Is mechanical energy conserved when the water rises in the tube? The tube is sufficiently long. If not, calculate the energy change.

SOLUTION:

In the equilibrium position (θ = 0° for water and glass)

$$2\pi rT\cos 0° = \pi r^2 h\rho g \text{ or } h = \frac{2T}{\rho gr}$$

$$\text{Work done by surface tension} = (2\pi rT) \times h = \frac{4\pi T^2}{\rho g}$$

The potential energy of water in the tube, $U = (\pi r^2 h\rho)gh/2$; it is multiplied by $h/2$ because the centre of gravity of the water and the capillary tube is at a height $h/2$

$$\therefore U = \frac{2\pi T^2}{\rho g}$$

Thus, it is seen that the mechanical energy is not conserved.

$$\therefore \text{ Mechanical energy loss } = \frac{4\pi T^2}{\rho g} - \frac{2\pi T^2}{\rho g} = \frac{4\pi T^2}{\rho g}$$

This energy is converted into heat.

MISCELLANEOUS

SOLVED EXAMPLES

1. The area of cross section of a steel wire (Y = 2.0 × 10^{11} N/m^2) is 0.1 cm^2. Find the force required to double its length.

Sol. When the length of wire is doubled then

l = L and strain = 1

$\therefore Y = \text{strain} = \frac{F}{A}$

$\therefore$ Force = Y × A = 2 × 10^{11} × 0.1 × 10^{-4} = 2 × 10^6 N

2. When the length of a wire having cross-section area 10^{-6} m^2 is stretched by 0.1%, then tension in it is 100 N. Find the Young's modulus of material of the wire.

Sol. Area of cross-section, A = $10^{-6} m^2$ (given)

Young's modulus of the material of the wire,

$$Y = \frac{\left(\frac{T}{A}\right)}{\frac{\Delta l}{l}} = \frac{\left(\frac{100}{10^{-6}}\right)}{\left(\frac{0.1}{100}\right)} = \frac{100}{10^{-6}} \times \frac{100}{0.1} = \frac{10^4}{10^{-7}} = 10^{11}\,\text{N/m}^2$$

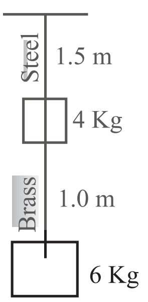
3. A wire is stretched by 0.01 m by a certain force F. Another wire of same material whose diameter and length are double to the original wire is stretched by the same force. Then, find its elongation.

Sol. As we know $l = \frac{FL}{\pi r^2 Y}$ $\therefore\ l \propto \frac{L}{r^2}$ (Y and F are constant)

$$\frac{l_2}{l_1} = \frac{L_2}{L_1} \times \left(\frac{r_1}{r_2}\right)^2 = (2) \times \left(\frac{1}{2}\right)^2 = \frac{1}{2}$$

$$\Rightarrow l_2 = \frac{l_1}{2} = \frac{0.01m}{2} = 0.005m$$

4. Two wires, one of steel and the other of brass are attached to two masses as shown in fig. The unloaded lengths of steel = 1.5 m, and brass = 1.0 m. The diameter of each wire is 0.25 cm. Y_{steel} = 2 × 10^{11} Pa and Y_{Brass} = 0.9 × 10^{11} Pa. Calculate the elongations of the steel and the brass wires. (Take g = 10 m/s^2).



Sol. The net downward force on the steel wire is

F_{steel} = (4 + 6) g = 10 × 10 = 100N

Therefore elongation of steel wire is

$$\Delta L_{steel} = \frac{(F_{steel}/A)\,L_{steel}}{Y_{steel}}$$

$$= \frac{100 \times 1.5}{3.14 \times \left(\frac{.25}{2} \times 10^{-2}\right)^2 \times 2 \times 10^{11}}$$

= 0.15 × 10^{-3} m = 0.15 mm

The tensile force on the brass wire is F_{brass} = 6g = 60 N

Therefore elongation of brass wire is

$$\Delta L_{brass} = \frac{(F_{brass}/A)\,L_{brass}}{Y_{brass}}$$

$$= \frac{60 \times 1.0}{3.14 \times \left(\frac{0.25}{2} \times 10^{-2}\right) \times 0.9 \times 10^{11}}$$

ΔL_{brass} = 0.13 × 10^{-3} m = 0.13 mm

5. A block of wood floats in water with two-third of its volume submerged. In oil the block floats with 0.90 of its volume submerged. Find the density of (a) wood and (b) oil. Density of water is 10^3 kg/m^3.

Sol. In case of floatation W = Th $\Rightarrow V\rho = V_{in}\sigma$

(a) For wood $V\rho = \frac{2}{3} V\sigma_w$

$\Rightarrow \rho = \frac{2}{3}\sigma_w = \frac{2}{2} \times 10^3 = 667\ \text{kg/m}^3$

(b) For oil, $V\rho = 0.9\ V\sigma_{oil} \Rightarrow \frac{2}{3} V\sigma_w = 0.9 V\sigma_{oil}$

$\therefore\ \sigma_{oil} = \frac{2}{3 \times 0.09}\sigma_w = \frac{2}{2.7} \times 10^3 = 740\ \text{kg/m}^3$

6. Water flows through a tunnel, a reservoir of dam towards the turbine installed in the power plant. Turbine is situated h m below the reservoir. If the ratio of cross-sectional areas of the tunnel at the reservoir and power station is η, find the speed of the water entering into the turbine.

Sol. Applying Bernoulli's theorem at reservoir and power plant for the flowing water, we obtain

$$P_0 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_0 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

$\Rightarrow v_2^2 = v_1^2 + 2g\,(h_1 - h_2)$.

Putting $(h_1 - h_2)$ =h we obtain

$v_2 = \sqrt{h_1 + h_2}$ ...(1)

Equation of continuity yields

$A_1 v_1 = A_2 v_2$ ...(2)

Eliminating v_1 from equation (1) and (2), we obtain

$$v_2 = \sqrt{\left(\frac{A_2}{A_1}v_2\right)^2 + 2gh}$$

$$\Rightarrow \; v_2 = \sqrt{\frac{2gh}{1-\left(\frac{A_2}{A_1}\right)^2}}$$

Putting $A_1/A_2 = \eta$,we obtain

$$v_2 = \eta\sqrt{\frac{2gh}{\eta^2 - 1}}$$

7. **Air flows horizontally with a speed v = 106 km/h. A house has plane roof of area A = 20m^2. Find the magnitude of aerodynamic lift of the roof.**

Sol. Air flows just above the roof and there is no air flow just below the roof inside the room. Therefore $v_1 = 0$ and $v_2 = v$. Applying Bermoulli's theorem at the points inside and outside the roof, we obtain.

$(1/2)\,\rho v_1^2 + \rho g h_1 + P_1 = (1/2)\,\rho v_2^2 + \rho g h_2 + P_2$.

Since $h_1 = h_2 = h$, $v_1 = 0$ and $v_2 = v_1$

$P_1 = P_2 + 1/2\,\rho v^2$

$P_1 - P_2 = \Delta P = 1/2\,\rho v^2$.

Since the area of the roof is A, the aerodynamic lift exerted on it = F = (ΔP) A

$\Rightarrow$ F = 1/2 ρAv^2

where ρ = density of air = 1.3 kg/m^3

A = 20 m^2, v = 29.44 m/sec.

$\Rightarrow$ F = {1/2 × 1.3 × 20 × (29.44)^2} N = 1.127 × 10^4 N.

8. **Mercury has an angle of contact equal to 140° with soda lime glass. A narrow tube of radius 1.00 mm made of this glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside ? Surface tension of mercury at the temperature of the experiment is 0.465 N m^{-1}. Density of mercury = 13.6 × 10^3 kg m^{-3}.**

Sol. Here θ = 140°, r = 1.00 mm = 10^{-3} m, S = 0.465 Nm^{-1}, ρ = 13.6 × 10^3 kg m^{-3}.

From formula $h = \dfrac{2S\cos\theta}{r\rho g}$

$$h = \frac{2\times 0.465\times\cos 140^\circ}{10^{-3}\times 13.6\times 10^3\times 9.8} \Rightarrow h = \frac{2\times 0.465\,(-\sin 50^\circ)}{13.6\times 9.8}$$

$\because$ cos 140° = cos (90 + 50)° = – sin 50°

$$\therefore \; = -\frac{2\times 0.465\times 0.7660}{13.6\times 9.8} = -5.34\times 10^{-3}\ \text{m} = -5.34\ \text{mm}$$

– *ve* sign shows that mercury dips down in the tube relative to outside liquid surface.

9. **A glass rod of diameter d_1 = 1.5 mm is inserted symmetrically into a glass capillary with inside diameter d_2 = 2.0 mm. Then the whole arrangement is vertically oriented and brought in contact with the surface of water. To what height will the liquid rise in the capillary? Surface tension of water is 73 × 10^{-3} N/m.**

Sol. If R is radius of meniscus, then

$$\frac{2T}{R} = h\rho g \text{ here } R = \frac{r_2 - r_1}{\cos\theta}$$

θ being angle of contact, r_1 = radius of glass rod, r_2 = radius of capillary.

$$\frac{2T\cos\theta}{r_2 - r_1} = h\rho g \quad\text{or}\quad h = \frac{2T\cos\theta}{(r_2 - r_1)\rho g}$$

$$\text{Here } r_1 = d_1/2,\ r_2 = \frac{d_2}{2} \quad \therefore\ h = \frac{4T\cos\theta}{(d_2 - d_1)\rho g}$$

Substituting given values and $\theta \cong 0^\circ$ for water-glass interface, we have

$$h = \frac{4\times 73\times 10^{-3}\cos 0^\circ}{(2.0-1.5)\times 10^{-3}\times 10^3\times 9.8} = 60\times 10^{-3}\text{m} = 6\text{cm}$$

10. **A vertical capillary with inside diameter 0.50 mm is submerged into water so that the length of its part emerging outside the water surface is equal to 25 mm. Find the radius of curvature of the meniscus. Surface tension of water is 73 × 10^{-3} N/m.**

Sol. In the capillary tube, the water should rise to a height

$$h = \frac{2T}{r\rho g} \quad \text{here } T = 73\times 10^{-3}\ \text{N/m},\ \rho = 10^3\,\text{kg/m}^3$$

$$\therefore\ h = \frac{2\times 73\times 10^{-3}}{0.25\times 10^{-3}\times 10^3\times 9.8} = 59\times 10^{-3}\,\text{m} = 59\ \text{mm}$$

Now $h > h'$ i.e., length is outside water surface.

Therefore radius of meniscus > radius of capillary r.

If R is the radius of meniscus, then we have

$$\frac{2T}{R} = h'\rho g \quad\text{or}\quad R = \frac{2T}{h'\rho g} \quad \text{here } h' = 25\ \text{mm} = 25\times 10^{-3}\ \text{m}$$

$$\therefore\ R = \frac{2\times 73\times 10^{-3}}{25\times 10^{-3}\times 10^3\times 9.8} = 0.6\times 10^{-3}\,\text{m} = 0.6\ \text{mm}$$

ADVANCED EXERCISE
BASED ON CONNECTING TOPICS

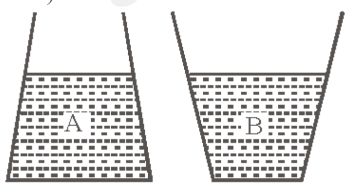
DIRECTIONS (Qs. 1-28): *This section contains multiple choice questions. Each question has 4 choices* (a), (b), (c) *and* (d)*, out of which ONLY ONE is correct.*

1. Two wires of same material and length but cross-sections in the ratio 1 : 2 are used to suspend the same loads. The extensions in them will be in the ratio of
(a) 1 : 2 (b) 2 : 1
(c) 4 : 1 (d) 1 : 4

2. The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied ?
(a) length = 50 cm , diameter = 0.5 mm
(b) length = 100 cm, diameter = 1 mm
(c) length = 200 cm, diameter = 2 mm
(d) length = 300 cm, diameter = 3 mm

3. An egg when placed in ordinary water sinks but floats when placed in brine. This is because
(a) density of brine is less than that of ordinary water
(b) density of brine is equal to that of ordinary water
(c) density of brine is greater than that of ordinary water
(d) None of these

4. Two vessels *A* and *B* of cross-sections as shown in figure contain a liquid up to the same height. As the temperature rises, the liquid pressure at the bottom (neglecting expansion of the vessels) will



(a) increase in *A*, decrease in *B*
(b) increase in *B*, decrease in *A*
(c) increase in both *A* and *B*
(d) decrease in both *A* and *B*

5. A water tank of height 10 m, completely filled with water is placed on a level ground. It has two holes one at 3 m and the other at 7 m from its base. The water ejecting from
(a) both the holes will fall at the same spot
(b) upper hole will fall farther than that from the lower hole
(c) upper hole will fall closer than that from the lower hole
(d) more information is required.

6. The rain drops falling from the sky neither injure us nor make holes on the ground because they move with
(a) constant acceleration
(b) variable acceleration
(c) variable speed
(d) constant terminal velocity

7. The angle of contact between pure water and pure glass, is
(a) 0º (b) 45º
(c) 90º (d) 135º

8. A wire fixed at the upper end stretches by length ℓ by applying a force F. The work done in stretching is
(a) $2F\ell$ (b) $F\ell$
(c) $\frac{F}{2\ell}$ (d) $\frac{F\ell}{2}$

9. A wire elongates by 1 mm when a load W is hanged from it. If the wire goes over a pulley and two weights W each are hung at the two ends, the elongation of the wire will be (in mm)
(a) 1 (b) 21
(c) zero (d) 1/2

10. The Bulk modulus for an incompressible liquid is
(a) zero (b) unity
(c) infinity (d) between 0 and 1

11. A beam of metal supported at the two edges is loaded at the centre. The depression at the centre is proportional to
(a) Y^2 (b) Y
(c) $1/Y$ (d) $1/Y^2$

12. An iron rod of length 2m and cross-sectional area of 50 mm^2 stretched by 0.5 mm, when a mass of 250 kg is hung from its lower end. Young's modulus of iron rod is
(a) 19.6×10^{20} N/m^2 (b) 19.6×10^{18} N/m^2
(c) 19.6×10^{10} N/m^2 (d) 19.6×10^{15} N/m^2

13. Which one of the following affects the elasticity of a substance ?
(a) Change in temperature
(b) Hammering and annealing
(c) Impurity in substance
(d) All of these

14. According to Hooke's law of elasticity, if stress is increased, then the ratio of stress to strain :
(a) becomes zero (b) remains constant
(c) decreases (d) increases

15. In a capillary tube, water rises to 3 mm. The height of water that will rise in another capillary tube having one-third radius of the first is
(a) 1 mm (b) 3 mm
(c) 6 mm (d) 9 mm

16. When an elastic material with Young's modulus Y is subjected to stretching stress S, elastic energy stored per unit volume of the material is

(a) $YS / 2$ (b) $S^2 Y / 2$
(c) $S^2 / 2Y$ (d) $S / 2i$

17. What per cent of length of wire increases by applying a stress of 1 kg weight/mm^2 on it?
$Y = 1 \times 10^{11}$ N/m^2 and 1 kg weight = 9.8 newton

(a) 0.0067% (b) 0.0098%
(c) 0.0088% (d) 0.0078%

18. Two soap bubbles are held by a tube. What will happen ?

(a) Air will travel from bigger to smaller bubble
(b) Air will not travel
(c) Air will travel through tube
(d) Nothing can be said

19. With the increase of temperature, the surface tension of the liquid

(a) may increase or decrease depending on the density of liquid
(b) remains the same
(c) always increases
(d) always decreases

20. In case of steel wire (or a metal wire), the limit is reached when

(a) the wire just break
(b) the load is more than the weight of wire
(c) elongation is inversely proportional to the tension
(d) None of these

21. The length of an iron wire is L and area of corss-section is A. The increase in length is l on applying the force F on its two ends. Which of the statement is correct

(a) Increase in length is inversely proportional to its length L
(b) Increase in length is proportional to area of cross-section A
(c) Increase in length is inversely proportional to A
(d) Increase in length is proportional to Young's modulus

22. A and B are two wires. The radius of A is twice that of B. They are stretched by the some load. Then the stress on B is

(a) equal to that on A (b) four times that on A
(c) two times that on A (d) half that on A

23. Two wires of equal lengths are made of the same material. Wire A has a diameter that is twice as that of wire B. If identical weights are suspended from the ends of these wires, the increase in length is

(a) four times for wire A as for wire B
(b) twice for wire A as for wire B
(c) half for wire A as for wire B
(d) one-fourth for wire A as for wire B

24. For a constant hydraulic stress on an object, the fractional change in the object volume $\left(\frac{\Delta V}{V}\right)$ and its bulk modulus (B) are related as

(a) $\frac{\Delta V}{V} \propto B$ (b) $\frac{\Delta V}{V} \propto \frac{1}{B}$
(c) $\frac{\Delta V}{V} \propto B^2$ (d) $\frac{\Delta V}{V} \propto B^{-2}$

25. Which of the following relation is true ?

(a) $3Y = K(1 -)$ (b) $K = \frac{9\eta Y}{Y + \eta}$
(c) $\sigma = (6K + \eta)Y$ (d) $\sigma = \frac{05.Y - \eta}{\eta}$

26. For a given material, the Young's modulus is 2. 4 times that of rigidity modulus. Its Poisson's ratio is

(a) 2.4 (b) 1.2
(c) 0.4 (d) 0.2

27. A 2 m long rod of radius 1 cm which is fixed from one end is given a twist of 0.8 radians. The shear strain developed will be

(a) 0.002 (b) 0.004
(c) 0.008 (d) 0.016

28. Small droplets of a liquid are usually more spherical in shape than larger drops of the same liquid because

(a) force of surface tension is equal and opposite to the force of gravity
(b) force of surface tension predominates the force of gravity
(c) force of gravity predominates the force of surface tension
(d) force of gravity and force of surface tension act in the same direction and are equal

DIRECTIONS (Qs. 29-34): *This section contains multiple choice questions. Each question has 4 choices* (a), (b), (c) *and* (d)*, out of which ONE or MORE may be correct.*

29. Two wires A and B have the same cross-section and are made of the same material, but the length of wire A is twice that of B. Then, for a given load :

(a) the extension of A will be twice that of B
(b) the strains in A and B will be equal
(c) the extensions of A and B will be equal
(d) the strain in A will be half that in B

30. Which of the following are correct ?

(a) According to Hooke's law, the ratio of stress and strain remains constant
(b) The shear modulus of a liquid is infinite
(c) Bulk modulus of a perfectly rigid body is infinity
(d) None of the above

31. Angle of contact between a liquid and a solid is a property of
(a) the material of the liquid
(b) the material of the solid
(c) the mass of the solid
(d) the shape of the solid

32. The rise of a liquid in a capillary tube depends on
(a) the inner radius of the tube
(b) the material of the tube
(c) the outer radius
(d) the length of the tube

33. When a drop of liquid splits up into number of drops
(a) energy is absorbed (b) energy is liberated
(c) area increases (d) volume increases

34. A liquid flows through a non-uniform pipe. The pressure in the pipe will be
(a) lower where the cross-section is smaller
(b) the same throughout the pipe
(c) higher where the cross-section is smaller
(d) higher where velocity of the liquid is smaller

DIRECTIONS (Qs. 35-36): *Following question has four statements (A, B, C and D) given in Column I and five statements (p, q, r, s and t) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in Column I with entries in Column II.*

35. Match the following.

Column - I	**Column - II**
(A) Bernoulli's theorem	(p) Lift of aeroplane
(B) Stoke's law	(q) Spinning of ball
(C) Archimede's principle	(r) Hydraulic lift
(D) Pascal's law	(s) Buoyant force
	(t) Terminal velocity

	A	B	C	D
(a)	p, q	t	s	r
(b)	t	p, r	s, t	p, q, r
(c)	p, s	q	r, s, t	r
(d)	p,	q, r	r	s

36. Match the following.

Column - I	**Column - II**
(A) Buoyancy	(p) capillarity
(B) Surface tension	(q) terminal velocity
(C) Elasticity	(r) law of floatation
(D) Viscosity	(s) Hooke's law
	(t) spherical shape of rain drops

	A	B	C	D
(a)	p,q	q	r, s	q, r
(b)	p, q, r, s	q	p, q, r, s	p, q, r, s
(c)	q	p, t	s	q
(d)	q, r	s	t	p, q, t

DIRECTIONS (Qs. 37-42): *Study the given paragraph(s) and answer the following questions.*

PASSAGE - I

In factories, rods are suspended from roof with the help of wires for certain mechanical operations. These operations are always limited by the elastic property of suspension wires and the rod. A rod of length 'l' is held horizontal by suspending it by two wires A and B of two different materials but equal length having areas of cross-sections 0.4 mm^2 and 0.6 mm^2. A weight W is hung from the rod. The value of Young's modulii of A and B respectively are $Y_A = 12 \times 10^{11}$ Nm^{-2} and $Y_B = 36 \times 10^{11}$ Nm^{-2}.

37. The weight is suspended to produce equal stress in two wires then the tensions in two wires T_A and T_B respectively are :
(a) $\frac{W}{2}, \frac{W}{2}$ (b) $\frac{2}{5}W, \frac{3}{5}W$
(c) $\frac{3}{5}W, \frac{2}{5}W$ (d) $\frac{W}{3}, \frac{2W}{3}$

38. In the above situation, the weight W is hanging from a point which divides the length of rod in the ratio $l_A : l_B$ as :
(a) 1 : 1 (b) 1 : 2
(c) 2 : 3 (d) 3 : 2

39. In the above questions what is the ratio of lengths $l_A : l_B$ for which the point of suspension of weight W will divide the length l, when strains produced in wires are equal ?
(a) 1 : 1 (b) 9 : 2
(c) 1 : 3 (d) 2 : 3

PASSAGE - II

Surface tension is the property of a liquid due to which free surface of liquid behaves as a stretched membrane and tends to have minimum possible surface area. The angle of contact determines the shape of meniscus. The shape of meniscus further tells about excess pressure which further explains capillary rise. Using the knowledge of surface tension, answer the following.

40. Which of the following statements is not correct ?
(a) Angle of contact increases with increase in temperature.
(b) Angle of contact increases with mixing of detergents in liquid.
(c) Angle of contact does not depend on inclination of wall of container
(d) Value of angle of contact is acute if it wets the wall of container.

41. The surface tension of liquid does not change with
(a) change in temperature of liquid
(b) electrification of liquid
(c) contamination of liquid
(d) change in nature of container for same liquid

42. Select the wrong statement.
(a) root cause of capillarity is the difference in pressure on two sides of free surface of liquid.
(b) rise of liquid in capillary tube against gravity violates the law of conservation of energy.
(c) height through which liquid rises or gets depressed depends on diameter of the tube
(d) ploughing of fields breaks the capillaries in soil block to stop rise of sub soil water to the surface of soil.

DIRECTIONS (Qs. 43-45): *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are correct and **Reason** is the correct explanation of **Assertion**.
(b) If both **Assertion** and **Reason** are correct, but **Reason** is not the correct explanation of **Assertion**.
(c) If **Assertion** is correct but **Reason** is incorrect.
(d) If **Assertion** is incorrect but **Reason** is correct.

43. **Assertion :** Steel is more elastic than rubber.
Reason : For same load, less strain is produced in strain.

44. **Assertion :** Rain drops fall through air with constant velocity.
Reason : Due to viscosity of air, rain drops acquire terminal velocity.

45. **Assertion :** Hydrostatic pressure is a scalar quantity.
Reason : Pressure is defined as force per unit area.

DIRECTIONS (Qs. 46-51): *Following are integer based/ Numeric based questions. Each question, when worked out will result in one integer or numeric value.*

46. A thick copper wire of density 1.5×10^3 kg m^{-3} and Young's modulus 5×10^6 N m^{-2}, 8 m in length is hung from the ceiling of a room. Find the increase in its length due to own weight.

47. A metallic rod breaks when strain produced is 0.2%. The Young's modulus of the material of the rod is 7×10^9 N/m^2. What should be its area of cross-section to support a load of 10^4 N ?

48. A spherical ball contracts in volume by 0.02% when subjected to a pressure of 100 atmosphere. Assuming one atmosphere = 10^5 N m^{-2}, what is the bulk modulus of the material of the ball ?

49. A material has poisson's ratio 0.5. If a uniform rod of it suffers a longitudinal strain of 2×10^{-3}, what is the percentage increase in volume?

50. **(a)** The bulk modulus of rubber is 9×10^8 N/m^2. To what depth below the surface of sea should the rubber ball be taken as to decrease its volume by 0.1%.
(b) A steel rail is 20 m long and has an area of cross section 40sq. cm. Between summer and winter its length changes by 1 cm. If it is laid in winter, what force parallel to its length is necessary to keep it from increasing the length in the summer?
($Y = 19 \times 10^{10}$ N/m^2).

51. A rubber cord catapult has a cross-sectional area 1 mm^2 and total unstretched length 10cm. It is stretched to 12 cm and then released to project a stone of mass 5 gm. Taking Young's modulus for rubber 5×10^8 N/m^2, find the velocity of projection of the stone.

SOLUTIONS

Brief Explanations of Selected Questions

ADVANCED EXERCISE
BASED ON CONNECTING TOPICS

1. (b) Let W newton be the load suspended. Then

$$Y = \frac{(W/A_1)}{(\ell_1/L)} = \frac{WL}{A_1 \ell_1} \quad ...(1)$$

and $$Y = \frac{(W/A_2)}{(\ell_2/L)} = \frac{WL}{A_2 \ell_2} \quad ....(2)$$

Dividing equation (1) by equation (2), we get

$$1 = \left(\frac{\ell_2}{\ell_1}\right)\left(\frac{A_2}{A_1}\right) = \left(\frac{\ell_2}{\ell_1}\right)\left(\frac{2}{1}\right)$$

$$\therefore \frac{\ell_1}{\ell_2} = \frac{2}{1} \text{ or } \ell_1 : \ell_2 = 2 : 1$$

2. (a) $$Y = \frac{T/A}{\Delta\ell/\ell}$$

$$\Delta\ell = \frac{T \times \ell}{A \times Y} = \frac{T}{Y} \times \frac{\ell}{A}$$

Hence, $\frac{T}{Y}$ is constant. Therefore, $\Delta\ell = \frac{\ell}{A}$

$\frac{\ell}{A}$ is largest in the first case.

3. (c) Brine due to its high density exerts an upthrust which can balance the weight of the egg.

4. (a) As temperature rises, the density decreases, height increases. In A, the top cross-section is smaller. Therefore, $h_A > h_B$

5. (a) Velocity of water from hole

$A = v_1 = \sqrt{2gh}$

Velocity of water from hole B

$= v_2 = \sqrt{2g(H_0 - h)}$

Time of reaching the ground from hole B

$= t_1 = \sqrt{2(H_0 - h)/g}$

Time of reaching the ground from hole A

$= t_2 = \sqrt{2h/g}$

6. (b)

7. (a) We know that angle of contact is the angle between the tangent to liquid surface at the point of contact and solid surface inside the liquid. In case of pure water and pure glass, the angle of contact is zero.

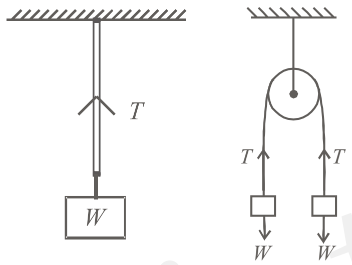
8. (d) Work done by constant force in displacing the object by a distance ℓ.

= change in potential energy

$$= \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

$$= \frac{1}{2} \times \frac{F}{A} \times \frac{\ell}{L} \times A \times L = \frac{F\ell}{2}$$

9. (a) Case (i) Case (ii)



At equilibrium, $T = W$

$$Y = \frac{W/A}{\ell/L} \quad .....(1)$$

Case (ii) At equilibrium $T = W$

$$\therefore Y = \frac{W/A}{\frac{\ell/2}{L/2}} \Rightarrow Y = \frac{W/A}{\ell/L}$$

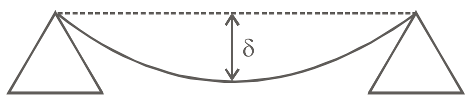
$\Rightarrow$ Elongation is the same.

10. (c) $$\text{Bulk Modulus} = \frac{\text{Pressure}}{\text{Volume Strain}} = \frac{\text{Pressure}}{0}$$

Bulk Modulus = ∞

[As liquid is incompressible, $\Delta V = 0$]

11. (c)



For a beam, the depression at the centre is given by,

$$\delta = \left(\frac{fL}{4Ybd^3}\right)$$

[f, L, b, d are constants for a particular beam]

12. (c) $$Y = \frac{F/A}{\Delta\ell/\ell} = \frac{\frac{250 \times 10}{50 \times 10^{-6}}}{\frac{0.5 \times 10^{-3}}{2}} = \frac{250 \times 9.8}{50 \times 10^{-6}} \times \frac{2}{0.5 \times 10^{-3}}$$

$\Rightarrow$ 19.6 × 10^{10} N/m^2

13. (d) The elasticity of a material depends upon the temperature of the material. Hammering & annealing reduces elastic property of a substance.

14. (b) The ratio of stress to strain is always constant. If stress is increased, strain will also increase so that their ratio remains constant.

15. (d) For rise in capillary, the formula is $h = \frac{2T}{r\rho g}$

So, for first capillary tube $h_1 = \frac{2T}{r_1 \rho g}$

For second, $h_2 = \frac{2T}{r_2 \rho g}$

$\frac{h_1}{h_2} = \frac{r_2}{r_1} \Rightarrow \frac{3}{h_2} = \frac{r_1}{3 \times r_1} \left[r_2 = \frac{r_1}{3} \right]$

$h_2 = 9$ mm

16. (c) Energy stored per unit volume

$= \frac{1}{2} \times \text{stress} \times \text{strain}$

$\frac{1}{2} \times \text{stress} \times (\text{stress/Young's modulus})$

$\frac{1}{2} \times (\text{stress})^2 / (\text{Young's modulus}) = \frac{S^2}{2Y}$

17. (b) Stress = 1 kg wt/mm^2 = 9.8 N/mm^2
= 9.8 × 10^6 N/m^2.

$Y = 1 \times 10^{11}$ N/m^2, $\frac{\Delta\ell}{\ell} \times 100 = ?$

$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\text{Stress}}{\Delta\ell / \ell}$

$\therefore \quad \frac{\Delta\ell}{\ell} = \frac{\text{Stress}}{Y} = \frac{9.8 \times 10^6}{1 \times 10^{11}}$

$\frac{\Delta\ell}{\ell} \times 100 = 9.8 \times 10^{-11} \times 100 \times 10^6$

$= 9.8 \times 10^{-3} = 0.0098$ %

18. (a) The excess pressure inside a soap bubble is given by, $p = \frac{4T}{r}$. As the excess of pressure is less in bigger bubble means pressure is more inside bigger bubble. So, air travels from bigger bubble to smaller.

19. (d) Surface-Tension is the property of liquid at rest. As we increase temperature, due to gain in kinetic energy of molecules, surface tension decreases.

20. (d) According to Hooke's Law, within the elastic limits stress is directly proportional to strain.

21. (c) $l = \frac{FL}{YA} \Rightarrow l \propto \frac{1}{A}$

22. (b) Stress $= \frac{\text{force}}{\text{Area}}$ $\therefore$ Stress $\propto \frac{1}{\pi r^2}$

$\frac{S_B}{S_A} = \left(\frac{r_A}{r_B} \right)^2 = (2)^2 \Rightarrow S_B = 4S_A$

23. (d) $l = \frac{FL}{AY} \Rightarrow l \propto \frac{1}{r^2}$ (F, L and Y are same)

$\frac{l_A}{l_B} = \left(\frac{r_B}{r_A} \right)^2 = \left(\frac{r_B}{2r_B} \right)^2 = \frac{1}{4} \Rightarrow l_A = 4l_B$ or $l_B = \frac{l_A}{4}$

24. (b) $B = \frac{\Delta p}{\Delta V / V} \Rightarrow \frac{1}{B} \propto \frac{\Delta V}{V}$ [Δp = constant]

25. (d) $Y = 2\eta(1+\sigma) \Rightarrow \sigma = \frac{0.5Y - \eta}{\eta}$

26. (d) $Y = 2\eta(1+\sigma)$

$2.4\eta = 2\eta(1+\sigma) \Rightarrow 1.2 = 1 + \sigma \Rightarrow \sigma = 0.2$

27. (b) $r\theta = L\phi \Rightarrow 10^{-2} \times 0.8 = 2 \times \phi \Rightarrow \phi = 0.004$

28. (b)

29. (a, b) We have, $Y = \frac{\text{Stress}}{\text{Strain}}$

As Y and stress are same for both A and B, strain must be the same.

Again, strain $= \frac{\Delta\ell}{\ell}$

For same strain, $\Delta\ell \propto \ell$

Hence extension in A is twice that in B.

30. (a, c)

According to Hooke's law, ratio of stress to strain is constant. Also for liquids, bulk modulus is zero and for a perfectly rigid body, modulus of rigidity is infinite.

31. (a, b)

Angle of contact depends on the nature of material of liquid and the container.

32. (a, b)

33. (a, c)

When a drop splits into a number of smaller drops, volume remains conserved but surface area and hence energy increases.

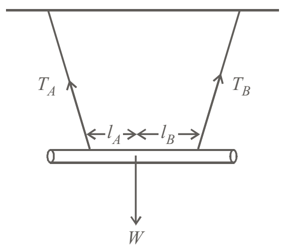
34. (a, d)

Pressure is smaller where velocity is higher and velocity is higher where area is smaller.

35. (a) (A) → (p, q); (B) → (t); (C) → (s); (D) → (r)

36. (c) (A) → (q); (B) → (p, t); (C) → (s); (D) → (q)

37. (b) For equal stress



$\frac{T_A}{a_A} = \frac{T_B}{a_B}$

or $T_A = \frac{0.4}{0.6} T_B = \frac{2}{3} T_B$ .... (1)

Also, $T_A + T_B = W$ .... (2)

On solving,

$T_A = \frac{2}{5} W$; $T_B = \frac{3}{5} W$

38. (d) For rotatory equilibrium,

$T_A l_A = T_B l_B$

$l_A : l_B = T_B : T_A = 3 : 2$

39. (b) For equal strains,

$$\frac{(\text{Stress})_A}{Y_A} = \frac{(\text{Stress})_B}{Y_B} \quad \text{or} \quad \frac{T_A}{a_A Y_A} \quad \frac{T_B}{a_B Y_B}$$

or $\frac{T_A}{T_B} = \frac{a_A Y_A}{a_B Y_B} = \left(\frac{0.4}{0.6}\right)\left(\frac{12\times10^{11}}{36\times10^{11}}\right) = \frac{2}{9}$

$\therefore \quad l_A : l_B = T_B : T_A = 9 : 2$

40. (b) Angle of contact decrease on adding impurities like detergents.

41. (d) Surface tension is property of liquid and does not depend on nature of container.

42. (b) Rise of liquid in capillarity does not violet the law of conservation of energy as work done by the force of surface tension is responsible for increase in potential energy of liquid.

43. (a) Material which extend less for same load are more elastic.

44. (a) Both assertion and reason are true and reason is correct explanation of assertion.

45. (b)

46. Use, $\Delta\ell = \frac{mg\ell}{YA} = \frac{\rho Al.gl}{YA} = \frac{\rho g \ell^2}{A}$

$\Delta\ell = 9.4$ mm

47. Maximum possible strain = 0.2/100

$\therefore \quad A = \frac{F}{Y\times strain} = \frac{10^4 \times 100}{(7\times10^9)\times0.2} = 7.1\times10^{-4}$

48. $B = -\frac{\Delta P}{\Delta V/V} = \frac{100\times10^5}{(0.02/100)} = 50\times10^9$ N/m^2

49. Here

$$\frac{\Delta V}{V} = \frac{\Delta(\pi r^2 \ell)}{\pi r^2 \ell} = \frac{r^2\,\Delta\ell + 2\,r\,\ell\,\Delta r}{r^2\,\ell}$$

or $\frac{\Delta V}{V} = \frac{\Delta \ell}{\ell} + 2\frac{\Delta r}{r}$

Now $\sigma = -\frac{\Delta r/r}{\Delta\ell/\ell}$ *or* $\frac{\Delta r}{r} = -\sigma\frac{\Delta\ell}{\ell}$;

$\therefore \quad \frac{\Delta r}{r} = -0.5\times(2\times10^{-3}) = -1\times10^{-3}$

Further, $\frac{\Delta V}{V} = (2\times10^{-3}) - 2\times(1\times10^{-3}) = 0$

$\therefore$ % increase in volume is 0%.

50. **(a)** $\Delta P = B\left(\frac{v}{V}\right)$ Here $\left(\frac{v}{V}\right) = \frac{0.1}{100}$

$\therefore \quad \Delta P = 9 \times 10^5$

Atmospheric pressure at sea leavel = 10 m of water column.

Let the ball is taken to a depth h. Then $\Delta P = (h - 10)\,\rho\,g$

$9 \times 10^5 = (h - 10)(1000)(10)$

$\therefore h = 100$ m

(b) Strain $= \frac{1 \text{ cm}}{20\times100 \text{ cm}} = 0.5\times10^{-3}$

Stress = Y × strain = $(19 \times 10^{10})(0.5 \times 10^{-3})$

= 9.5×10^7 N/m^2

Force = Area of cross section × stress

= $(40 \times 10^{-4}) \times (9.5 \times 10^7) = 3.8 \times 10^5$ Newton.

51. Here, elastic potential energy = K.E. of stone

$\therefore$ 1/2 × Y × (strain)^2 × volume of cord = 1/2 mv^2

or $v = \sqrt{\left\{\frac{Y\times(\text{strain})^2\times\text{volume of cord}}{m}\right\}}$

Given that strain $= \frac{12-10}{10} = \frac{1}{5}, Y = 5\times10^8\,\text{N/m}^2$

Volume of the cord

$= (1 \times 10^{-6}) \times 10 \times 10^{-2} = 1 \times 10^{-7}$ m^3

Mass m of the stone = 5×10^{-3} kg

Hence, $v = \sqrt{\left\{\frac{(5\times10^8)\times(1/5)^2\times(1\times10^{-7})}{5\times10^{-3}}\right\}}$

$= \frac{1}{5}\times\sqrt{(10000)} = \frac{1}{5}\times100 = 20\,\text{m/s}$

Chapter 9

Heat and Thermodynamics

INTRODUCTION

Throw a ball vertically upwards in air. It rises up and again falls back downwards. The question arises here that why does the ball fall down; Why does it not go upwards ? This question got an answer in the year 1665 when Newton provided the theory of gravitation to the world. According to his theory, earth attracts every body towards itself with a force known as 'gravity'. Due to the force of gravity the ball doesn't go upwards but it falls downwards after covering some vertical distance.

Actually, every object attracts every other object towards itself with a force. This force is called the gravitational force. Gravitational force is one among the four fundamental forces. It is always attractive in nature. This chapter is basically an outline of the theory of gravitation, acceleration due to gravity (the acceleration produced in an object due to the force of gravity), motion of a satellite and the Kepler's laws (laws governing the motion of a planet).

HEAT

We know that when a hotter body is kept in contact with a colder body for some time, the colder body becomes hot and hotter body cools down till the bodies have the same temperature i.e, the bodies are said to be in **thermal equilibrium**. This is because heat energy has transferred from hotter body to the colder body. **Hot and cold are relative terms**. Heat is a form of energy which is also called thermal energy. *Heat or thermal energy is the sum of all types of kinetic energies (translational, vibrational, rotational) of all the molecules of the body*.

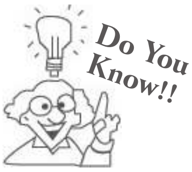
The **SI unit** of heat energy is joule (J), practical unit of heat energy is Calorie.

"One calorie is the amount of heat required to raise temperature of one gram of water from 14.5°C to 15.5°C."

1 Calorie = 4.186 joule

If W amount of work done is converted into Q amount of heat, then $J = \frac{W}{Q}$; J - mechanical equivalent of heat

J is not a physical quantity it is just a conversion factor.



Just as water flows from a higher to lower level, heat flows from a body at higher temperature to that at lower temperature.

TEMPERATURE

Temperature is defined as the degree of hotness or coldness of a body. Spontaneous transfer of heat takes place from higher temperature to lower temperature. Temperature is a measure of average translational kinetic energy of a molecule.

To measure temperature of a body we use an instrument called **thermometer**. The design of a thermometer is based on some thermometric property of material used in thermometer. Thermometric property is a property of a material which varies linearly with temperature. For example – length, volume, pressure, resistance etc.

The common temperature scales are Celsius, Fahrenheit and Kelvin. The lower fixed point is ice point and upper fixed point is boiling point of water.

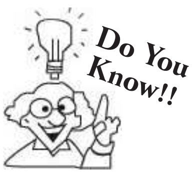
$$\frac{C-0}{100-0} = \frac{F-32}{212-32} = \frac{K-273}{373-273} \Rightarrow \frac{C}{5} = \frac{F-32}{9} = \frac{K-273}{5}$$

If ΔC, ΔF and ΔK are change in temperatures, then $\Delta C = \frac{5}{9}\Delta F = \Delta K$

ABSOLUTE TEMPERATURE

The lowest temperature of –273.16 °C at which a gas is supposed to have zero volume and zero pressure and at which entire molecular motion stops is called absolute zero temperature. A new scale of temperature starting with –273.16°C by Lord Kelvin as zero. This is called Kelvin scale or absolute scale of temperature.

T(K) = t°C + 273.16



Normal temperature of human body is 310.15 k (= 37°C = 98.6°F).

Relation Between Thermometric Property and Temperature

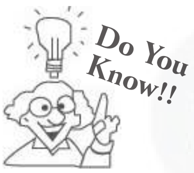
If x_0, x_{100} and x_t be the value of thermometric properties at temperature 0°C, 100°C and t°C then

$$t°C = \frac{x_t - x_0}{x_{100} - x_0} \times 100$$

Other than thermometer, thermocouple and total radiation pyrometer are also used for measurement of temperature. Total radiation pyrometer is used for measurements of very high temperatures. It is based on Stefan's Boltzmann law for radiation. The advantage of total radiation pyrometer is that the body whose temperature is to be measured is not kept in contact with it. It can be used to measure temperatures above 800°C. Ranges of different thermometers are given below.

Range of Various Thermometers

Types of thermometer	Range	
	Lower limit	Upper limit
1. Mercury thermometer	– 20°C	– 333°C
2. Alcohol thermometer	– 110°C	– 55°C
3. Bimetallic thermometer	– 50° C	– 555°C
4. Gas thermometer	– 268°C	– 1000°C
5. Platinum resistance thermometer	– 200°C	– 1500°C
6. Thermoelectric thermometer	– 200°C	– 3000°C
7. Pyrometer	800°C	– No limit



At a temperature of 273.16 k and a pressure of 0.46 cm of Hg water exist on all three phase i.e., ice, liquid water and vapour. This is called triple point of water.

CHECK Point

- **What is the temperature for which the reading on Celsius and Fahrenheit scales are same?**

Solution

Let x be the temperature, which has the same reading on Celsius and Fahrenheit scales.

Then, $C = F = x$

From the relation : $\frac{C}{5} = \frac{F-32}{9}$, we have

$\frac{x}{5} = \frac{x-32}{9}$ or x = – 40°

Thus, – 40°C and – 40°F represent the same temperature.

THERMAL EXPANSION

When a body (almost all) is heated it expands. The expansion can take place in the length, area or volume of the body. Depending upon the expansion in length, area or volume we have three types of expansion.

(i) Linear Expansion:

Let l_1 be the length of a wire at temperature 'θ_1' when temperature is increased to θ_2, length increases to l_2 then

$\Delta l \propto \Delta\theta$ ($\Delta l = l_1 - l_2$ change in length & change in temperature $\Delta\theta = \theta_1 - \theta_2$)

Also, $\Delta l \propto l_1$ so, $\Delta l = \alpha l_1 \Delta\theta$

$$\Rightarrow \alpha = \frac{\Delta l}{l_1 \Delta\theta} \quad \text{or} \quad l_2 = l_1(1 + \alpha\,\Delta\theta)$$

Where α is **coefficient of linear expansion**. Its unit is /°C or /K. It depends upon the nature of material. The value of 'α' also depends on temperature but very slightly infact, the change is extremely small even for over a temperature range of 1000°C.

(ii) Superficial or Areal Expansion:

Increase in surface area of a solid when temperature is increased. If A_1 and A_2 be the surface area at temperature θ_1 and θ_2 respectively then

$$\Delta A \propto \Delta\theta \ \ (\Delta A = A_2 - A \ \& \ \ \Delta\theta = \theta_2 - \theta_1)$$

$$\Delta A = \beta A_1 \Delta\theta \ \Rightarrow \ \beta = \frac{\Delta A}{A_1 \Delta\theta}$$

or, $$A_2 = A_1 (1 + B\Delta\theta)$$

'β' is **coefficient of superficial expansion** of a solid. Its unit is /°C and /K, it depends upon nature of material.

(iii) Cubical or Volume Expansion:

Increase in volume of a substance on heating. If V_1 and V_2 are volumes of a substance at temperature q_1 and q_2 respectively, then

$$\Delta V = V_1 \gamma \Delta\theta$$

$$(\Delta V = V_2 - V_1, \Delta\theta = \theta_1 - \theta_2)$$

$$\gamma = \frac{\Delta V}{V_1 \Delta\theta} \ \text{ or, } V_2 = V_1(1 + \gamma\Delta\theta)$$

Where 'γ' is **coefficient of cubical expansion** of solid. Its unit is /°C or /K and it depends upon the nature of material.

The relation between α, β and γ is $\alpha = \frac{\beta}{2} = \frac{\gamma}{3} \quad \Rightarrow \alpha : \beta : \gamma = 1 : 2 : 3$

Do You Know!!

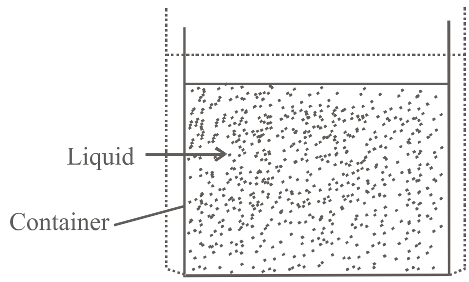
The coefficient of linear, superficial and cubical expansion (α, β and γ) are not constant. Their values are different in different temperature ranges.

IDEA BOX — idea box

As the size of degree is same on celsius and absolute scales, the units of α, β and γ may be written as $°C^{-1}$ (in place of K^{-1}) without affecting their values.

EXPANSION OF LIQUID

When we heat a liquid which is kept inside a container then liquid as well as the container both expand. In this case the observed expansion of liquid will be apparent expansion. But if the container were not expand then the expansion will be real expansion.



Coefficient of real expansion $\gamma_r = \dfrac{\text{real increase in volume}}{\text{original volume} \times \Delta\theta}$

Coefficient of apparent expansion $\gamma_a = \dfrac{\text{apparent increase in volume}}{\text{original volume} \times \Delta\theta}$

If γ_g is coefficient of volume expansion of material of container then

$$\gamma_r = \gamma_g + \gamma_a$$

Anomalous Expansion of Water

Almost all liquids expand on heating but water when heated from 0°C to 4°C its volume decreases and hence density increases until its temperature reaches 4°C. Its density is maximum at 4°C on further heating its density decreases. This behaviour of water is called anomalous behaviour of water.

CALORIMETRY

We know that the spontaneous transfer of heat is from a hot body to colder body. If heat exchange with the surrounding is negligible then *the total heat lost by a hot body is always equal to the heat gained by the cold body,* this is the **principle of calorimetry or, law of mixture.**

Specific Heat Capacity

When we supply heat to a body, its temperature rises. If m is mass, $\Delta\theta$ is temperature rise and Q is the heat supplied, then

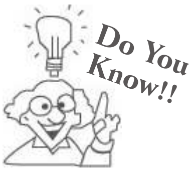
$$Q \propto M \Rightarrow Q \propto \Delta\theta \quad \text{or } Q = Ms\Delta\theta \Rightarrow s = \frac{Q}{M\Delta\theta}$$

Where '*s*' is constant called specific heat which depends upon the nature of material and its surrounding.

The **S.I. unit** of specific heat is $\frac{\text{J}}{\text{Kg°C}}$ and other unit is $\frac{\text{cal}}{\text{g°C}}$.

If $M = 1$ and $\Delta\theta = 1$ then $s = Q$

So, specific heat capacity of a material is equal to the heat required to raise temperature of unit mass from 14.5°C *to* 15.5°C. Specific heat for H_2 is maximum i.e. 3.5 cal g^{-1} $°C^{-1}$ and minimum for radon and actinium 0.22 cal g^{-1} $°C^{-1}$. Specific heat of water is 1 cal g^{-1} $°C^{-1}$. Specific heat depends upon the state of substance for example for water it is 1 cal g^{-1} $°C^{-1}$, for ice 0.5 cal g^{-1} $°C^{-1}$ and for steam 0.47 cal g^{-1} $°C^{-1}$.



When two substances are mixed together, the final temperature of the mixture can never be less than the temperature of the colder substance or more than the temperature of the hotter substance.

Molar Heat Capacity and Heat Capacity

Molar heat capacity of a substance is defined as the amount of heat required to raise the temperature of one mole of a substance by unit degree. Its unit is J $mol^{-1}°C^{-1}$ or Cal $mol^{-1}°$ C^{-1}

$$s_m = \frac{Q}{n\Delta\theta} \qquad n = \text{no. of moles}$$

Heat capacity of a substance is amount of heat required to raise temperature of a body by unit degree. It is represented by *C*, its unit is $J°C^{-1}$ or cal $°C^{-1}$. Heat capacity depends upon nature of material and its mass.

$$C = \frac{Q}{\Delta\theta} = \text{ms}$$

Water Equivalent and Latent Heat

Water Equivalent of a body is defined as the mass of water which has the same heat capacity as that of the body. It is represented by *W*. Its unit is gram.

$Ws_w = ms_b$: s_w - specific heat of water

s_b - specific heat of body; m - mass of body

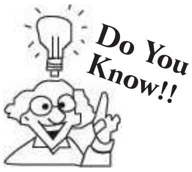
Latent Heat or Hidden Heat : When state of a substance changes, change of state takes place at constant temperature (m.pt. or B. pt.) heat is released or absorbed and is given by $Q = mL$ where ***L* is latent heat**. The **S.I. unit** of latent heat is **J/kg**.

Latent heat of fusion or melting (L_f): It is the amount of heat required to change unit mass of solid into liquid state at its melting point. It is represented by L_f, its unit is Jkg^{-1} or $Calg^{-1}$, for ice its value is 80 cal g^{-1}.

$$Q = mL_f$$

Latent heat of vaporisation or boiling (L_v): It is the amount of heat required to change unit mass of liquid into its vapors at its boiling point. It is represented by L_v, its unit is Jkg^{-1}or $Calg^{-1}$. For water $L_v = 540$ cal g^{-1}.

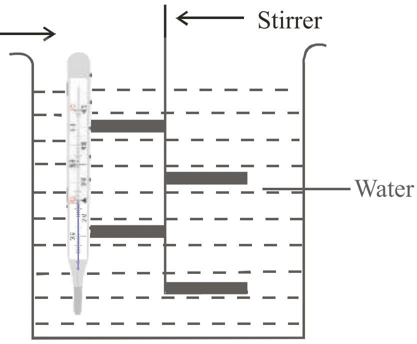
$$Q = mL_v$$



Thermal capacity of a body and its water equivalent are numerically equal but have different units. Thermal capacity is measured in JK^{-1} (or cal $°C^{-1}$), water equivalent is measured in kg (or g).

CALORIMETER

Calorimeter is a device used to measure specific heat of a solid. Solid is heated to high temperature then put into calorimeter, after some equilibrium reaches.



According to principle of calorimetry

Heat lost by solid = Heat gain by water + Calorimeter

$$m_1 s_1(\theta_1 - \theta_f) = m_2 s_2(\theta_f - \theta_2) + m_3 s_3(\theta_f - \theta_2)$$

$$m_1 s_1(\theta_1 - \theta_f) = (m_2 s_2 + m_3 s_3)\ (\theta_f - \theta_2)$$

When water equivalent (w) of calorimeter is given

$$m_1 s_1(\theta_1 - \theta_f) = (m_2 + w) s_2\ (\theta_f - \theta_2)$$

M_1 - mass of solid

s_1 - specific heat of solid

θ_1 - initial temperature of solid

M_2 - mass of water

s_2 - specific heat of water

θ_2 - initial temperature of water

θ_f - equilibrium temperature

m_3 - mass of calorimeter

s_3 - specific heat of calorimeter

θ_2 - temperature of calorimeter

$\theta_2 < \theta_f < \theta_1$

ILLUSTRATION : 1

The design of some physical apparatus requires that there be a constant difference in length at any temperature between an iron and copper of cylinders laid side by side. What should be the length of cylinders at 0°C for the difference in length to be 20 cm at all temperatures. (Given : $\alpha_{iron} = 1.1 \times 10^{-5}$°C^{-1}, $\alpha_{copper} = 1.7 \times 10^{-5}$°C^{-1})

SOLUTION :

We know that for the above condition to be satisfied

$l_1\alpha_1 = l_2\alpha_2$; l_1 - length of iron cylinder; l_2 - length of copper cylinder

$l_1 - l_2 = 20$ or, $l_1 = l_2 + 20$,

$(l_2 + 20) \times 1.1 \times 10^{-5} = l_2 \times 1.7 \times 10^5$

$1.1\ l_1 + 22 = 1.7 l_2$

Solving we get, $l_2 = \frac{110}{3}$cm, $\quad l_1 = \frac{170}{3}$cm

ILLUSTRATION : 2

Three liquids *A, B* and *C* are at temperatures 30°C, 40°C and 50°C respectively. When 2g of *A* and 4 g of *B* are mixed final temperature is 36°C and when 3 g of *B* and 4 g of *C* are mixed final temperature is 44°C. Find the final temperature when equal masses of *A* and *C* are mixed.

SOLUTION :

Let s_A, s_B and s_C be the specific heats of A, B and C, then

Heat lost by B = Heat gained by A

$4 \times s_B \times (40 - 36) = 2 \times s_A \times (50 - 36)$ $\qquad (Q = ms\ \Delta\theta)$

$4s_B = 3s_A$

Similarly, Heat gained by B = Heat lost by C

$3 \times s_B \times (44 - 40) = 4 \times s_C \times (50 - 44) \Rightarrow S_B = 2S_C$

Now, $m \times s_A\ (\theta_f - 30) = ms_C\ (50 - \theta_f)$

$$\frac{4}{3} s_B(\theta_f - 30) = \frac{s_B}{2}(50 - \theta_f) \ \Rightarrow\ \theta_f = 35.45°C$$

ILLUSTRATION : 3

A 100 g of copper calorimeter contains 150 g of oil at 20°C. An aluminium block of mass 60 g is heated upto 400°C and put into the calorimeter, find equilibrium temperature. s_{cu} = 0.39 J g^{-1} °C^{-1}, s_{Al} = 0.88 Jg^{-1} °C^{-1} and s_{oil} = 1.6 Jg^{-1} °C^{-1}.

SOLUTION :

Using the formula, $M_1 s_1(\theta_1 - \theta_f) = (m_2 s_2 + m_3 s_3)(\theta_f - \theta_2)$

$60 \times 0.88\,(400 - \theta_f) = (100 \times 0.39 + 150 \times 1.6)\,(\theta_f - 20)$

$52.8\,(400 - \theta_f) = 279\,(\theta_f - 20) \Rightarrow \theta_f = 80.47°C$

ILLUSTRATION : 4

50 g of ice at – 20°C is heated, untill whole of it evaporates. How much heat energy is needed? Latent heat of fusion L_f = 80 cal g^{-1}, specific heat of ice = 0.5 cal g^{-1}°C^{-1}, latent heat of vaporisation = 540 cal g^{-1}°C^{-1}.

SOLUTION :

Q_1 = heat required to bring ice from – 20°C to 0°C $Q_1 = ms_{ice}.\Delta\theta$

$Q_1 = 50 \times 0.5 \times (20) = 500$ cal ....(i)

Q_2 = heat required to change ice into water at 0°C $Q_2 = mL_F$

$Q_2 = 50 \times 80 = 4{,}000$ cal ....(ii)

Q_3 = heat required to change temp. of water from 0°C to 100°C $Q_3 = ms_w \times \Delta\theta$

$Q_3 = 50 \times 1 \times (100 - 0) = 5{,}000$ cal ....(iii)

Q_4 = heat required to vaporise water at 100°C to steam $Q_4 = mL_V$

$Q_4 = 50 \times 540 = 27{,}000$ cal ....(iv)

Total heat needed = $Q_1 + Q_2 + Q_3 + Q_4 = 36{,}500$ cal

ILLUSTRATION : 5

A mixture of 250 g water and 200 g ice at 0°C is kept in a calorimeter of water equivalent 50 g. If 200 g of steam at 100°C is passed through this mixture, calculate the final temperature and weight of the contents of calorimeter. Given, latent heat of fusion of ice = 80 cal/g, latent heat of vaporisation = 540 cal/g.

SOLUTION :

Here, the heat contained by the steam (mass 200g) is much more than the heat required to bring (water + ice + calorimeter) to 100°C. Hence the final temperature would be 100°C.

In order to find how much steam is condensed, let us use principle of calorimetry.

Heat required by 200 g ice to convert into water at 0°C,

$Q_1 = 200 \times 80 = 16{,}000$ cal.

Now, we have (250 + 200) g = 450 g of water + calorimeter which has to be brought at temperature 100°C,

$Q_2 = (450 + 50) \times 1 \times 100$

$Q_2 = 50{,}000$ cal

So the total heat rquired = 50,000 + 16,000 = 66,000 cal.

Suppose x g of steam get condensed, so the energy lost by it, $x \times 540$ cal,

So $540x = 66{,}000 \Rightarrow x = 122.2$ g

So the weight of contents = 200 + 250 + 122.22 = 572.22 g

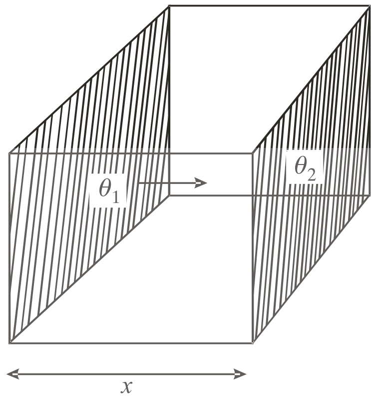
HEAT TRANSFER

Heat energy can be transferred from a body at higher tamperature to a body at lower temperature by three different ways viz. *conduction*, *convection* and *radiation*.

Conduction

***Conduction** is the process in which heat is transmitted from one point to the other through the substance without the actual motion of the particles.* When one end of a metal is heated, the molecules at the hot end start vibrating with higher amplitudes (kinetic energy) and transmit this K.E. to the next molecule and so on. However, the molecules still remain in their mean positions of equilibrium. This process of conduction is prominent in the case of solids.

Coefficient of Thermal Conductivity (*K*)



Consider a cube of side x and area of each face A. The opposite faces of the cube are maintained at temperatures θ_1 and θ_2 where $\theta_1 > \theta_2$. Heat gets conducted in the direction of the fall of temperature.

Quantity of heat Q conducted across the two opposite faces is given by

$Q \propto A$

$Q \propto (\theta_1 - \theta_2)$; $\quad Q \propto t$

$\propto \frac{1}{x}$

$$\therefore \quad Q \propto \frac{A(\theta_1 - \theta_2)}{x}t$$

or $\quad Q = \frac{KA(\theta_1 - \theta_2)\,t}{x} \quad$ or $\quad K = \frac{Qx}{A(\theta_1 - \theta_2)\,t}$

Here K is a constant called the **coefficient of thermal conductivity** of the material of the cube and t stands for time-interval. Quite often x is written as d also representing depth of penetration of heat.

We can also write it as $H = KA\left|\frac{dT}{dx}\right|$

where H = heat flow per second,

$\frac{\Delta Q}{\Delta t}$ = heat current

T = θ, temperature

Also K is sometimes defined as follows.

If $\quad A = 1\ m^2, (\theta_1 - \theta_2) = 1\ ^\circ C,\ t = 1\ s,\ x = 1\ cm,$

then $Q = K$

Thus, the **coefficient of thermal conductivity** K is defined as *the amount of heat flowing in unit time across the opposite faces of a cube of side having unit length maintained at unit temperature difference.*

Do You Know!!

The quantity $\frac{dx}{KA}$ *is called* ***thermal resistance*** *of a material. It is analogous to electrical resistance given by* $R = \frac{\ell}{\sigma A}$, *where* $\sigma = \frac{1}{\rho}$ *is electrical conductivity.*

idea box

If two slabs of thicknesses d_1, d_2; coefficients of thermal conductivity K_1, K_2 and the end faces at temperatures T_1, T_2 are placed in contact, the rate of flow of heat across them is same.

i.e. $\quad \frac{K_1 A(T_1 - T)}{d_1} = \frac{K_2 A(T - T_2)}{d_2}$

Here, T is the temperature at their common surface.

Also, the temperature at the common surface is given by $T = \frac{A(T_1 - T_2)}{d_1/K_1 + d_2/K_2}$

CONVECTION

Convection *is the process in which heat is transmitted from one place to the other by the actual movement of the vibrating particles.* It is prominent in the case of liquids and gases. Land and sea breezes and trade winds are formed due to convection. Convection plays an important part in ventilation, gas filled electric lamps and heating of buildings by hot water circulation.

It is the process of transfer of heat in a fluid by the movement of the fluid itself. There are two distinct types of convection:

(i) *Natural (or free) convection*, when the motion of the fluid is due solely to the presence of the hot body in it giving rise to temperature and hence density gradients, the fluid thus moving under the influence of gravity. This is natural or free convection.

(ii) *Forced convection*, in which a relative motion between the hot body and the fluid is maintained by some external agency (e.g. a drought), the relative velocity being such as to make the contribution of the gravity currents negligible.

RADIATION

***Radiation** is the process in which heat is transmitted from one place to the other directly without the necessity of any intervening medium.* We get heat radiations directly from the sun without affecting the intervening medium. Heat radiations can pass through vacuum. Heat radiations are a part of the electromagnetic spectrum.

It is mode of heat transfer in which no material medium is needed. In the year 1792, Prevost proposed that all bodies radiate thermal radiation at all temperatures which depends upon the nature of emitting surface, its area and temperature.

Radiation has the following properties:

(a) Radiant energy travels in straight line and when some object is placed in the path, it's shadow is formed at the detector.

(b) It is reflected and refracted or can be made to interfere. The reflection or refraction are exactly as in case of light.

(c) It can travel through vacuum.

(d) Intensity of radiation follows the law of inverse square.

(e) Thermal radiation can be polarised in the same way as light by transmission through a nicol prims.

Reflectance, Absorptance and Transmittance

Let Q be the thermal radiations incident on a body in a given time, Q_a, Q_r and Q_t be the amount of heat radiations absorbed, reflected and transmitted respectively, then

$$Q = Q_a + Q_r + Q_t$$

$$1 = \frac{Q_a}{Q} + \frac{Q_r}{Q} + \frac{Q_t}{Q} = a + r + t$$

where, a = absorptance, r = reflectance and t = transmittance

If $t = 0$ then $a = 1 - r$ i.e. good reflectors and bad absorbers and vice-versa. Absorptance, reflectance and transmittance depends upon (i) nature of the surface of body (ii) wavelength of incident radiation but not on the nature of material of body.

Emissive power, Absorptive power and Kirchhoff's law

Emissive Power : It is the total amount of thermal energy emitted per unit time, per unit area of the body for all wavelength. It is denoted by 'E' its SI unit is Watt m^{-2} or $Js^{-1}\ m^{-2}$.

Absorptive Power : It is defined as the ratio of radiation absorbed by a body to the incident radiation.

i.e., Absorptive power, $a = \dfrac{\text{energy absorbed}}{\text{energy incident}}$

Kirchhoff's Law : According to Kirchhoff's law the ratio of emissive power to absorptive power is the same for all bodies at a given temperature and it is equal to the emissive power of a black body at that temperature.

$$\frac{E\,(\text{body})}{a\,(\text{body})} = E(\text{black body})$$

Significance of Kirchhoff's law is that good absorbers are good emitters. It also explains existence of Fraunhoffer lines.

Stefan's Boltzmann Law

According to this law the total amount of heat energy radiated per unit time, per unit area by a perfectly black body is directly proportional to the fourth power of its absolute temperature.

$\rho = \sigma AT^4$ where, Stefan's constant $\sigma = 5.67 \times 10^{-8}\ Jm^{-2}\ s^{-1}K^4$

If it is not a perfectly black body, then

$$\rho = \sigma eAT^4 \,;\ e = \text{emissivity} = \frac{E(\text{body})}{E(\text{black body})}$$

If T_s is the temperature of surrounding then body will absorb radiation for surrounding.

$P_a = \sigma eAT_s^4$

So, net heat radiation loss $P_{net} = \sigma eA\,(T^4 - T_s^4)$

Wien's Displacement Law

When a body radiates thermal energy it is a mixture of different wavelengths, but for a particular wavelength, intensity of thermal radiation is maximum, which is referred as λ_m . According to **Wien's displacement law, wavelength corresponding to highest intensity (λ_m) is inversely proportional to the absolute temperature of the body.**

i.e., $$\lambda_m \propto \frac{1}{T} \quad \Rightarrow T\lambda_m = b$$

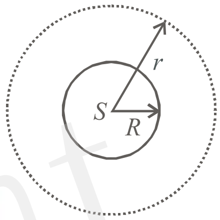
b = Wien's constant = 0.288 cm-K

Solar constant and temperature of the Sun

Solar constant is the amount of solar radiation received per unit area per unit time on the surface of the earth when unit area is held in a direction perpendicular to the direction of sun rays.

$S = 1388 \text{Wm}^{-2}$ or $2 \text{ Cals}^{-1} \text{ cm}^{-2} \text{ min}^{-1}$

Solar luminosity is defined as the amount of energy emitted per second by the sun in all direction, it is represented by L_S.
r = distance between the earth and the sun, R = radius of the sun
Energy radiated by the sun = energy received by the sphere of radius r



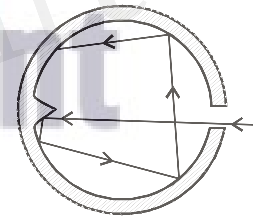
$$\sigma(4\pi R^2)T^4 = 4\pi r^2 S \Rightarrow T = \left(\frac{r^2 S}{R^2\sigma}\right)^{1/4}$$

The surface temperature of the sun is about 5800 K.

Ferry's Black Body

A black body absorbs the entire thermal radiation incident on it, practically there is no body which absorbs 100(%) percent radiations incident on it. Ferry designed a black body which a spherical enclosure painted black from inside with a small hole in the wall.

Any radiation through this hole goes inside and get absorbed after multiple reflections. There is cone directly opposite to the hole due to which incident radiation is not reflected back through the hole.



Newton's Law of Cooling

The rate of cooling of a body is directly proportional to the temperature difference between the body and surrounding and exposed area.

$$\frac{d\theta}{dt} = -KA(\theta - \theta_0)$$

where, K is a constant, A = area, θ = temperature of body = $\frac{\theta_i + \theta_f}{2}$; θ_0 = temperature of surrounding

THERMODYNAMIC PROCESSES

Thermodynamics is the branch of science in which the conversion of heat into mechanical work and vice-versa is studied.
Thermodynamic process is said to take place if some change occurs in the state of a thermodynamic system, i.e. the thermodynamic variables of the system – pressure, volume, temperature and entropy change with time.
In practice, the following types of thermodynamic processes can take place :

(a) **Isothermal process.** A thermodynamic process that takes place at constant temperature.

(b) **Isobaric process.** A thermodynamic process that takes place at constant pressure.

(c) **Isochoric process.** A thermodynamic process that takes place at constant volume.

(d) **Adiabatic process.** A thermodynamic process in which no heat enters or leaves the system.

(e) **Cyclic process.** A thermodynamic process in which the system returns to its original state.

Sign conventions for the study of thermodynamic processes
The following sign conventions are adopted in the study of thermodynamical process :

(i) Heat gained by a system is taken as positive while that lost by a system is taken as negative.

(ii) The work done by a system is taken as positive while that done on the system is taken as negative.

(iii) Increase in the internal energy of a system is taken as positive. Decrease in the internal energy of system is taken as negative.

The internal energy of a gas is sum of internal energy due to moleculer motion (called internal kinetic energy U_K) and internal energy due to molecular configuration (called internal potential energy $U_{P.E.}$) i.e.,

$$U = U_K + U_{P.E.} \quad ....(1)$$

(a) In ideal gas, as there is no intermolecular attraction, hence

For n mole ideal gas $U = U_K = \frac{3n}{2}RT \quad ....(2)$

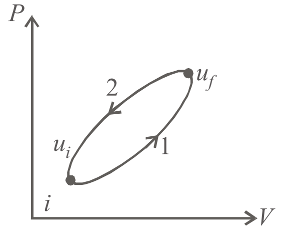
(b) Internal energy is path independent i.e., point function.

(c) In cyclic process, there is no change in internal energy i.e.,

$dU = U_f - U_i = 0$

$\Rightarrow U_f - U_i$

(d) Internal energy of an ideal gas depends only on temperature.



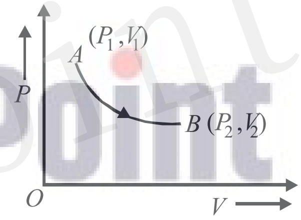
Indicator Diagram or P-V Diagram

The equation of state of a gas is $PV = \mu RT$. Out of the three variables P, V and T, if any two are known, the third can be calculated. So, two thermodynamic variables are sufficient to describe the behavior of a thermodynamic system.

P-V diagram is a graph between the volume V and the pressure P of the system. The volume is plotted against X-axis while the pressure is plotted against Y-axis.

Fig. shows an indicator or P-V diagram. The point A represents the initial stage of the system. P_1 and V_1 are the initial pressure and initial volume respectively of the system. The point B represents the final state of the system. P_2 and V_2 are the final pressure and final volume respectively of the system. The points between A and B represent the intermediate states of the system.

The indicator diagram helps us to calculate the amount of work done by the gas or on the gas during expansion or compression.



Work Done by a Thermodynamic System

A gas in a cylinder with a movable piston is a simple example of a thermodynamic system.

Figure shows a gas confined to a cylinder that has a movable piston at one end. If the gas expands against the piston, it exerts a force on the piston and displace it through a distance and does work on the piston.

If the piston compresses the gas as it is moved inward, work is also done–in this case on the gas. The work associated with such volume changes can be determined as follows.

Let pressure of gas on the piston be P.

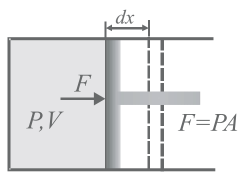
Then the force on the piston due to gas is $F = PA$, where A is the area of the piston.

When the piston is pushed outward an infinitesimal distance dx, the work done by the gas is

$$dW = F \, . \, dx = PA \, dx$$

Since the change in volume of the gas is $dV = Adx$,

$$\therefore \quad dW = PdV$$



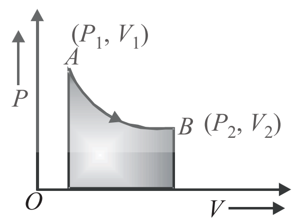
A = area of piston

For a finite change in volume from V_i to V_f, this equation is then integrated between V_i to V_f to find the net work

$$W = \int dW = \int_{V_i}^{V_f} PdV$$

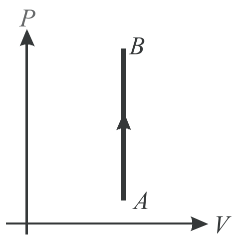
The work done by a gas is also equal to the area under P-V graph.

Following different cases are possible.

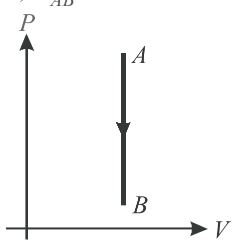


Shaded area = work done

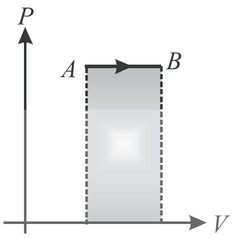
When volume is constant : V = constant, $W_{AB} = 0$



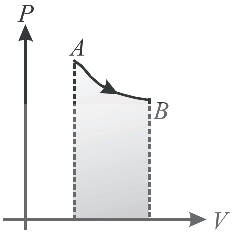
or



When volume is increasing (Expansion) : V is increasing, $W_{AB} > 0$, W_{AB} = Shaded area

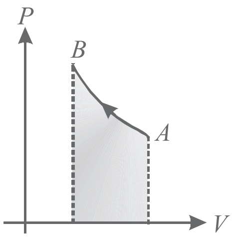


or

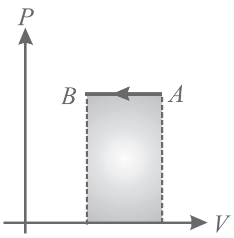


When volume is decreasing (Compression)

V is decreasing, $W_{AB} < 0$, W_{AB} = – Shaded area



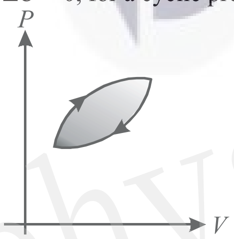
or



CYCLIC PROCESS

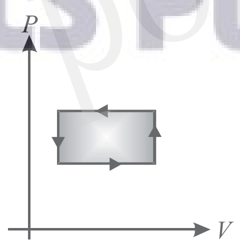
It is a thermodynamic process in which a system is taken through a series of changes and finally brought back to its initial state. In this process, the change in internal energy of the system is zero.

$\Delta U = 0$, for a cyclic process.



(a)

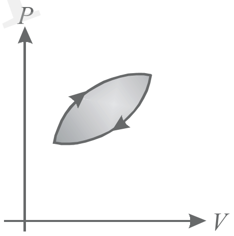
$W_{\text{clockwise cycle}}$ = + Shaded area



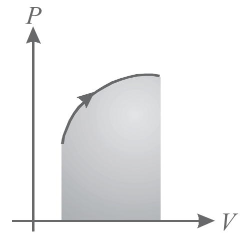
(b)

$W_{\text{anticlockwise cycle}}$ = – Shaded area

Work Done in Clockwise Cycle

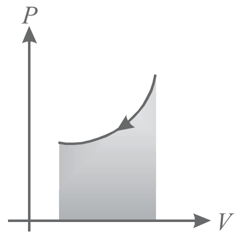


=

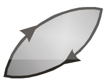


$W_{AB\,\text{I}}$ Positive

=



$W_{AB\,\text{II}}$ Negative

$W_{\text{cyclic}} = W_{AB\,\text{I}}$ Positive + $W_{AB\,\text{II}}$ Negative  area of closed path

FIRST LAW OF THERMODYNAMICS

According to **first law of thermodynamics**, *if some quantity of heat is supplied to a system capable of doing external work, then the quantity of heat absorbed by the system is equal to the sum of the increase in the internal energy of the system and the external work done by the system.*

i.e., $\delta Q = \delta U + \delta W$

The first law of thermodynamics is essentially a restatement of the law of conservation of energy, i.e., energy can neither be created nor be destroyed but may be converted from one form to another.

In applying the first law of thermodynamics, all the three quantities, i.e., δQ, dU and δW must be expressed in the same units, i.e., either in units of work or in units of heat.

Sign conventions for the first law of thermodynamics :

Quantity	Definition	Meaning of +ve sign	Meaning of –ve sign
Q	Heat flow	Heat flows into the system	Heat flows out of the system
W	Work done	Surroundings do positive work on the system	Surroundings do negative work on the system
ΔU	Internal energy change	Internal energy increases	Internal energy decreases

Limitations of First Law of Thermodynamics

(i) It does not explain the direction of heat flow.

(ii) It does not explain how much amount of heat given will be converted into work.

Significance : The first law of thermodynamics tells us that it is impossible to get work from any machine without giving it an equivalent amount of energy.

Adiabatic Process

It is that thermodynamic process in which pressure, volume and temperature of the system change but there is no exchange of heat between the system and the surroundings.

A process has to be sudden and quick to be adiabatic.

Equation of state : $PV = \mu RT$

Equation for adiabatic process PV^γ **= constant**

Let initial state of system is (P_1, V_1, T_1) and after adiabatic change final state of system is (P_2, V_2, T_2) then we can write,

$P_1 V_1^\gamma = P_2 V_2^\gamma = K$ (*here K is const.*)

Work done $W = \int_{V_1}^{V_2} PdV$ or $W = \frac{1}{(\gamma - 1)}[P_1V_1 - P_2V_2]$

$$W = \frac{\mu R}{(\gamma - 1)}(T_1 - T_2) \qquad (\because PV = \mu RT)$$

Adiabatic Relation Between P and V for Ideal Gas

$PV^\gamma = constant$

If P_1, V_1 be the initial and P_2, V_2 be the final pressures and volumes respectively of the gas for an adiabatic change.

Then $P_1V_1^\gamma = P_2V_2^\gamma$

Adiabatic Relation Between V and T for Ideal Gas

$TV^{\gamma - 1} = constant$

If V_1, T_1 be the initial and V_2, T_2 be the final volumes and temperatures respectively of the gas for an adiabatic change, then

$T_1V_1^{\gamma - 1} = T_2V_2^{\gamma - 1}$

Adiabatic Relation Between P and T for Ideal Gas

$T^\gamma P^{1 - \gamma} = constant$

If P_1, T_1 be the original and P_2, T_2 be the final pressures and temperatures respectively of gas for an adiabatic change, then

$T_1^\gamma P_1^{1 - \gamma} = T_2^\gamma P_2^{1 - \gamma}$

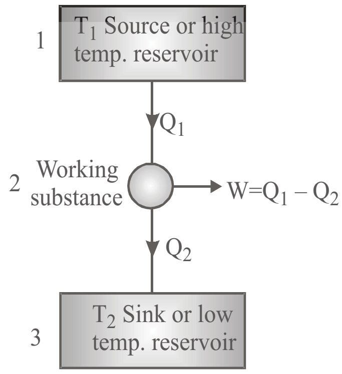
HEAT ENGINES

Heat engine is a device which converts heat energy into work. A heat engine, in general, consists of three parts :

(1) A source or high temperature reservoir at temperature T_1.

(2) A working substance.

(3) A sink or low temperature reservoir at temperature T_2.



In a cycle of heat engine the working substance extracts heat Q_1 from source, does some work W and rejects remaining heat Q_2 to sink.

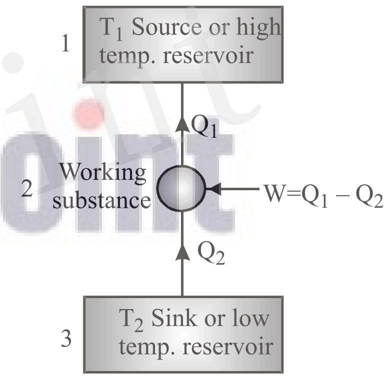
Efficiency of heat engine, $\eta = \dfrac{Work\ \ done\ \ (W)}{Heat\ \ taken\ \ from\ \ soruce\ \ (Q_1)}$

$$\eta = \frac{T_1 - T_2}{T_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

The useful work is done in third stroke called work stroke or power stroke.

The efficiency of internal combustion engine is approximately 40% to 60%.

REFRIGERATORS AND HEAT PUMPS



A refrigerator is the reverse of a heat engine. A heat pump is the same as a refrigerator.

The coefficient of performance of a refrigerator or heat pump

$$\alpha = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} \quad [\because W = Q_1 - Q_2]$$

ILLUSTRATION : 6

A gram molecule of a gas at 127°C expands isothermally, until volume is doubled. Find the amount of work done and heat absorbed.

SOLUTION :

Here, $T = 127 + 273 = 400\ K$

Suppose that $V_1 = V$ then, $V_2 = 2V$

For 1 mole of a gas, work done during isothermal change

$$W = 2.303\ R\ T \log \frac{V_2}{V_1} = 2.303 \times R \times 400 \log \frac{2V}{V}$$

$= 2.303 \times R \times 400 \log 2 = 2.303 \times R \times 400 \times 0.3010$

or $W = 277.28\ R$ erg .... (i)

Now, $R = \dfrac{76 \times 13.6 \times 980 \times 22,400}{273} = 8.31$ erg mole^{-1} °C^{-1}

Therefore, the equation (i) gives

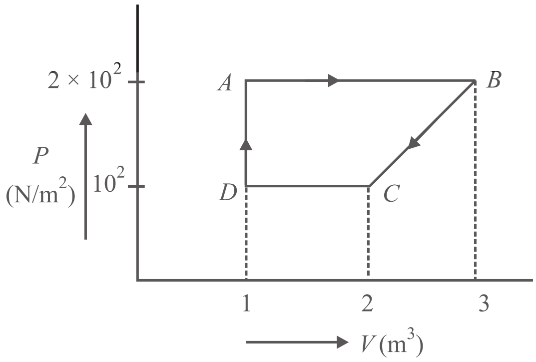
$W = 277.28 \times 8.31 \times 10^7 = 2.304 \times 10^{10}$ erg

For isothermal change, $dQ = dW$ ($\because$ dU is zero)

Therefore, heat absorbed $= \dfrac{2.30 \times 10^{10}}{4.2 \times 10^7} = 548.67$ cal

ILLUSTRATION : 7

A cyclic process is shown in Fig. Find the work done during isobaric expansion *A – B*.



SOLUTION :

Isobaric expansion is represented by curve *AB*

Work done = area under *AB*

$= 2 \times 10^2 \times (3 - 1) = 4 \times 10^2 = 400$ J.

ASSUMPTIONS OF KINETIC THEORY OF GASES

The theory is based on following assumptions as regards to the motion of molecules and the nature of the gases.

Assumptions of the kinetic theory of gases

(1) All the molecules of a gas are identical . The molecules of different gases are different.

(2) The molecules are rigid and perfectly elastic spheres of very small diameter.

(3) Gas molecules occupy very small space. The actual volume occupied by the molecule is very small compared to the total volume of the gas. Therefore volume of the gas is equal to volume of the vessel.

(4) The molecules of gases are in a state of random motion, i.e., they are constantly moving with all possible velocities lying between zero and infinity in all possible directions.

(5) Normally no force acts between the molecules. Hence they move in straight line with constant speeds.

(6) The molecules collide with one another and also with the walls of the container and change there direction and speed due to collision. These collisions are perfectly elastic i.e., there is no loss of kinetic energy in these collisions.

(7) The molecules do not exert any force of attraction or repulsion on each other except during collision. So, the molecules do not posses any potential energy. Their energy is wholly kinetic.

(8) The collisions are instantaneous i.e., the time spent by a molecule in a collision is very small as compared to the time elapsed between two consecutive collisions.

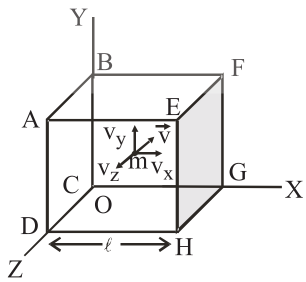
(9) Though the molecules are constantly moving from one place to another, the average number of molecules per unit volume of the gas remains constant.

(10) The molecules inside the vessel keep on moving continuously in all possible directions, the distribution of molecules in the whole vessel remains uniform.

(11) The mass of a molecule is negligibly small and the speed is very large, there is no effect of gravity on the motion of the molecules. If this effect were there, the density of the gas would have been greater at the bottom of the vessel.

Expression for the Pressure Exerted by a Gas

A gas exerts pressure on the walls of the containing vessel due to continuous collisions of the molecules against the wall.



Consider a gas enclosed in a cube shape container having each side length ℓ.

The area of each face of the cube is $A = \ell^2$.
The volume of the cube is $V = \ell^3$.
Let the total number of molecules of the gas inside the cube = N
The mass of each molecule = m
Suppose that the three intersecting edges of the cube are along the rectangular co-ordinate axes X, Y, and Z with the origin O at one corner of the cube.
Consider a molecule which has a velocity

$$\vec{v} = v_x\,\hat{i} + v_y\,\hat{j} + v_z\,\hat{k}$$

If this molecule collides the face EFGH, it will rebound with its x component of velocity $(-v_x)$. There will be no effect on v_y or v_z and after collision molecule will move with constant velocity along straight path along –ive X-axis.

∴ Change in momentum of the molecule
= final momentum – initial momentum = $-mv_x - (mv_x) = -2mv_x$

Since the total momentum is conserved, the momentum imparted by the molecule to the wall of the container in this impact to the face EFGH = $2mv_x$.

∴ Change in momentum of the surface EFGH = $+2mv_x$

Some time later, the molecule strikes the opposite wall ABCD and eventually returns to the wall EFGH.
Between two successive collisions with the same face EFGH, the molecule covers a distance 2ℓ.

Therefore, the time between two successive collisions at the same face EFGH = $\dfrac{2\ell}{v_x}$.

Hence in one second, the number of collisions of the molecule with face EFGH = $\dfrac{v_x}{2\ell}$.

Change in momentum of surfaces EFGH per second due to this molecule
= [change in momentum in one collision] × [number of collisions per second]

$$= 2mv_x \times \frac{v_x}{2\ell} = \frac{mv_x^2}{\ell}$$

To find the total change in momentum per second at the wall, we add the contributions of all these molecules.
Change in momentum per second

$$= \frac{m}{\ell}\left(v_{x1}^2 + v_{x2}^2 + v_{x3}^2 + ... + v_{xN}^2\right)$$

By Newton's second law of motion the change in momentum per second is equal to the force.

$$\therefore \quad F_x = \frac{m}{\ell}\left(v_{x1}^2 + v_{x2}^2 + v_{x3}^2 + ... + v_{xN}^2\right)$$

So, pressure on this wall

$$P_x = \frac{\text{Force}}{\text{Area}} = \frac{F_x}{\ell^2} = \frac{m}{\ell^3}\left(v_{x1}^2 + v_{x2}^2 + v_{x3}^2 + ... + v_{xN}^2\right)$$

Similarly, on the walls normal to Y and Z axes

$$P_y = \frac{m}{\ell^3}\left(v_{y1}^2 + v_{y2}^2 + ... + v_{yN}^2\right)$$

and $$P_z = \frac{m}{\ell^3}\left(v_{z1}^2 + v_{z2}^2 + ... + v_{zN}^2\right)$$

Since molecule are in random motion so they exert same pressure on all the phases of cubical vessel. As the choice of the axes has been arbitrary, so $P_x = P_y = P_z = P$

Therefore, $$P = \frac{P_x + P_y + P_z}{3}$$

$$= \frac{m}{3\ell^3}\left[\begin{array}{r}(v_{x1}^2 + v_{y1}^2 + v_{z1}^2) + (v_{x2}^2 + v_{y2}^2 + v_{z2}^2) \\ + ...(v_{xN}^2 + v_{yN}^2 + v_{zN}^2)\end{array}\right]$$

$$P = \frac{mN}{3\ell^3}\left[\frac{v_1^2 + v_2^2 + ... + v_N^2}{N}\right] = \frac{Nm}{3V}v_{rms}^2$$

$\because \quad PV = \frac{1}{3} Nm.v_{rms}^2$

or $\quad P = \frac{1}{3} \frac{M}{V} \overline{v^2} \qquad (v_{rms}^2 = \overline{v^2})$

$P = \frac{1}{3} \rho v_{rms}^2 \;\; (\because \frac{Nm}{V} = \rho$ density of gas)

Relation between pressure and kinetic energy

From kinetic theory of gases,

$$P = \frac{1}{3} \frac{Nm}{V} v_{rms}^2 \quad \text{or} \quad PV = \frac{2}{3} N \left(\frac{1}{2} mv_{rms}^2\right)$$

$\therefore \quad N\left(\frac{1}{2} mv_{rms}^2\right) = \frac{3}{2} PV$

But, $\frac{1}{2} mv_{rms}^2$ = average K.E. of a gas molecule.

$\therefore$ *Total K.E. of a gas* $E = N\left(\frac{1}{2} mv_{rms}^2\right) = \frac{3}{2} PV$

$\therefore$ *K.E. per unit volume of the gas* $E = \frac{3}{2} P$

The pressure exerted by a gas is numerically equal to $\frac{2}{3}$ rd of the kinetic energy of the molecules present per unit volume of the gas.

Different Types of Speeds of Gas Molecules– RMS, Most Probable and Average

Root mean square speed(v_{rms}) : *The square root of mean of square speed is called root mean square speed or rms speed.*

$\because \quad P = \frac{1}{3} \frac{M}{V} \overline{v^2} \quad \text{or} \quad 3PV = M\,\overline{v^2}$

$$\overline{v^2} = \frac{3PV}{M}$$

$$\sqrt{\overline{v^2}} = \frac{3P}{\rho} \quad \left(\because \rho = \frac{M}{V}\right)$$

$$v_{rms} = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M_w}} = \sqrt{\frac{3kT}{m}}$$

$(\because R = N_A k$ and $M_w = m \,.\, N_A)$

(a) For the molecules of a given gas, the root mean square velocity is proportional to the square root of its temperature in Kelvin scale.

i.e., $v_{rms} \propto \sqrt{T}$; $v_{rms} = \sqrt{\frac{T}{M_w}}$

For two different temperatures of a gas $\frac{v_{rms_2}}{v_{rms_1}} = \sqrt{\frac{T_2}{T_1}}$

(b) For two different gases $\frac{v_{rms_2}}{v_{rms_1}} = \sqrt{\frac{T_2}{T_1} \times \frac{M_{w1}}{M_{w2}}}$

(c) For given temperature lighter the gas, larger the root mean square velocity of the molecules as

$$v_{rms} \propto \sqrt{\frac{1}{m}}$$

(d) If v_{rms} is equal or greater than escape velocity v_e than gas will escape from earth or any other planet and so a planet or satellite will have atmosphere only and only if

$$v_{rms} < v_e \; (= \sqrt{2gR})$$

In earth's atmosphere hydrogen molecules acquires this velocity due to elastic collisions with other molecules and escape, so free hydrogen is not present in atmosphere.

Most probable speed (v_{mp}) : *It is the speed which maximum number of molecules in a gas have at constant temperature* and is given by

$$v_{mp} = \sqrt{\frac{2RT}{M_w}} = \left(\sqrt{\frac{2}{3}}\right) v_{rms} = 0.816\ v_{rms}$$

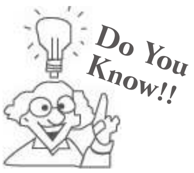
Average speed (v_{av}) : *It is the arithmetic mean of the speeds of molecules in a gas at a given temperature.*

i.e, $$v_{av} = \frac{v_1 + v_2 + v_3 + ... + v_N}{N}$$

And according to kinetic theory of gases,

$$v_{av} = \sqrt{\frac{8RT}{\pi M_w}} = \left(\sqrt{\frac{8}{3\pi}}\right) v_{rms} = 0.92\ v_{rms}$$

$$v_{rms} > v_{av} > v_{mp}$$



$$v_{rms} : v_{av} : v_{mp} = \sqrt{3} : \sqrt{\frac{8}{\pi}} : \sqrt{2} \quad \text{or,} \quad v_{rms} : v_{av} : v_{mp} = \sqrt{3} : \sqrt{2.5} : \sqrt{2}$$

MEAN FREE PATH

The distance covered by the molecules between two successive collisions is called the **free path**.

Mean free path : *The average distance covered by the molecules between two successive collisions is called the mean free path.*

i.e., $$\lambda = \frac{1}{\sqrt{2}\ .\ \pi n d^2} = \frac{K_B T}{\sqrt{2}\pi d^2 P}$$

where, n = number of molecules per unit volume

d = diameter of each molecule

K_B = Boltzmann's constant

T = temperature

P = pressure

Mean free path depends on the diameter of molecule (d) and the number of molecules per unit volume n.

At N.T.P., λ for air molecules is 0.01 μm.

ILLUSTRATION : 8

Calculate the temperature at which rms velocity of hydrogen will be equal to 10 kms^{-1}. (R = 8.314 mol^{-1} K^{-1})

SOLUTION :

Here, $M = 2 \times 10^{-3}$ kg, $T = ?$ $v_{rms} = 10,000$ ms^{-1}

As we know, $v_{rms} = \sqrt{\frac{3RT}{M}}$

$$10^4 = \sqrt{\frac{3 \times 8.314 \times T}{2 \times 10^{-3}}} \Rightarrow 10^8 = \frac{3 \times 8.314 \times T}{2 \times 10^{-3}} \quad \text{or,} \quad T = 8018.6 \text{ K}$$

ILLUSTRATION : 9

Three molecules have velocities 1 ms^{-1}, 2 ms^{-1} and 3 ms^{-1}, find v_{av} and v_{rms}.

SOLUTION :

Average velocity, $V_{av} = \frac{1+2+3}{3} = 2\,\text{ms}^{-1}$

$$v_{rms} = \sqrt{\frac{1^2 + 2^2 + 3^2}{3}} = \sqrt{\frac{14}{3}} \Rightarrow v_{rms} = 2.16\,\text{ms}^{-1}$$

MISCELLANEOUS SOLVED EXAMPLES

1. If the volume of a block of a metal changes by 0.12% when it is heated through 20ºC, what is the coefficient of linear expansion of metal ?

Sol. Coefficient of cubical expansion of metal is given by

$$\gamma = \frac{\Delta V}{Vt} \quad \text{Here } \gamma = \frac{\Delta V}{Vt} = \frac{0.12}{100t}, \; t = 20^\circ C$$

$$\therefore \quad \gamma = \frac{0.12}{100 \times 20} = 6.0 \times 10^{-5} \text{ per } ^\circ C$$

Coefficient of linear expansion

$$\alpha = \frac{\gamma}{3} = \frac{6.0 \times 10^{-5}}{3} = 2.0 \times 10^{-5} \text{ Per } ^\circ C$$

2. From what height must a block of ice be dropped into a well of water so that 5% of it may melt? Given : both ice and water are at 0ºC, L = 80 cal g^{-1}, J = 4.2 J cal^{-1} and g = 980 cm s^{-2}.

Sol. Let m be the mass of ice. Let h be the height from which block of ice is dropped. Work done,

W = mgh = m × 980 × h erg

$$\text{Mass of ice to be melted } = \frac{5}{100} \times m$$

$$\text{Heat required, } Q = \frac{5}{100} \times m \times 80 \text{ cal or } Q = 4 \text{ m cal}$$

$[\because L = 80 \text{ cal g}^{-1}]$ $[\because W = JQ]$

Now, m × 980 × h = J × 4 m

$$h = \frac{4.2 \times 10^7 \times 4m}{m \times 980} \text{ cm} = 1714.3 \text{ m}$$

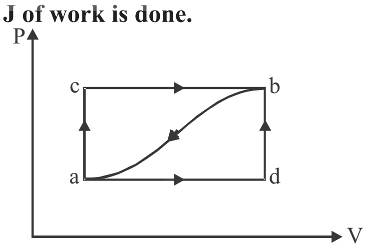
$[\because J = 4.2 \times 10^7 \text{ erg cal}^{-1}]$

3. The energy radiated from a black body at a temperature of 727°C is E. By what factor the radiated energy shall increase if the temperature is raised to 2227°C?

Sol. According to Stefan-Boltzmann law

$$\frac{E_2}{E_1} = \left[\frac{T_2}{T_1}\right]^4 = \left[\frac{2227+273}{727+273}\right]^4 = \left[\frac{2500}{1000}\right]^4 = 39$$

4. When a system is taken from state a to state b, in fig. along the path a → c → b, 60 J of heat flows into the system, and 30 J of work is done.



Find :

(i) How much heat flows into the system along the path a → d → b if the work is 10 J?

(ii) When the system is returned from b to a along the curved path, the work done by the system is –20 J. Does the system absorb or liberate heat, and by how much?

Sol. For the path a → c → b,

$dU = dQ - dW = 60 - 30 = 30$ J or $U_b - U_a = 30$ J

(i) Along the path a → d → b, dQ = dU + dW = 30 + 10 = 40 J

(ii) Along the curved path b → a,

$dQ = (U_a - U_b) + W = (-30) + (-20) = -50$ J,

of heat flows out the system

5. Find temperature, at which rms velocity of a gas is double that of the velocity at 27°C.

Sol. According to question, V_{rms} at 27°C and $2V_{rms}$ at T K

$$v_{rms} = \sqrt{\frac{3R}{M}} \quad \text{and} \quad 2v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$2 \times \sqrt{\frac{3R \times 300}{M}} = \sqrt{\frac{3RT}{M}} \Rightarrow T = 1200 \text{ K}$$

6. **A body cools in 7 minute from 60°C to 40°C. What will be its temperature after the next 7 minute? The temperature of the surroundings is 10°C. Assume that Newton's law of cooling holds good throughout the process.**

Sol. Newton's law of cooling can be written as :

$$\frac{T_1 - T_2}{t} = k\left[\frac{T_1 + T_2}{2} - T_0\right]$$

In first case ; $T_1 = 60$°C, $T_2 = 40$°C, $T_0 = 10$°C and $t = 7$ minute

$$\therefore \quad \frac{60-40}{7} = k\left[\frac{60+40}{2} - 10\right]$$

$$\text{or} \quad k = \frac{1}{14}$$

In second case; $T_1 = 40$°C and $T_2 = ?$, $T = 7$ minute

$$\therefore \quad \frac{40 - T_2}{7} = \frac{1}{14}\left[\frac{40 + T_2}{2} - 10\right]$$

$$\text{or} \quad 80 - 2T_2 = 20 + \frac{T_2}{2} - 10$$

$$\text{or} \quad T_2 = 28^\circ C$$

ADVANCE EXERCISE
BASED ON CONNECTING TOPICS

DIRECTIONS (Qs. 1–44) : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. What temperature is the same on celsius scale as well as on Fahrenheit scale?
(a) – 212°C (b) – 40°C
(c) – 32°C (d) 32°C

2. The temperature of a piece of metal is increased from 27°C to 84°C. The rate of radiated energy emitted from the metal will increase approximately
(a) 16 times (b) 8 times
(c) 4 times (d) 2 times

3. The length of a metallic rod is 5 m at 0°C and becomes 5.01 m, on heating upto 100°C. The linear expansion of the metal will be
(a) $2.33 \times 10^{-5}/°C$ (b) $6.0 \times 10^{-5}/°C$
(c) $4.0 \times 10^{-5}/°C$ (d) $2.0 \times 10^{-5}/°C$

4. The temperature of two bodies *A* and *B* are respecitvely 727°C and 327°C. The ratio of the rates of heat radiated $(H_A : H_B)$ by them, is
(a) : 81 (b) 25 : 9
(c) 5 : 3 (d) 727 : 372

5. A body initially at 80°C cools to 64°C in 5 minutes and to 52°C in 10 minutes. The temperature of the body after 15 minutes will be
(a) 42.7°C (b) 35°C
(c) 47°C (d) 40°C

6. A gas with specific heat ratio $\gamma = \frac{5}{3}$ is compressed suddenly to 1/8 of its initial volume. If the pressure is *P*, then the final pressure is
(a) 8P (b) 16P
(c) 24P (d) 32P

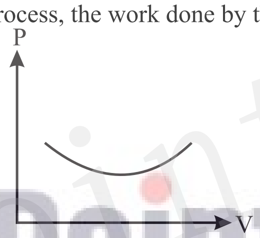
7. An ideal gas at 27°C is compressed adiabatically to $\frac{8}{27}$ of its original volume. If $\gamma = \frac{5}{3}$, then the rise in temperature is
(a) 575 K (b) 450 K
(c) 225 K (d) 375 K

8. The work done in which of the following processes is equal to the internal energy of the system?
(a) adiabatic process (b) isothermal process
(c) isochoric process (d) None of these

9. Two rods of the same length and diameter having thermal conductivities K_1 and K_2 are joined in parallel. The equivalent thermal conductivity of the combination is
(a) $\frac{K_1K_2}{K_1+K_2}$ (b) K_1+K_2
(c) $\frac{K_1+K_2}{2}$ (d) $\sqrt{K_1K_2}$

10. The velocity of the molecules of a gas at temperature 120 *K* is *v*. At what temperature will the velocity be 2 *v* ?
(a) 120 *K* (b) 240 *K*
(c) 480 *K* (d) 1120 *K*

11. The *P-V* diagram of process on a system is shown in figure. During the process, the work done by the system



(a) increases continuously
(b) decreases continuously
(c) first increases, becomes maximum and then decreases
(d) first decreases, becomes minimum and then increases

12. From what height should a piece of ice fall so that it melts completely? Only one-quarter of the heat produced absorbed by the ice. The latent heat of ice is 3.4×10^5 J/kg and g = 10 N/kg.
(a) 136 km (b) 140 km
(c) 68 km (d) None of these

13. The tempertaure of equal masses of three different liquids *A*, *B* and *C* are 12°C, 19°C and 28°C respectively. The temperature when A and B are mixed is 16°C and when *B* and *C* are mixed is 23°C. The temperature when *A* and *C* are mixed is
(a) 18.2°C (b) 22°C
(c) 20.2°C (d) 25.2°C

14. When an ideal gas $\left(\gamma = \frac{5}{3}\right)$ is heated under constant pressure, then what percentage of given heat energy will be utilised in doing external work?
(a) 40% (b) 30%
(c) 60% (d) 20%

15. A solid cube and a solid sphere of the same material have equal surface area. Both are at the same temperature 120°C, then
(a) both the cube and the sphere cool down at the same rate
(b) the cube cools down faster than the sphere
(c) the sphere cools down faster than the cube
(d) whichever is having more mass will cool down faster

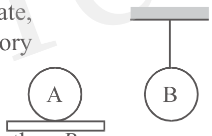
16. At what temperature is the r.m.s velocity of a hydrogen molecule equal to that of an oxygen molecule at 47°C?
(a) 80 K (b) –73 K
(c) 3 K (d) 20 K

17. At 0°C a body emits
(a) no radiation
(b) only visible light
(c) only microwave radiation
(d) all wavelengths.

18. The temperature of a cavity of fixed volume is doubled. Which of the following is true for the black-body radiation inside the cavity?
(a) Its energy and the number of photons both increases 8 times
(b) Its energy increases 8 times and the number of photons increases 16 times
(c) Its energy increases 16 times and the number of photons increases 8 times
(d) Its energy and the number of photons both increase 16 times

19. Consider the following statements
(i) The coefficient of linear expansion has dimension K^{-1}
(ii) the coefficient of volume expansion has dimension K^{-1}
Choose the correct statement ?
(a) Both 'i' and 'ii' are correct
(b) 'i' is correct but 'ii' is wrong
(c) 'ii' is correct but 'i' is wrong
(d) 'i' and 'ii' both wrong

20. Consider two identical iron spheres, one which lie on a thermally insulating plate, while the other hangs from an insulatory thread equal amount of heat is supplied to the two spheres.



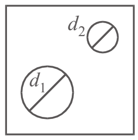
(a) Temperature of A will be greater than B
(b) Temperature of B will be greater than A
(c) Their temperature will be equal
(d) Can't be predicted

21. Radiation from which of the following sources, approximates black body radiation best?
(a) A tungsten lamp
(b) Sodium flame
(c) Hot lamp black
(d) A hole in a cavity, maintained at constant tempera-ture

22. Which of the following will have maximum total kinetic energy at temperature 300 K ?
(a) 1 kg, H_2
(b) $\frac{1}{2}kg\ H_2 + \frac{1}{2}kg\ He$
(c) $\frac{1}{2}kg\ H_2 + \frac{3}{4}kg\ He$
(d) 1 kg, He

23. Two rods of thermal conductivities K_1 and K_2, cross-sections A_1 and A_2 and specific heats S_1 and S_2 are of equal lengths. The temperatures of two ends of each rod are T_1 and T_2. The rate of flow of heat at the steady state will be equal if
(a) $\frac{K_1}{A_1S_1} = \frac{K_2}{A_2S_2}$ (b) $K_1A_1 = K_2A_2$
(c) $K_1S_1 = K_2S_2$ (d) $A_1S_1 = A_2S_2$

24. Two holes of unequal diameters d_1 and d_2 ($d_1 > d_2$) are cut in a metal sheet. If the sheet is heated,



(a) both d_1 and d_2 will decrease
(b) both d_1 and d_2 will increase
(c) d_1 will increase, d_2 will decrease
(d) d_1 will decrease, d_2 will increase

25. A black body is at temperature of 500 K. It emits energy at rate which is proportional to
(a) $(500)^4$ (b) $(500)^3$
(c) $(500)^2$ (d) 500

26. According to kinetic theory of gases, at absolute zero temperature
(a) water freezes
(b) liquid helium freezes
(c) molecular motion stops
(d) liquid hydrogen freezes

27. At constant volume, temperature is increased then
(a) collision on walls will be less
(b) number of collisions per unit time will increase
(c) collisions will be in straight lines
(d) collisions will not change.

28. With the increases of temperature of a black body the peak of the spectral energy distribution of the radiation shift towards lower wavelength. This is given by
(a) Kirchoff's law
(b) Wein's displacement law
(c) Stefan's law
(d) Rayleigh-Jeans law

29. If the temperature of a black body increases from 7°C to 287°C, then the rate of energy radiation increases by
(a) $\left(\frac{287}{7}\right)^4$ times (b) 16 times
(c) 8 times (d) 4 times

30. When an ideal diatomic gas is heated at constant pressure, the fraction of the heat energy supplied which increases the internal energy of the gas is
(a) 2/5 (b) 3/5
(c) 3/7 (d) 5/7

31. An ideal gas in a cylinder is compressed adiabatically to one-third of its initial volume. During this process 20 J work is done on the gas by compressing agent. Which of the following statements is true for this case?
(a) Change in the internal energy in this process is zero
(b) The internal energy increases by 20 J
(c) The internal energy decreases by 20 J
(d) Temperature of the gas decreases

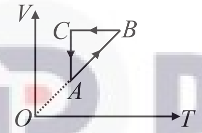
32. There are four objects A, B, C and D. It is observed that A and B are in thermal equilibrium and C and D are also in thermal equilibrium. However, A and C are not in thermal equilibrium. We can conclude that

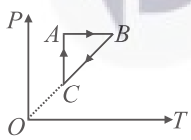
(a) B and D are in thermal equilibrium
(b) B and D could be in thermal equilibrium but might not be A and D
(c) B and D cannot be in thermal equilibrium
(d) The zeroth law of thermodynamics does not apply here because there are more than three objects

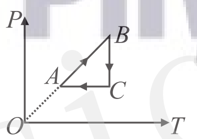
33. Which of the following process is possible according to the first law of thermodynamics ?

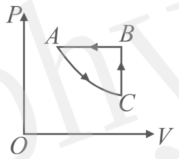
(a) $W > 0$, $Q < 0$ and $dU = 0$
(b) $W > 0$, $Q < 0$ and $dU > 0$
(c) $W > 0$, $Q < 0$ and $dU < 0$
(d) $W < 0$, $Q > 0$ and $dU < 0$

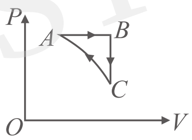
34. An ideal gas undergoes a thermodynamic cycle as shown in figure. Which of the following graphs represents the same cycle



(a)


(b)


(c)


(d)


35. A heat engine absorbs at a temperature T_1 and rejects it at a temperature T_2. The efficiency of the engine can be maximized if :

(a) T_1 is increased, T_2 is decreased
(b) T_1 and T_2 both are increased
(c) T_1 and T_2 both are decreased
(d) T_1 is decreased, T_2 is increased

36. The thermodynamic state of a system changes from a state i to f by different processes. Which of the following depends on whether to process $i \rightarrow f$ is reversible or irreversible?

(a) Change in internal energy (ΔU)
(b) Change in volume (ΔV)
(c) Change in temperature (ΔT)
(d) None of the above

37. A diatomic gas initially at 18ºC is compressed adiabatically to one eighth of its original volume. The temperature after compression will be

(a) 18ºC (b) 668.4ºK
(c) 395.4ºC (d) 144ºC

38. If the ratio of specific heat of a gas at constant pressure to that at constant volume is γ, the change in internal energy of a mass of gas, when the volume changes from V to $2V$ at constant pressure P, is

(a) $\dfrac{R}{(\gamma-1)}$ (b) PV
(c) $\dfrac{PV}{(\gamma-1)}$ (d) $\dfrac{\gamma PV}{(\gamma-1)}$

39. We consider a thermodynamic system. If ΔU represents the increase in its internal energy and W the work done by the system, which of the following statements is true?

(a) $\Delta U = -W$ in an adiabatic process
(b) $\Delta U = W$ in an isothermal process
(c) $\Delta U = -W$ in an isothermal process
(d) $\Delta U = W$ in an adiabatic process

40. If 1 g of steam is mixed with 1 g of ice, then the resultant temperature of the mixture is

(a) 270ºC (b) 230ºC
(c) 100ºC (d) 50ºC

41. An ideal gas A and a real gas B have their volumes increased from V to $2V$ under isothermal conditions. The increase in internal energy

(a) will be same in both A and B
(b) will be zero in both the gases
(c) of B will be more than that of A
(d) of A will be more than that of B

42. For hydrogen gas $C_p - C_v = a$ and for oxygen gas $C_p - C_v = b$, so the relation between a and b is given by

(a) $a = 16\,b$ (b) $16\,b = a$
(c) $a = 4\,b$ (d) $a = b$

43. If for a gas, $\dfrac{R}{C_v} = 0.67$, the gas is made up of molecules which are

(a) diatomic
(b) mixture of diatomic and polyatomic molecules
(c) monoatomic
(d) polyatomic

44. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ration $\dfrac{C_p}{C_v} = \gamma$ for the gas is

(a) 2 (b) 3/2
(c) 5/3 (d) 4/3

DIRECTIONS (Qs. 45–61) : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

45. Reading of temperature may be same on
(a) Celsius and Kelvin scale
(b) Fahrenheit and Kelvin scale
(c) Celsius and Fahrenheit scale
(d) All the three scales

46. Two identical beakers are filled with water to the same level at 4°C. If one say *A* is heated while the other *B* is cooled, then
(a) Water level in *A* will rise
(b) Water level in *A* will fall
(c) Water level in *B* will rise
(d) Water level in *B* will fall

47. The coefficient of linear expansion of a metal rod depends upon
(a) the original length of the rod
(b) the nature of the metal
(c) the change in temperature of the rod
(d) the specific heat of the metal

48. The density of a liquid depends upon
(a) the mass of the liquid
(b) the temperature of the liquid
(c) the nature of the liquid
(d) the volume of the liquid

49. The heat capacity of a body depends on
(a) the heat given
(b) the material of the body
(c) the mass of the body
(d) the temperature raised

50. Which of the following statements are correct ?
(a) Temperature of a body can be raised without heating it
(b) Temperature of a body may not change when it is heated
(c) Whole of work can be converted into heat
(d) Whole of heat can be converted into work

51. A body can be cooled by
(a) conduction (b) radiation
(c) convection (d) None of these

52. The energy radiated by a body depends on
(a) mass of body (b) temperature of body
(c) area of body (d) nature of surface

53. Which of the following properties are suitable for a cooking utensil ?
(a) Low specific heat (b) Low conductivity
(c) High specific heat (d) High conductivity

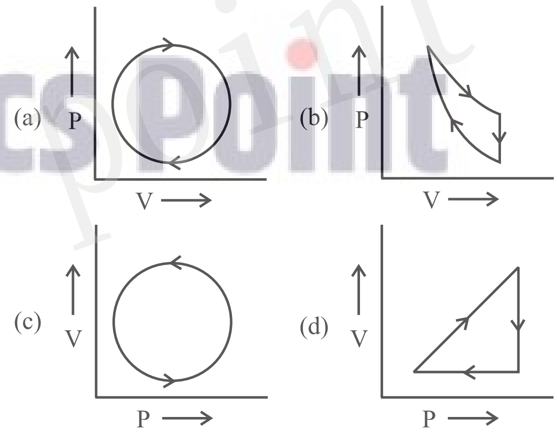
54. The rate of cooling of a body by radiation depends on
(a) specific heat of body
(b) area of body
(c) temperature of body and surroundings
(d) mass of body

55. The internal energy of a perfect gas is independent of
(a) temperature (b) volume
(c) pressure (d) None of these

56. If heat is supplied to an ideal gas in an isothermal process
(a) the gas will do positive work
(b) the internal energy of the gas will remain same
(c) this process is not possible
(d) the gas will do negative work

57. In which processes, there is no change in internal energy of the system ?
(a) Free expansion (b) Cyclic
(c) Isothermal (d) Adiabatic

58. The following are the *P*-*V* diagrams for the cyclic processes for a gas. In which of these processes is heat absorbed by the gas ?



59. During the melting of a slab of ice at 273 K at the atmospheric pressure :
(a) positive work is done on the ice-water system by the atmosphere
(b) the internal energy of the ice-water system decreases
(c) positive work is done by the ice-water system on the atmosphere
(d) the internal energy of the ice-water system increases

60. Which of the following quantities depend on temperature only for a given ideal gas ?
(a) The ratio of pressure and density of the gas
(b) Product *PV* of the gas
(c) Root mean square speed of the gas
(d) Internal energy of the gas

61. The first law of thermodynamics incorporates the concepts of
(a) conservation of heat
(b) equivalence of heat and work
(c) conservation of work
(d) conservation of energy

DIRECTIONS (Qs. 62–63) : *Following question has four statements (A, B, C and D) given in Column I and five statements (p, q, r, s and t .....) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in Column I with entries in Column II.*

62. Match the following for an ideal gas.
Here ΔU = change in internal energy, ΔQ = Heat supplied or evolved by the system, W = work done.

	Column I		**Column II**
(A)	Isothermal process	(p)	$\Delta Q = \Delta U + W$
(B)	Adiabatic process	(q)	$\Delta U = 0$
(C)	Isochoric process	(r)	$\Delta Q = 0$
(D)	Isobaric process	(s)	$W = 0$
		(t)	$\Delta Q = W$

	A	B	C	D
(a)	p, q, t	p, r	p, s	p
(b)	r, t	s	p, r	q
(c)	p, s	q	r, s, t	r
(d)	p,	q, r	r	s

63. Match the following

	Column I		**Column II**
(A)	Heating water above 4°C	(p)	Volume increases
(B)	Cooling water below 4°C	(q)	Volume decreases
(C)	Heating an iron rod	(r)	Density decreases
(D)	Heating a piece of rubber	(s)	Internal energy increases
		(t)	Internal energy decreases

	A	B	C	D
(a)	p,q	q	r, s	q, r
(b)	p, q, r, s	q	p, q, r, s	p, q, r, s
(c)	p, r, s	p, r, t	p, r, s	q, s
(d)	r, t	p, t	q, r, s	p, r, t

DIRECTIONS (Qs. 64–69) : *Study the given paragraph(s) and answer the following questions.*

PASSAGE - I

There are three modes through which heat can be transfered from one body to the other viz conduction, convection and radiation. In conduction, heat is transfered without actual transfer of material between two bodies in direct or indirect contact. In convection heat flows from one point to the other alongwith actual flow of material, and in case of radiation, heat energy travels in the form of electromagnetic radiations called thermal radiations. On the basis of above information, answer the following :

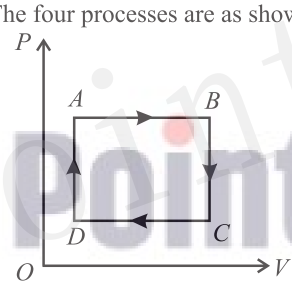
64. The fastest mode of transfer of heat is
(a) conduction (b) convection
(c) radiation (d) can't be predicted

65. Which of the following is NOT an example of convection?
(a) Land breez
(b) Burning of hand when kept above the candle flame
(c) Boiling of upper surface of water when a beaker is placed on a gas stove.
(d) Reaching of heat of the sun to the earth.

66. The rate at which a body recieves thermal energy through radiation is
(a) directly proportional to the surface area of the body.
(b) inversly proportional to the surface area of the body.
(c) independent of the surface area of the body.
(d) none of the above.

PASSAGE - II

A gas is taken through four different processes to complete a cyclic process. The four processes are as shown in the graph.



67. Work done in the process $B \to C$ is
(a) positive
(b) negative
(c) zero
(d) may be positive or negative

68. The change in internal energy over the complete cycle is
(a) positive (b) negative
(c) zero (d) non-zero

69. In the complete cycle, the heat is
(a) absorbed by the system
(b) evolved by the system
(c) neither absorbed nor evolved
(d) can't be predicted

DIRECTIONS (Qs. 70–79) : *Each of these questions contains an **Assertion** followed by **Reason**. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.
(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.
(c) If **Assertion** is **correct** but **Reason** is **incorrect**.
(d) If **Assertion** is **incorrect** but **Reason** is **correct**.

70. **Assertion :** A beaker is completely filled with water at 4°C. It will over flow, both when heated or cooled.
Reason : There is expansion of water below and above 4°C.

71. **Assertion :** Melting of solid causes no change in internal energy.
Reason : Latent heat is the heat required to melt unit mass of a solid.

72. **Assertion :** Specific heat capacity is the cause of formation of land and sea breeze.
Reason : The specific heat of water is more than land.

73. **Assertion :** Snow is better insulator than ice.
Reason : Snow contains air packet and air is good insulator of heat.

74. **Assertion :** It is hotter over the top of a fire than at the same distance from the sides.
Reason : Air surrounding the fire coducts more heat upwards.

75. **Assertion :** Like light radiations thermal radiations are also electromagnetic radiations.
Reason : The thermal radiations require no medium for propagation.

76. **Assertion :** Air quickly leaking out of a ballon becomes cooler.
Reason : The leaking air undergoes adiabatic expansion.

77. **Assertion :** First law of thermodynamics is a restatement of the principle of conservation of energy.
Reason : Heat is a form of energy.

78. **Assertion :** The internal energy of an isothermal process of an ideal gas does not change.
Reason : The internal energy of an ideal gas depends only on temperature of the gas.

79. **Assertion :** The bulb of one thermometer is spherical while that of the other is cylindrical. Both have equal amount of mercury. The response of the cylindrical bulb thermometer will be quicker.
Reason : Heat conduction in a body is directly proportional to cross-sectional area.

DIRECTIONS (Qs. 80–84) : *Following are integer based/ Numeric based questions. Each question, when worked out will result in one integer or numeric value.*

80. What would be the final temperature when 100g of 25°C water is mixed with 75g of 40°C water?

81. At what temperature, pressure remaining constant, will the r.m.s. velocity of a gas be half of its value at 0°C?

82. Calculate the temperature at which rms velocity of hydrogen will be equal to 10 kms^{-1}. (R = 8.314 $mole^{-1}$ K^{-1})

83. A copper sphere cools from 62°C to 50°C in 10 minutes and to 42°C in the next 10 minutes. Calculate the temperature of the surroundings.

84. Behaving like a black body sun emits maximum radiation at wavelength 0.48μm. The mean radius of the sun is 6.96×10^8m. Stefan's constant is $5.67 \times 10^{-8} wm^{-2}k^{-4}$ and Wien's constant is 0.293 cm-k. Find the loss of mass per second by the emission of radiation from sun.

SOLUTIONS

Brief Explanations of Selected Questions

ADVANCED EXERCISE
BASED ON CONNECTING TOPICS

1. (b) $\frac{C}{5} = \frac{F-32}{9}$ Here C = F

$\frac{C}{5} = \frac{C-32}{9} \quad \Rightarrow \quad 9C = 5C - 160$

$4C = -160 \quad \Rightarrow \quad C = -40°C.$

Thus at $-40°C$ and $-40°$ F the temperture is same.

2. (d) $E = \sigma T^4 \qquad \therefore \frac{E}{E'} = \frac{T^4}{T'^4}$

$\Rightarrow \frac{E}{E'} = \left(\frac{273+27}{273+84}\right)^4 \Rightarrow \frac{E'}{E} = \left(\frac{357}{300}\right)^4 = 2$

$\therefore E' = 2E$

Rate of radiated energy will be 2 times.

3. (d) $l = 5\text{m} \qquad t_1 = 0°C$

$l_2 = 5.0\text{m} \qquad t_2 = 100°C$

$\alpha = \frac{l_2 - l_1}{l_1(t_2 - t_1)} = \frac{5.01 - 5}{5 \times 100} = 2 \times 10^{-5} /°C$

4. (a) $\frac{E_1}{E_2} = \frac{T_1^4}{T_2^4} = \left(\frac{T_1}{T_2}\right)^4$

or, $\frac{E_1}{E_2} = \left(\frac{273+727}{273+327}\right)^4 == \left(\frac{1000}{600}\right)^4 = \frac{625}{81}$

5. (a) From Newton's law of cooling :–

$\frac{\theta_1 - \theta_2}{t} = k\left(\frac{\theta_1 + \theta_2}{2} - \theta_0\right)$ where θ_1 is higher temperature θ_2 is lower temperature.

$\frac{80-64}{5} = k(72 - \theta_0)$ .... (i)

Where, θ_0 is temperature of surroundings

$\frac{54-52}{10} = k\left(\frac{52+\theta}{2} - \theta_0\right)$ .... (ii)

Dividing (i) and (ii) we get θ_0

$\frac{52-\theta}{15} = k\left(\frac{52+\theta}{2} - \theta_0\right)$ .... (iii)

Thus θ is found.

6. (d) The gas relation for adiabatic proccess,

$P_1V_1^\gamma = P_2V_2^\gamma$

$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^\gamma$

$\frac{P_1}{P_2} = \left(\frac{1}{8}\right)^{5/3} = \frac{1}{2^{3\times\frac{5}{3}}} = \frac{1}{32}$

or final pressure, $P_2 = 32\,P$.

7. (d) For an adiabatic process,

$TV^{\gamma-1}$ = constant

Here, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$T_1 = 27°C = 300\,K.$

$V_2 = \frac{8}{27}V_1, \qquad \gamma = \frac{5}{3}$

$\therefore \quad T_2 = T_1\left(\frac{v_1}{v_2}\right)^{\gamma-1} = 300\left(\frac{27}{8}\right)^{5/3-1} = 300\left(\frac{27}{8}\right)^{\frac{2}{3}}$

$= 300 \times \frac{9}{4} = 675\,K$

$\therefore \quad \Delta T = T_2 - T_1 = 675 - 300 = 375\,K$

8. (a) In adiabatic process

$\Delta Q = 0$

$\therefore \quad \Delta W = -\Delta U$

9. (c) In parallel combination, the equivalent thermal conductivity is given by

$K = \frac{K_1A_1 + K_2A_2 + K_3A_3 + ... + K_nA_n}{A_1 + A_2 + A_3 + ... + A_n}$

For two rods of equal area,

$K = \frac{(K_1 + K_2)A}{2A} \qquad (\text{if } A_1 = A_2 = A)$

$\Rightarrow K = \frac{K_1 + K_2}{2}$

10. (c) $v \propto \sqrt{T}$

$\therefore \quad \frac{v'}{v} = \sqrt{\frac{T'}{T}}$

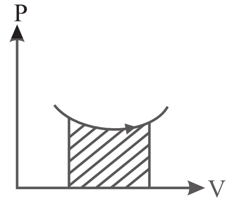
Given $v' = 2\,v$ or, $\frac{2}{1} = \sqrt{\frac{T'}{T}}$

$\therefore \quad T' = 4\,T = 4 \times 120K = 480\,K$

11. (a) During the process,

$W = P\Delta V$; ΔV increasing,

so work done increases.



12. (a) Let the piece of ice be fall from a height h. Then

$mL = \frac{1}{4}mgh \quad \Rightarrow h = \frac{4L}{g} = \frac{4\times3.4\times10^5}{10}$

$= 136 \times 10^3$ m $= 136$ km.

13. (c) Heat gain = heat lost

$C_A(16-12) = C_B(19-16) \Rightarrow \frac{C_A}{C_B} = \frac{3}{4}$

and $C_B(23-19) = C_c(28-23) \Rightarrow \frac{C_B}{C_C} = \frac{5}{4}$

$\Rightarrow \frac{C_A}{C_C} = \frac{15}{16}$ ...(i)

If θ is the temperature when A and C are mixed then,

$C_A(\theta-12) = C_C(28-\theta) \Rightarrow \frac{C_A}{C_B} = \frac{28-\theta}{\theta-12}$ ...(ii)

On solving equations (i) and (ii) θ = 20.2°C

14. (a) $\Delta Q = \Delta U + \Delta W$

$\Rightarrow \frac{\Delta W}{\Delta Q} = 1 - \frac{\Delta U}{\Delta Q} = 1 - \frac{nC_V dT}{nC_P dT}$

$\Rightarrow \frac{\Delta W}{\Delta Q} = 1 - \frac{C_V}{C_P} = 1 - \frac{3}{5} = \frac{2}{5} = 0.4$

15. (b) Rate of cooling of a body $R = \frac{\Delta\theta}{t} = \frac{A\varepsilon\sigma(T^4 - T_0^4)}{mc}$

$\Rightarrow R \propto \frac{A}{m} \propto \frac{\text{Area}}{\text{Volume}}$

$\Rightarrow$ For the same surface area. $R \propto \frac{1}{\text{Volume}}$

$\because$ Volume of cube < Volume of sphere

$\Rightarrow R_{\text{cube}} > R_{\text{Sphere}}$ i.e., cube, cools down with faster rate.

16. (d) $v_{rms} = \sqrt{\frac{8RT}{\pi M}}$

For v_{rms} to be equal $\frac{T_{H_2}}{M_{H_2}} = \frac{T_{O_2}}{M_{O_2}}$

Here $M_{H_2} = 2$; $M_{O_2} = 32$;

$T_{O_2} = 47 + 273 = 320$ K

$\therefore \frac{T_{H_2}}{2} = \frac{320}{32} \Rightarrow T_{H_2} = 20$K

17. (d) At any temperature other than 0 K, each body emits all wavelengths.

18. (c) $E \propto T^4$

19. (a) Coefficient of linear expansion is given by

$\alpha = \frac{d\ell}{\ell.dt}$ and coefficient of volume expansion is by

$\gamma = \frac{dV}{V.dt}$.

So from above formula it is clear that both have units of per Kelvin.

20. (b) Temperature of B will be higher because, due to expansion, centre of mass of B will come down. Same heat is supplied but in B, Potential energy is decreased therefore internal energy gain will be more.

21. (d)

22. (a) In case of H_2 degree of freedom is greatest and number of moles n is highest.

So, this is the case of maximum kinetic energy.

23. (b) Rate of heat flow for one rod

$= \frac{K_1A_1(T_1 - T_2)}{d}(d \to \text{Length})$

Rate of heat flow for other rod

$= \frac{K_2A_2(T_1 - T_2)}{d}$

In steady state, $\frac{K_1A_1(T_1 - T_2)}{d}$

$= \frac{K_2A_2(T_1 - T_2)}{d} \Rightarrow K_1A_1 = K_2A_2$

24. (b) When a body is heated, the distance between any two points on it increases. The increase is in the same ratio for any set of two point.

25. (a) According to Stefan's Law E = σeAT^4

$E \propto T^4$; so, $E \propto (500)^4$

26. (c)

27. (b) As the temperature increases, the average velocity increases. So, the collisions are faster.

28. (b) According to Wein's displacement law, λT = constant.

29. (b) E = σT^4

$\therefore \frac{E_1}{E_2} = \frac{\sigma(273+7)^4}{\sigma(273+287)^4} = \frac{1}{16}$

or $E_2/E_1 = 16$

30. (d) Increase in internal energy = $C_V\,dT$, net energy given $= C_P\,dT$

$$\therefore \frac{\text{increase in internal energy}}{\text{energy supplied}} = \frac{C_V}{C_P} = \frac{5}{7} \quad \left(\gamma = \frac{C_P}{C_V} = 1.4\right)$$

31. (b) During adiabatic process heat exchange is zero. Therefore, by first law of thermodynamics
$dU = -\,dW$
since work has been done on the system, internal energy will increase.

32. (c) Thermal equilibrium means same temperature.

33. (c) **34.** (a)

35. (a) With increase in T_1 and decrease in T_2, $\frac{T_2}{T_1}$ decreases and so
$\eta = \left(\frac{1-T_2}{T_1}\right)$ increase.

36. (a) Change in ΔU, ΔV and ΔT depend on initial and final states.

37. (b) Initial temperature (T_1) = 18°C = 291 K
Let Initial volume $(V_1) = V$
Final volume $(V_2) = \frac{V}{8}$
According to adiabatic process,
$TV^{\gamma-1}$ = constant
According to question, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$$\Rightarrow T_2 = 291\left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$\Rightarrow T_2 = 291(8)^{\frac{7}{5}-1} = 291 \times 2.297 = 668.4\text{K}$$

$$\left[\text{For diatomic gas } \gamma = \frac{Cp}{Cv} = \frac{7}{5}\right]$$

38. (c) Change in internal energy is equal to work done in adiabatic system
$\Delta W = -\Delta U$ (Expansion in the system)

$$= -\frac{1}{\gamma-1}(P_1V_1 - P_2V_2)$$

$$\Delta U = \frac{1}{1-\gamma}(P_2V_2 - P_1V_1)$$

Here, $V_1 = V$, $V_2 = 2V$

$$\therefore \quad \Delta U = \frac{1}{1-\gamma}[P \times 2V - PV] = \frac{PV}{1-\gamma}$$

$$\Rightarrow \Delta U = -\frac{PV}{\gamma-1}$$

39. (a) $\Delta Q = \Delta U + W$
For adiabatic process, $\Delta Q = 0$
$\Delta U = -W$

40. (c) Heat required by ice at 0°C to reach a temperature of 100°C = $mL + mc\Delta\theta$
= 1 × 80 + 1 × 1 × (100 – 0) = 180 cal
Heat available with 1 g steam to condense into 1 g of water at 100°C = 536 cal.
Obviously the whole steam will not be condensed and ice will attain a temperature of 100°C; so the temperature of mixture = 100°C.

41. (b) Under isothermal conditions, there is no change in internal energy

42. (d) Both are diatomic gases and $C_p - C_v = R$ for all gases.

43. (c) Since, $\frac{R}{C_V} = 0.67 \Rightarrow C_V = \frac{3}{2}R$, hence gas is monoatomic.

44. (b) $p^{1-\gamma}T^{\gamma}$ = constant
or $p \propto T^{[\gamma/(\gamma-1)]}$ Here, $p \propto T^3$
or $\frac{\gamma}{\gamma-1} = 3$ or $\gamma = \frac{3}{2}$

45. (b, c) Reading of Celsius and Kelvin scale cannot be same as they have a constant difference.

46. (a, c) At 4°C, water expands either it is cooled or heated.

47. (a, b, c) **48.** (b, c)

49. (b, c)

50. (a, b, c) In an adiabatic process, temperature changes without supplying heat.
In an isothermal process, such as phase change processes, temperature remains constant even when heat is supplied.
Whole work can be converted into heat but reverse is not true.

51. (a, b, c) A body can lose heat by any of the three processes.

52. (b, c, d)

53. (a, d) A cooking utensil must conduct light and heat up quickly.

54. (a, b, c, d)

55. (b, c) The internal energy of an ideal gas depends only on its temperature.

56. (a, b) In isothermal process, internal energy of the ideal gas remains unchanged, hence heat supplied is used for work done by the system.

57. (a, b, c)

58. (a, b, c)

59. (c, d) On melting, volume of ice decreases, hence is work done alone on the system.
Again, internal energy of liquid molecules is more than solid molecules, hence internal energy of system increases in the process.

60. (a, b, c, d).

61. (b, d)

62. (a) (A) → (p, q, t); (B) → (p, r); (C) → (p, s); (D) → (p)

63. (c) (A) → (p, r, s); (B) → (p, r, t); (C) → (p, r, s); (D) → (q, s)

64. (c)

65. (d) Heat of the sun reaches the earth by the phenomenone of radiation.

66. (a) $P \propto A$

67. (c) In process $B \to C$, volume of the system is constant hence no work is done in the process.

68. (c) Change in internal energy for a cyclic process is always zero.

69. (a) As the cycle is clockwise work done and hence heat is positive i.e. heat is absorbed by the system.

70. (a) Both assertion and reason are true and reason is correct explanation of assertion.

71. (d) When a solid melts, its internal energy increases.

72. (a) Due to high specific, water takes more time for heating as well as for cooling which is root cause of land and sea breeze.

73. (a) Both assertion and reason are true and reason is correct explanation of assertion.

74. (a) Due to convection, hot lighter molecules move upwards hence sideway molecules are not much heated.

75. (b) Thermal radiations are electromagnetic radiations which do not require any material medium for their propagation.

76. (a) Any fast process is adiabatic and in adiabatic expansion internal energy and hence temperature of system decreases.

77. (b)

78. (a) Both assertion and reason are true and reason is correct explanation of the assertion.

79. (b)

80. Here we have different masses of water that are mixed together. We equate the heat gained by the cool water to the heat lost by the warm water. We can express this equation formally, and then let the expressed terms lead to a solution:

Heat gained by cool water = heat lost by warm water

$Cm_1\Delta T_1 = Cm_2\Delta T_2$

ΔT_1 doesn't equal ΔT_2 because of different masses of water. We can see that ΔT_1 will be the final temperature T minus 25°, since T will be greater than 25°. ΔT_2 is 40° minus T, because T will be less than 40°. Then,

$c\,(100)(T - 25) = c\,(75)(40 - T)$

$100\,T - 2500 = 3000 - 75\,T$

$T = 31.4°C$

81. $v_{rms} \propto \sqrt{T}$

At 0°C $v_{rms} \propto \sqrt{273}$

At temperature T, $\frac{v_{rms}}{2} \propto \sqrt{T}$

$\therefore\ \frac{1}{2} = \frac{\sqrt{T}}{\sqrt{273}}$ or $T = \frac{273}{4} = 68.2\text{ K} \approx 69\text{K}$

T = 69 – 273 = – 204°C

82. $v_{rms} = \sqrt{\frac{3RT}{M}}$

$M = 2 \times 10^{-3}$ kg , T = ?, $v_{rms} = 10{,}000\text{ ms}^{-1}$

$10^4 = \sqrt{\frac{3 \times 8.314 \times T}{2 \times 10^{-3}}} \Rightarrow 10^8 = \frac{3 \times 8.314 \times T}{2 \times 10^{-3}}$

$T = 8018.6$ K

83. By Newton's law of cooling,

$$\frac{\theta_1 - \theta_2}{t} = -k\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right] \quad ....(1)$$

A sphere cools from 62°C to 50°C in 10 min.

$$\frac{62 - 50}{10} = -k\left[\frac{62 + 50}{2} - \theta_0\right] \quad ....(2)$$

Now, sphere cools from 50°C to 42°C in next 10 min.

$$\frac{50 - 42}{10} = -k\left[\frac{50 + 42}{2} - \theta_0\right] \quad ....(3)$$

Dividing eq^n. (2) by (3) we get,

$\frac{56 - \theta_0}{46 - \theta_0} = \frac{1.2}{0.8}$ or $0.4\theta_0 = 10.4$ hence $\theta_0 = 26°C$

84. Using wien's displacement law

$T = \frac{b}{\lambda_m} = \frac{0.293 \times 10^{-2}}{0.48 \times 10^{-6}} = 6104$ K

Energy given out by sun per second

$= A\sigma T^4 = 4\pi\,(6.96 \times 10^8)^2 \times 5.67 \times 10^{-8}\,(6104)^4$

$= 49.285 \times 10^{25}$J

Loss of mass per second

$m = \frac{E}{c^2} = \frac{49.285 \times 10^{25}}{9 \times 10^{16}} = 5.4 \times 10^9$ kg/s