Physics Point

SESSION 2026 - 27

NCERT Class 12 Physics Solutions Chapter-Wise

Complete chapter-wise coverage with step-by-step solutions, key formulae and concept notes for CBSE Class 12.

14	112	100+
CHAPTERS	SOLVED PROBLEMS	KEY FORMULAE

WHAT THIS BOOK CONTAINS

Electrostatics & Current
Chapters 1 - 3

Magnetism & Induction
Chapters 4 - 7

EM Waves & Optics
Chapters 8 - 10

Modern Physics & Electronics
Chapters 11 - 14

How to Use This Book

This book follows the rationalised NCERT Class 12 Physics syllabus for the 2026–27 session, which is organised into fourteen chapters across two parts.

Every chapter is set out in the same way:

1. Chapter overview – a short statement of the concepts the chapter covers, so you know exactly what is being tested.

2. Key formulae and results – every relation you need for that chapter, collected in one table for quick revision before an examination.

3. Solved problems – each question is worked through in full, showing the data given, the formula applied, the substitution, and the final answer with its correct unit. Conceptual questions are answered in the structured form expected by CBSE examiners.

> **Note on the questions.** The problems in this book are modelled on the NCERT exercises and follow the same physics and the same data, but the statements have been rewritten in our own words. Read each question alongside your NCERT textbook rather than in place of it.

> **Values used throughout.** Unless a question states otherwise, we use $1/4\pi\varepsilon_0 = 9 \times 10^9$ N m^2 C^{-2}, $\varepsilon_0 = 8.854 \times 10^{-12}$ C^2 N^{-1} m^{-2}, $\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1}, $e = 1.6 \times 10^{-19}$ C, $m_e = 9.11 \times 10^{-31}$ kg, $h = 6.626 \times 10^{-34}$ J s, $c = 3 \times 10^8$ m s^{-1} and $N_A = 6.022 \times 10^{23}$ mol^{-1}. Answers are rounded to three significant figures, so small differences from other sources are only rounding.

Advice before the examination. Work each problem on paper before reading the solution. In the CBSE board examination, marks are awarded for the formula stated, the substitution shown and the unit written — not only for the final number. Present your work in the same sequence used here.

Contents

CHAPTER 1

Electric Charges and Fields

This chapter builds the foundation of electrostatics: quantisation and conservation of charge, Coulomb's law, the superposition principle, the electric field and field lines, electric dipoles and their behaviour in a uniform field, electric flux, and Gauss's theorem with its standard applications.

KEY FORMULAE AND RESULTS

Coulomb's law (vacuum)	$F = (1/4\pi\varepsilon_0) \cdot q_1 q_2/r^2$, $1/4\pi\varepsilon_0 = 9 \times 10^9$ N m² C⁻²
Quantisation of charge	$q = n e$, $n = 0, \pm1, \pm2, \ldots$ and $e = 1.6 \times 10^{-19}$ C
Electric field of a point charge	$E = (1/4\pi\varepsilon_0) \cdot q/r^2$, $F = qE$
Electric dipole moment	$p = q \times 2a$, directed from $-q$ to $+q$
Field of a dipole	Axial: $E = (1/4\pi\varepsilon_0) \cdot 2p/r^3$; Equatorial: $E = (1/4\pi\varepsilon_0) \cdot p/r^3$ $(r >> a)$
Torque on a dipole	$\tau = pE \sin\theta$ (vector form: $\tau = p \times E$)
Electric flux	$\varphi_E = \int E \cdot dS = EA \cos\theta$
Gauss's theorem	$\varphi_E = q_{enclosed}/\varepsilon_0$, $\varepsilon_0 = 8.854 \times 10^{-12}$ C² N⁻¹ m⁻²
Standard results from Gauss's law	Infinite line: $E = \lambda/2\pi\varepsilon_0 r$ • Infinite sheet: $E = \sigma/2\varepsilon_0$ • Shell (outside): $E = q/4\pi\varepsilon_0 r^2$, (inside): $E = 0$

Q1 Two point charges of 2×10^{-7} C and 3×10^{-7} C are placed 30 cm apart in vacuum. Find the magnitude of the electrostatic force between them and state whether it is attractive or repulsive.

SOLUTION

Given: $q_1 = 2 \times 10^{-7}$ C, $q_2 = 3 \times 10^{-7}$ C, $r = 30$ cm $= 0.30$ m.

Formula: By Coulomb's law, $F = (1/4\pi\varepsilon_0)\, q_1 q_2/r^2$.

Working:

$$F = 9 \times 10^9 \times (2 \times 10^{-7})(3 \times 10^{-7}) / (0.30)^2$$

$$F = 9 \times 10^9 \times 6 \times 10^{-14} / 0.09 = 5.4 \times 10^{-4} / 0.09$$

Both charges are positive, so the force acts along the line joining them and pushes them apart.

Answer: $F = 6 \times 10^{-3}$ N, repulsive.

Q2 Two small charged spheres carry charges of 0.4 μC and −0.8 μC. The attractive force between them is 0.2 N. (a) Find the separation between the spheres. (b) What force does the second sphere experience due to the first?

SOLUTION

Given: $q_1 = 0.4\ \mu C = 4 \times 10^{-7}$ C, $q_2 = -0.8\ \mu C = -8 \times 10^{-7}$ C, F = 0.2 N.

(a) Rearranging Coulomb's law for r:

$$r^2 = (1/4\pi\varepsilon_0)\ |q_1q_2| / F = 9 \times 10^9 \times (4 \times 10^{-7})(8 \times 10^{-7}) / 0.2$$

$$r^2 = 9 \times 10^9 \times 3.2 \times 10^{-13} / 0.2 = 2.88 \times 10^{-3} / 0.2 = 1.44 \times 10^{-2}\ m^2$$

$$r = 0.12\ m = 12\ cm$$

(b) Electrostatic forces obey Newton's third law. The two spheres form an action–reaction pair, so the second sphere experiences a force of the same magnitude in the opposite direction.

Answer: (a) r = 0.12 m = 12 cm. (b) 0.2 N, directed towards the first sphere (attractive).

Q3 Compare the electrostatic and gravitational forces between an electron and a proton by evaluating the ratio of the Coulomb force to the gravitational force between them.

SOLUTION

Given: $e = 1.602 \times 10^{-19}$ C, $m_e = 9.11 \times 10^{-31}$ kg, $m_p = 1.67 \times 10^{-27}$ kg, $G = 6.674 \times 10^{-11}\ N\ m^2\ kg^{-2}$.

Formula: Both forces vary as $1/r^2$, so r cancels in the ratio:

$$F_e/F_g = (1/4\pi\varepsilon_0)\ e^2 / (G\ m_e\ m_p)$$

Working:

$$\text{Numerator} = 8.99 \times 10^9 \times (1.602 \times 10^{-19})^2 = 2.31 \times 10^{-28}$$

$$\text{Denominator} = 6.674 \times 10^{-11} \times 9.11 \times 10^{-31} \times 1.67 \times 10^{-27} = 1.02 \times 10^{-67}$$

$$F_e/F_g = 2.31 \times 10^{-28} / 1.02 \times 10^{-67}$$

The ratio is dimensionless and enormously large, which is why gravitational attraction is completely ignored in atomic physics.

Answer: $F_e/F_g \approx 2.3 \times 10^{39}$.

Q4 A polythene strip rubbed with wool acquires a charge of -3×10^{-7} C. (a) How many electrons are transferred, and in which direction? (b) Estimate the mass transferred along with the charge.

SOLUTION

(a) Charge is quantised, so q = n e.

$$n = |q|/e = 3 \times 10^{-7} / 1.6 \times 10^{-19} = 1.875 \times 10^{12}$$

The polythene becomes negative, so electrons move *from the wool to the polythene*.

(b) Mass transferred = n × m_e:

$$m = 1.875 \times 10^{12} \times 9.11 \times 10^{-31} = 1.71 \times 10^{-18} \text{ kg}$$

This is utterly negligible compared with the mass of the strip, so no mass change is detectable in practice.

Answer: (a) 1.875×10^{12} electrons, wool → polythene. (b) ≈ 1.71×10^{-18} kg.

Q5 Two identical insulated copper spheres A and B each carry a charge of 6.5×10^{-7} C, with their centres 50 cm apart. (a) Find the mutual force. (b) What happens to the force if the charge on each sphere is doubled and the separation is halved?

SOLUTION

(a) Treating the spheres as point charges at their centres:

$$F = 9 \times 10^{9} \times (6.5 \times 10^{-7})^2 / (0.50)^2$$

$$F = 9 \times 10^{9} \times 4.225 \times 10^{-13} / 0.25 = 3.80 \times 10^{-3} / 0.25$$

$$F = 1.52 \times 10^{-2} \text{ N} \quad \text{(repulsive)}$$

(b) Doubling each charge multiplies the product q_1q_2 by 4. Halving r divides r^2 by 4, multiplying F by another 4.

$$F' = 16 F = 16 \times 1.52 \times 10^{-2} = 0.243 \text{ N}$$

Answer: (a) 1.52×10^{-2} N (repulsive). (b) F becomes 16 times larger, i.e. 0.243 N.

Q6 Charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C are located at the points (0, 0, −15 cm) and (0, 0, +15 cm) respectively. Find the total charge and the electric dipole moment of the system.

SOLUTION

Total charge: $q_A + q_B = 2.5 \times 10^{-7} - 2.5 \times 10^{-7} = 0$.

Dipole moment: The separation is 2a = 15 + 15 = 30 cm = 0.30 m.

$p = q \times 2a = 2.5 \times 10^{-7} \times 0.30 = 7.5 \times 10^{-8}$ C m

By convention the dipole moment points from the negative charge to the positive charge, i.e. from +15 cm towards −15 cm — along the **negative z-direction**.

Answer: Total charge = 0; $p = 7.5 \times 10^{-8}$ C m directed along the −z axis.

Q7 An electric dipole of moment 4×10^{-9} C m is aligned at 30° to a uniform electric field of magnitude 5×10^{4} N C^{-1}. Calculate the torque acting on the dipole.

SOLUTION

Given: $p = 4 \times 10^{-9}$ C m, $E = 5 \times 10^{4}$ N C^{-1}, $\theta = 30°$.

Formula: $\tau = pE \sin\theta$

$\tau = 4 \times 10^{-9} \times 5 \times 10^{4} \times \sin 30°$

$\tau = 2 \times 10^{-4} \times 0.5 = 1 \times 10^{-4}$ N m

Note that the net *force* on the dipole is zero in a uniform field; only a torque acts, tending to align p with E.

Answer: $\tau = 1 \times 10^{-4}$ N m.

Q8 A uniform electric field $E = 3 \times 10^{3}\ \hat{i}$ N C^{-1} exists in a region. Find the flux through a square of side 10 cm whose plane is parallel to the y-z plane. What is the flux if the normal to the square makes an angle of 60° with the x-axis?

SOLUTION

Given: $E = 3 \times 10^{3}$ N C^{-1} along +x, side = 0.10 m, $A = 0.10 \times 0.10 = 1 \times 10^{-2}$ m^2.

Case 1 — plane parallel to y-z plane: the outward normal is along the x-axis, so $\theta = 0°$.

$\varphi = EA \cos 0° = 3 \times 10^{3} \times 1 \times 10^{-2} = 30$ N m^2 C^{-1}

Case 2 — normal at 60° to the x-axis:

$\varphi = EA \cos 60° = 30 \times 0.5 = 15$ N m^2 C^{-1}

Answer: $\varphi_1 = 30$ N m^2 C^{-1}; $\varphi_2 = 15$ N m^2 C^{-1}.

Q9 The net outward flux through the surface of a cube of side 10 cm is 8.0×10^3 N m^2 C^{-1}. (a) Find the net charge inside the cube. (b) If the net flux were zero, could you conclude that there is no charge inside?

SOLUTION

(a) By Gauss's theorem, $\varphi = q/\varepsilon_0$, hence $q = \varepsilon_0\varphi$.

$$q = 8.854 \times 10^{-12} \times 8.0 \times 10^3 = 7.08 \times 10^{-8}\ \text{C}$$

$$q \approx 0.07\ \mu\text{C}$$

Note that the side of the cube is not required — Gauss's law depends only on the enclosed charge.

(b) No. Zero net flux only tells us that the *net* enclosed charge is zero. Equal amounts of positive and negative charge could still be present inside.

Answer: (a) $q \approx 7.08 \times 10^{-8}$ C = 0.07 μC. (b) No — only the net charge is zero.

Q10 A point charge of 2.0 μC is placed at the centre of a cubical box of side 9 cm. Find the electric flux through one face of the cube.

SOLUTION

Reasoning: The charge is at the centre, so by symmetry the total flux is shared equally among the six faces.

$$\varphi_{total} = q/\varepsilon_0$$

$$\varphi_{face} = q / 6\varepsilon_0 = 2.0 \times 10^{-6} / (6 \times 8.854 \times 10^{-12})$$

$$\varphi_{face} = 2.0 \times 10^{-6} / 5.312 \times 10^{-11} = 3.76 \times 10^4\ \text{N m}^2\ \text{C}^{-1}$$

The side length is again irrelevant — enlarging the cube does not change the enclosed charge.

Answer: $\varphi_{face} \approx 3.76 \times 10^4$ N m^2 C^{-1}.

Q11 An infinite straight wire carries a uniform linear charge density λ. The electric field at a point 20 cm from the wire is 9×10^4 N C^{-1} directed radially inward. Find λ.

SOLUTION

Formula: For an infinite line charge, a coaxial cylindrical Gaussian surface gives

$$E = \lambda / (2\pi\varepsilon_0 r) = 2 \times 9 \times 10^9 \times \lambda / r$$

Working: With $E = 9 \times 10^4$ N C^{-1} and $r = 0.20$ m,

$\lambda = E\, r / (2 \times 9 \times 10^9) = (9 \times 10^4 \times 0.20) / (1.8 \times 10^{10})$

$\lambda = 1.8 \times 10^4 / 1.8 \times 10^{10} = 1 \times 10^{-6}$ C m^{-1}

The field points *inward*, which means the charge is negative.

Answer: $|\lambda| = 1 \times 10^{-6}$ C m^{-1} = 10 μC m^{-1}; the wire is negatively charged.

Q12 A conducting sphere of radius 2.4 m carries a uniform surface charge density of 80.0 μC m^{-2}. Find (a) the total charge on the sphere and (b) the total electric flux leaving its surface.

SOLUTION

(a) Surface area of the sphere:

$A = 4\pi r^2 = 4\pi(2.4)^2 = 4\pi \times 5.76 = 72.38$ m^2

$q = \sigma A = 80.0 \times 10^{-6} \times 72.38 = 5.79 \times 10^{-3}$ C

(b) By Gauss's theorem:

$\varphi = q/\varepsilon_0 = 5.79 \times 10^{-3} / 8.854 \times 10^{-12}$

$\varphi = 6.54 \times 10^8$ N m^2 C^{-1}

Answer: (a) $q \approx 5.79 \times 10^{-3}$ C.   (b) $\varphi \approx 6.54 \times 10^8$ N m^2 C^{-1}.

CHAPTER 2

Electrostatic Potential and Capacitance

Electrostatic potential energy and potential, equipotential surfaces, potential due to point charges and dipoles, conductors in electrostatic equilibrium, dielectrics and polarisation, capacitance, combinations of capacitors and the energy stored in a capacitor.

KEY FORMULAE AND RESULTS

Potential of a point charge	$V = (1/4\pi\varepsilon_0) \cdot q/r$ (scalar — add algebraically)
Potential energy of two charges	$U = (1/4\pi\varepsilon_0) \cdot q_1q_2/r_{12}$
Relation between E and V	$E = -dV/dr$; work done $W = q(V_B - V_A)$
Dipole in a field	$U = -pE\cos\theta$; stable at $\theta = 0°$, unstable at $\theta = 180°$
Capacitance	$C = Q/V$; parallel plate: $C = \varepsilon_0A/d$; with dielectric: $C = K\varepsilon_0A/d$
Capacitors in series	$1/C_s = 1/C_1 + 1/C_2 + \ldots$ (charge is the same on each)
Capacitors in parallel	$C_p = C_1 + C_2 + \ldots$ (voltage is the same across each)
Energy stored	$U = ½CV^2 = ½QV = Q^2/2C$; energy density $u = ½\varepsilon_0E^2$

Q1 Two charges 5×10^{-8} C and -3×10^{-8} C are placed 16 cm apart. Locate the point(s) on the line joining them where the electrostatic potential is zero. Take the potential at infinity to be zero.

SOLUTION

Set-up: Let the positive charge be at A and the negative charge at B, with AB = 16 cm. Let P be at a distance x (in cm) from A.

Case 1 — P between the charges:

$$V = (1/4\pi\varepsilon_0)[\ 5 \times 10^{-8}/x - 3 \times 10^{-8}/(16 - x)\ ] = 0$$

$$5/x = 3/(16 - x) \ \Rightarrow\ 80 - 5x = 3x \ \Rightarrow\ 8x = 80 \ \Rightarrow\ x = 10 \text{ cm}$$

Case 2 — P outside, beyond the negative charge:

$$5/x = 3/(x - 16) \ \Rightarrow\ 5x - 80 = 3x \ \Rightarrow\ 2x = 80 \ \Rightarrow\ x = 40 \text{ cm}$$

So the second null point lies 40 cm from A, i.e. 24 cm beyond B. (There is no null point on the far side of A, because the larger charge is nearer there.)

Answer: V = 0 at 10 cm from the positive charge (between them) and at 40 cm from the positive charge (outside, beyond the negative charge).

Q2 A regular hexagon of side 10 cm has a charge of 5 μC at each of its six vertices. Calculate the electrostatic potential at the centre of the hexagon.

SOLUTION

Key geometric fact: In a regular hexagon, the distance from the centre to each vertex equals the side length. Hence r = 10 cm = 0.10 m for all six charges.

Formula: Potential is a scalar, so the six contributions simply add:

$$V = 6 \times (1/4\pi\varepsilon_0)\ q/r$$

$$V = 6 \times 9 \times 10^9 \times 5 \times 10^{-6} / 0.10$$

$$V = 6 \times 4.5 \times 10^5 = 2.7 \times 10^6\ \text{V}$$

Note: by symmetry the electric *field* at the centre is zero, even though the potential is not.

Answer: $V = 2.7 \times 10^6$ V.

Q3 Two charged conducting spheres of radii a and b are connected to each other by a wire. Find the ratio of the electric fields at their surfaces, and explain why the charge density on the sharper end of a conductor is higher.

SOLUTION

Reasoning: Once connected, the two spheres form one conductor, so they must be at the same potential.

$$V = (1/4\pi\varepsilon_0)\ Q_1/a = (1/4\pi\varepsilon_0)\ Q_2/b \ \Rightarrow\ Q_1/Q_2 = a/b$$

The surface field of a sphere is $E = (1/4\pi\varepsilon_0)\ Q/r^2$. Therefore

$$E_1/E_2 = (Q_1/a^2) \div (Q_2/b^2) = (a/b)(b^2/a^2) = b/a$$

Interpretation: The smaller sphere has the larger surface field. Since $\sigma = \varepsilon_0 E$, the surface charge density is also larger where the radius of curvature is smaller. A sharp point behaves like a sphere of very small radius, so charge accumulates there and the field can become large enough to ionise the surrounding air — the principle behind corona discharge and lightning conductors.

Answer: $E_1/E_2 = b/a$ — the field (and charge density) is greater on the sphere of smaller radius.

Q4 A parallel plate capacitor has plates of area 6×10^{-3} m^2 separated by 3 mm of air. (a) Find its capacitance. (b) If it is connected to a 100 V supply, what charge does each plate acquire?

SOLUTION

Given: $A = 6 \times 10^{-3}$ m^2, $d = 3$ mm $= 3 \times 10^{-3}$ m, $V = 100$ V.

(a) $C = \varepsilon_0 A/d$

$C = 8.854 \times 10^{-12} \times 6 \times 10^{-3} / 3 \times 10^{-3}$

$C = 8.854 \times 10^{-12} \times 2 = 1.771 \times 10^{-11}$ F ≈ 17.7 pF

(b) $Q = CV$

$Q = 1.771 \times 10^{-11} \times 100 = 1.77 \times 10^{-9}$ C

Answer: (a) C ≈ 17.7 pF. (b) Q ≈ 1.77×10^{-9} C = 1.77 nC.

Q5 A 3 mm thick sheet of mica of dielectric constant K = 6 exactly fills the gap of the capacitor in the previous question. Find the capacitance and charge (a) if the 100 V supply remains connected, and (b) if the supply is disconnected before the mica is inserted.

SOLUTION

New capacitance: $C' = K\,C = 6 \times 17.7$ pF = 106.2 pF (about 106 pF). This is the same in both cases.

(a) Supply connected — V stays at 100 V:

$Q' = C'V = 1.062 \times 10^{-10} \times 100 = 1.06 \times 10^{-8}$ C

The battery supplies extra charge, so Q increases six-fold while V is unchanged.

(b) Supply disconnected — Q stays at 1.77×10^{-9} C:

$V' = Q/C' = 1.77 \times 10^{-9} / 1.062 \times 10^{-10} = 16.7$ V

Here the charge is trapped, so the potential difference falls to V/K = 100/6 ≈ 16.7 V.

Answer: C′ ≈ 106 pF in both cases. (a) Q ≈ 1.06×10^{-8} C at 100 V. (b) Q unchanged at 1.77 nC, V falls to ≈ 16.7 V.

Q6 Three capacitors of 2 pF, 3 pF and 4 pF are connected in series across a 100 V supply. Find the equivalent capacitance, the charge on each capacitor and the potential difference across each.

SOLUTION

Equivalent capacitance:

$1/C_s = 1/2 + 1/3 + 1/4 = (6 + 4 + 3)/12 = 13/12$

$C_s = 12/13 = 0.923$ pF

Charge: In series the same charge flows through every capacitor.

$Q = C_s V = 0.923 \times 10^{-12} \times 100 = 9.23 \times 10^{-11}$ C

Potential differences: V = Q/C for each capacitor.

$V_1 = 9.23 \times 10^{-11} / 2 \times 10^{-12} = 46.2$ V

$V_2 = 9.23 \times 10^{-11} / 3 \times 10^{-12} = 30.8$ V

$V_3 = 9.23 \times 10^{-11} / 4 \times 10^{-12} = 23.1$ V

Check: 46.2 + 30.8 + 23.1 = 100.1 ≈ 100 V (rounding). The smallest capacitor carries the largest share of the voltage.

Answer: $C_s = 0.923$ pF; $Q = 9.23 \times 10^{-11}$ C on each; V = 46.2 V, 30.8 V and 23.1 V respectively.

Q7 The same three capacitors (2 pF, 3 pF, 4 pF) are now connected in parallel across 100 V. Find the equivalent capacitance and the charge on each capacitor.

SOLUTION

Equivalent capacitance:

$C_p = 2 + 3 + 4 = 9$ pF

Charges: In parallel the potential difference across each capacitor is the full 100 V, and Q = CV.

$Q_1 = 2 \times 10^{-12} \times 100 = 2 \times 10^{-10}$ C

$Q_2 = 3 \times 10^{-12} \times 100 = 3 \times 10^{-10}$ C

$Q_3 = 4 \times 10^{-12} \times 100 = 4 \times 10^{-10}$ C

Check: Total charge = 9×10^{-10} C = $C_p V$. ✓

Answer: $C_p = 9$ pF; $Q = 2 \times 10^{-10}$ C, 3×10^{-10} C and 4×10^{-10} C.

Q8 How much energy is stored in a 12 pF capacitor connected to a 50 V battery?

SOLUTION

Formula: $U = ½CV^2$

$U = ½ \times 12 \times 10^{-12} \times (50)^2$

$U = ½ \times 12 \times 10^{-12} \times 2500$

$U = 1.5 \times 10^{-8}$ J

Answer: $U = 1.5 \times 10^{-8}$ J.

Q9 A 600 pF capacitor is charged by a 200 V supply, then disconnected and connected across another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process, and where does it go?

SOLUTION

Before connection:

$U_i = \frac{1}{2}CV^2 = \frac{1}{2} \times 600 \times 10^{-12} \times (200)^2 = 1.2 \times 10^{-5}$ J

$Q = CV = 600 \times 10^{-12} \times 200 = 1.2 \times 10^{-7}$ C

After connection: Charge is conserved and the two equal capacitors share it equally, so the common potential is

$V' = Q / (C + C) = 1.2 \times 10^{-7} / 1200 \times 10^{-12} = 100$ V

$U_f = \frac{1}{2} \times 1200 \times 10^{-12} \times (100)^2 = 6 \times 10^{-6}$ J

Energy lost:

$\Delta U = 1.2 \times 10^{-5} - 6 \times 10^{-6} = 6 \times 10^{-6}$ J

Exactly half the stored energy disappears. It is dissipated as heat in the connecting wires (and as a small amount of electromagnetic radiation) during the transient current surge.

Answer: Energy lost = 6×10^{-6} J (half the initial energy), dissipated mainly as heat in the connecting wires.

Q10 Explain why the electrostatic field inside the material of a charged conductor is zero, and why the field just outside its surface is normal to the surface with magnitude σ/ε_0.

SOLUTION

Field inside is zero: In electrostatic equilibrium no charge moves. If a field existed inside the conducting material, the free electrons would experience a force and drift, contradicting equilibrium. Therefore E = 0 everywhere inside the bulk of the conductor. A consequence is that all excess charge must reside on the outer surface, and the whole conductor is one equipotential volume.

Field outside is normal to the surface: If the field had a component along the surface, free charges on the surface would move along it. In equilibrium that cannot happen, so the tangential component must vanish and E must be perpendicular to the surface.

Magnitude: Take a small "pill-box" Gaussian surface straddling the surface, with flat faces of area ΔS parallel to it. The inner face contributes no flux ($E = 0$ inside) and the curved side contributes none (E is normal). So

> $\varphi = E\,\Delta S = q_{enc}/\varepsilon_0 = \sigma\Delta S/\varepsilon_0$

> $E = \sigma/\varepsilon_0$, directed outward for $\sigma > 0$ and inward for $\sigma < 0$.

> **Answer:** $E = 0$ inside (else charges would move); just outside, E is normal to the surface with magnitude σ/ε_0.

CHAPTER 3

Current Electricity

Electric current and drift of electrons, Ohm's law and its limitations, resistivity and its temperature dependence, series and parallel combinations, EMF and internal resistance, Kirchhoff's rules, the Wheatstone bridge, the metre bridge and the potentiometer.

KEY FORMULAE AND RESULTS

Current and drift velocity	$I = nAev_d$; $v_d = (e\tau/m)E$; mobility $\mu = v_d/E$
Ohm's law and resistance	$V = IR$; $R = \rho L/A$; conductivity $\sigma = 1/\rho = ne^2\tau/m$
Temperature dependence	$R_T = R_0[1 + \alpha(T - T_0)]$
Combinations	Series: $R_s = R_1 + R_2 + \ldots$ • Parallel: $1/R_p = 1/R_1 + 1/R_2 + \ldots$
EMF and internal resistance	$\varepsilon = V + Ir$, so $V = \varepsilon - Ir$ (terminal p.d. while discharging)
Kirchhoff's rules	Junction rule: $\Sigma I = 0$ • Loop rule: $\Sigma \Delta V = 0$
Wheatstone bridge (balanced)	$P/Q = R/S$ (no current through the galvanometer)
Metre bridge / potentiometer	$X = R \cdot l/(100 - l)$; $\varepsilon_1/\varepsilon_2 = l_1/l_2$
Power	$P = VI = I^2R = V^2/R$

Q1 A storage battery of EMF 12 V has an internal resistance of 0.4 Ω. What is the maximum current that can be drawn from it?

SOLUTION

Reasoning: The current is largest when the external resistance is zero (a short circuit), so the only resistance in the circuit is the internal resistance.

$$I_{max} = \varepsilon/r$$

$$I_{max} = 12 / 0.4 = 30 \text{ A}$$

Note: This is a limiting value only. Drawing such a current would heat the battery severely and is dangerous in practice.

Answer: $I_{max} = 30$ A.

Q2 A battery of EMF 10 V is connected to a resistor R. The current in the circuit is 0.5 A and the terminal potential difference of the battery is 8 V. Find the internal resistance of the battery and the value of R.

SOLUTION

Internal resistance: The terminal p.d. is $V = \varepsilon - Ir$.

$$8 = 10 - (0.5)r$$

$$0.5\,r = 2 \;\Rightarrow\; r = 4\,\Omega$$

External resistance: The full terminal p.d. appears across R.

$$R = V/I = 8 / 0.5 = 16\,\Omega$$

Check: $I = \varepsilon/(R + r) = 10/(16 + 4) = 0.5$ A. ✓

Answer: $r = 4\,\Omega$, $R = 16\,\Omega$.

Q3 Three resistors of 1 Ω, 2 Ω and 3 Ω are joined in series and connected to a 12 V battery of negligible internal resistance. Find the total resistance, the current, and the potential drop across each resistor.

SOLUTION

Total resistance:

$$R_s = 1 + 2 + 3 = 6\,\Omega$$

Current: The same current flows through all three resistors.

$$I = V/R_s = 12/6 = 2\text{ A}$$

Potential drops: $V = IR$ for each.

$$V_1 = 2 \times 1 = 2\text{ V},\quad V_2 = 2 \times 2 = 4\text{ V},\quad V_3 = 2 \times 3 = 6\text{ V}$$

Check: 2 + 4 + 6 = 12 V. ✓

Answer: $R_s = 6\,\Omega$, $I = 2$ A, and the drops are 2 V, 4 V and 6 V.

Q4 The same three resistors (1 Ω, 2 Ω, 3 Ω) are now joined in parallel across the 12 V battery. Find the equivalent resistance, the current in each branch and the total current.

SOLUTION

Equivalent resistance:

$1/R_p = 1/1 + 1/2 + 1/3 = (6 + 3 + 2)/6 = 11/6$

$R_p = 6/11 = 0.545\ \Omega$

Branch currents: Each resistor has the full 12 V across it.

$I_1 = 12/1 = 12$ A, $I_2 = 12/2 = 6$ A, $I_3 = 12/3 = 4$ A

Total current:

I = 12 + 6 + 4 = 22 A

Check: $I = V/R_p = 12 \div 0.545 = 22$ A. ✓

Answer: $R_p = 6/11\ \Omega \approx 0.545\ \Omega$; branch currents 12 A, 6 A, 4 A; total 22 A.

Q5 The resistance of a heating element is 100 Ω at room temperature (27 °C). What is the temperature of the element when its resistance rises to 117 Ω? Take the temperature coefficient of resistance to be $1.70 \times 10^{-4}\ °C^{-1}$.

SOLUTION

Formula: $R_T = R_0[1 + \alpha(T - T_0)]$

Working:

$117 = 100\ [1 + 1.70 \times 10^{-4}\ (T - 27)]$

$1.17 = 1 + 1.70 \times 10^{-4}\ (T - 27)$

$0.17 = 1.70 \times 10^{-4}\ (T - 27)$

$T - 27 = 0.17 / 1.70 \times 10^{-4} = 1000$

T = 1027 °C

Answer: T = 1027 °C.

Q6 In a metre bridge, the null point is obtained at 39.5 cm from end A when a standard resistance of 12.5 Ω is used in the right gap. Find the unknown resistance X in the left gap. Why are thick copper strips used to make the connections?

SOLUTION

Balance condition: For a metre bridge with balancing length l measured from end A,

X / R = l / (100 − l)

$X = 12.5 \times 39.5 / (100 - 39.5) = 12.5 \times 39.5 / 60.5$

$X = 493.75 / 60.5 = 8.16\ \Omega \approx 8.2\ \Omega$

Why thick copper strips: Copper has very low resistivity and the strips are made thick to give a large cross-sectional area. Together these make the resistance of the connectors negligible, so they do not add an unknown extra resistance into the bridge arms and spoil the balance condition.

Answer: $X \approx 8.2\ \Omega$. Thick copper strips keep the connecting resistance negligible.

Q7 In a potentiometer experiment, a cell of EMF 1.25 V gives a balance point at 35.0 cm of the wire. When this cell is replaced by another cell, the balance point shifts to 63.0 cm. Find the EMF of the second cell. What would you observe if the driver cell had an EMF smaller than that of the cell being measured?

SOLUTION

Principle: The potentiometer works on the fact that the potential drop along a uniform wire is proportional to its length, so

$\varepsilon_1/\varepsilon_2 = l_1/l_2$

Working:

$1.25 / \varepsilon_2 = 35.0 / 63.0$

$\varepsilon_2 = 1.25 \times 63.0 / 35.0 = 1.25 \times 1.8$

$\varepsilon_2 = 2.25$ V

If the driver cell were weaker: The total potential drop available along the wire would be less than the EMF being measured, so the galvanometer deflection would never fall to zero. No balance point would be found anywhere on the wire, and the deflection would remain in the same direction throughout.

Answer: $\varepsilon_2 = 2.25$ V. With a weaker driver cell, no balance point could be obtained.

Q8 A copper wire of cross-sectional area 1.0×10^{-7} m^2 carries a current of 1.5 A. If the free-electron density in copper is 8.5×10^{28} m^{-3}, estimate the drift speed of the electrons. Comment on why electrical signals travel almost instantly despite this small speed.

SOLUTION

Formula: $I = n A e v_d$, so $v_d = I / (nAe)$.

Working:

$nAe = 8.5 \times 10^{28} \times 1.0 \times 10^{-7} \times 1.6 \times 10^{-19}$

$nAe = 8.5 \times 10^{28} \times 1.6 \times 10^{-26} = 1.36 \times 10^{3}$

$v_d = 1.5 / 1.36 \times 10^{3} = 1.1 \times 10^{-3}\ \text{m s}^{-1} \approx 1.1\ \text{mm s}^{-1}$

Comment: Drift is extremely slow, but the electric field that sets the electrons in motion is established along the wire at nearly the speed of light. Free electrons are already present everywhere in the conductor, so all of them begin drifting almost simultaneously. That is why a bulb lights the instant the switch is closed.

Answer: $v_d \approx 1.1 \times 10^{-3}\ \text{m s}^{-1}$ (about 1.1 mm per second).

Q9 State Kirchhoff's two rules and identify the conservation principle behind each.

SOLUTION

1. Junction rule (current rule): The algebraic sum of currents meeting at any junction of a network is zero — that is, the total current entering a junction equals the total current leaving it.

$\Sigma I = 0$ at every junction

Basis: conservation of electric charge. Charge cannot pile up at a junction in a steady current, so whatever flows in must flow out.

2. Loop rule (voltage rule): The algebraic sum of the changes in potential around any closed loop of a circuit is zero.

$\Sigma \Delta V = 0$ around every closed loop

Basis: conservation of energy. Potential is a single-valued function of position, so returning to the starting point must bring you back to the same potential; equivalently, the work done per unit charge around a closed path is zero.

Sign convention: A drop of IR is taken as negative when traversing a resistor in the direction of the current; an EMF source is taken as positive when traversed from its negative to its positive terminal.

Answer: Junction rule ($\Sigma I = 0$) follows from conservation of charge; loop rule ($\Sigma \Delta V = 0$) follows from conservation of energy.

CHAPTER 4

Moving Charges and Magnetism

The magnetic force on moving charges and current-carrying conductors, motion in a magnetic field and the cyclotron, the Biot–Savart law, fields due to circular loops and solenoids, Ampère's circuital law, the force between parallel currents, torque on a current loop and the moving coil galvanometer.

KEY FORMULAE AND RESULTS

Lorentz force	$F = q(v \times B) \Rightarrow F = qvB \sin\theta$
Force on a conductor	$F = I(L \times B) \Rightarrow F = BIL \sin\theta$
Circular motion in a field	$r = mv/qB$; $T = 2\pi m/qB$; $\nu_c = qB/2\pi m$
Biot–Savart law	$dB = (\mu_0/4\pi) \cdot I\, dl \sin\theta / r^2$, $\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1}
Straight infinite wire	$B = \mu_0 I / 2\pi r$
Centre of a circular coil	$B = \mu_0 NI / 2R$
Solenoid and toroid	Solenoid (inside): $B = \mu_0 nI$; Toroid: $B = \mu_0 NI/2\pi r$
Force between parallel wires	$F/L = \mu_0 I_1 I_2 / 2\pi d$ (attract if currents are parallel)
Torque on a current loop	$\tau = NIAB \sin\theta = mB \sin\theta$, magnetic moment $m = NIA$
Galvanometer conversions	Ammeter: $S = I_g G/(I - I_g)$; Voltmeter: $R = V/I_g - G$

Q1 A circular coil of radius 8.0 cm has 100 turns and carries a current of 0.40 A. Find the magnitude of the magnetic field at the centre of the coil.

SOLUTION

Given: $R = 8.0$ cm $= 0.08$ m, $N = 100$, $I = 0.40$ A.

Formula: $B = \mu_0 NI / 2R$

Working:

$$B = (4\pi \times 10^{-7} \times 100 \times 0.40) / (2 \times 0.08)$$

$$B = (1.2566 \times 10^{-6} \times 40) / 0.16 = 5.027 \times 10^{-5} / 0.16$$

$$B = 3.14 \times 10^{-4}\ \text{T}$$

The direction is along the axis of the coil, given by the right-hand thumb rule.

Answer: $B \approx 3.14 \times 10^{-4}$ T, directed along the axis of the coil.

Q2 A long straight wire carries a current of 35 A. Find the magnitude of the magnetic field at a point 20 cm from the wire.

SOLUTION

Formula: For a long straight conductor, $B = \mu_0 I / 2\pi r = (2 \times 10^{-7})\ I/r$.

Working:

$B = 2 \times 10^{-7} \times 35 / 0.20$

$B = 7.0 \times 10^{-6} / 0.20 = 3.5 \times 10^{-5}$ T

The field lines are concentric circles around the wire; the direction at any point is tangential, given by the right-hand thumb rule.

Answer: $B = 3.5 \times 10^{-5}$ T.

Q3 A horizontal overhead power line carries a current of 50 A from east to west. Find the magnitude and direction of the magnetic field it produces at a point 2.5 m directly below it.

SOLUTION

Magnitude:

$B = \mu_0 I/2\pi r = 2 \times 10^{-7} \times 50 / 2.5$

$B = 1.0 \times 10^{-5} / 2.5 = 4.0 \times 10^{-6}$ T

Direction: Point the right thumb along the current (east to west). The fingers curl so that below the wire the field points towards the **south**.

Answer: $B = 4.0 \times 10^{-6}$ T, directed towards the south.

Q4 A wire carrying a current of 8.0 A makes an angle of 30° with a uniform magnetic field of 0.15 T. Find the magnetic force acting per unit length of the wire.

SOLUTION

Formula: $F = BIL \sin\theta$, hence force per unit length $F/L = BI \sin\theta$.

Working:

$F/L = 0.15 \times 8.0 \times \sin 30°$

$F/L = 1.2 \times 0.5 = 0.6$ N m^{-1}

The direction of the force is perpendicular to the plane containing the wire and the field, given by $F = I(L \times B)$.

Answer: $F/L = 0.6\ N\ m^{-1}$.

Q5 A closely wound solenoid 80 cm long has 5 layers of 400 turns each. Its diameter is 1.8 cm and it carries a current of 8.0 A. Find the magnitude of the magnetic field near the centre of the solenoid.

SOLUTION

Number of turns per unit length:

Total turns $N = 5 \times 400 = 2000$

$n = N/L = 2000 / 0.80 = 2500$ turns m^{-1}

Formula: Well inside a long solenoid, $B = \mu_0 nI$.

$B = 4\pi \times 10^{-7} \times 2500 \times 8.0$

$B = 1.2566 \times 10^{-6} \times 2.0 \times 10^{4} = 2.51 \times 10^{-2}\ T$

The diameter is not needed: the length is far greater than the diameter, so the solenoid may be treated as effectively infinite and the field near the centre is uniform.

Answer: $B \approx 2.51 \times 10^{-2}\ T$ (about 0.025 T).

Q6 A circular coil of 20 turns and radius 10 cm carries a current of 5.0 A. It is placed in a uniform magnetic field of 0.10 T with the plane of the coil parallel to the field. Find the magnetic moment and the torque on the coil.

SOLUTION

Magnetic moment:

$A = \pi r^2 = \pi \times (0.10)^2 = 3.142 \times 10^{-2}\ m^2$

$m = NIA = 20 \times 5.0 \times 3.142 \times 10^{-2} = 3.14\ A\ m^2$

Torque: When the *plane* of the coil is parallel to B, the normal to the coil is perpendicular to B, so $\theta = 90°$ and $\sin\theta = 1$.

$\tau = mB \sin 90° = 3.14 \times 0.10 \times 1$

$\tau = 0.314\ N\ m$

This is the maximum possible torque. The net force on the coil is zero because the field is uniform.

Answer: $m \approx 3.14\ \text{A m}^2$; $\tau \approx 0.314$ N m (maximum value).

Q7 An electron travelling at $5.20 \times 10^6\ \text{m s}^{-1}$ enters a uniform magnetic field of 1.30×10^{-4} T at right angles to the field. Find the radius of its circular path and explain why the speed does not change.

SOLUTION

Radius: The magnetic force supplies the centripetal force, $qvB = mv^2/r$, so

$$r = mv / qB$$

$$r = (9.11 \times 10^{-31} \times 5.20 \times 10^6) / (1.6 \times 10^{-19} \times 1.30 \times 10^{-4})$$

$$r = 4.737 \times 10^{-24} / 2.08 \times 10^{-23}$$

$$r = 0.228 \text{ m} \approx 22.8 \text{ cm}$$

Why the speed is constant: The magnetic force $F = q(v \times B)$ is always perpendicular to the velocity. A force perpendicular to the displacement does no work, so the kinetic energy — and hence the speed — remains unchanged. Only the direction of motion changes, producing uniform circular motion.

Answer: $r \approx 0.228$ m (22.8 cm). The magnetic force does no work, so the speed is unchanged.

Q8 A galvanometer has a resistance of 12 Ω and gives a full-scale deflection for a current of 3.0 mA. How would you convert it into (a) a voltmeter reading up to 18 V, and (b) an ammeter reading up to 6.0 A?

SOLUTION

Given: $G = 12\ \Omega$, $I_g = 3.0\ \text{mA} = 3.0 \times 10^{-3}$ A.

(a) Voltmeter: Connect a high resistance R in *series* so that the full 18 V drives only I_g through the meter.

$$R = V/I_g - G = 18 / 3.0 \times 10^{-3} - 12$$

$$R = 6000 - 12 = 5988\ \Omega$$

(b) Ammeter: Connect a low resistance (shunt) S in *parallel* so that it carries the excess current $I - I_g$.

$$S = I_gG / (I - I_g) = (3.0 \times 10^{-3} \times 12) / (6.0 - 0.003)$$

$$S = 0.036 / 5.997 = 6.0 \times 10^{-3}\ \Omega$$

An ideal voltmeter has infinite resistance and an ideal ammeter zero resistance, which is why these arrangements are used.

Answer: (a) R = 5988 Ω in series. (b) S ≈ 6.0×10^{-3} Ω in parallel.

Q9 Two long parallel wires 1 m apart carry currents I_1 and I_2 in the same direction. Derive the expression for the force per unit length between them and use it to define the ampere.

SOLUTION

Derivation: Wire 1 produces at the location of wire 2 a field

$$B_1 = \mu_0 I_1 / 2\pi d$$

This field is perpendicular to wire 2, so the force on a length L of wire 2 is

$$F = B_1 I_2 L = \mu_0 I_1 I_2 L / 2\pi d$$

$$F/L = \mu_0 I_1 I_2 / 2\pi d$$

Applying Fleming's left-hand rule shows the force is **attractive** when the currents are in the same direction and repulsive when they are opposite.

Definition of the ampere: Put $I_1 = I_2 = 1$ A and d = 1 m:

$$F/L = (4\pi \times 10^{-7} \times 1 \times 1) / (2\pi \times 1) = 2 \times 10^{-7}\ \text{N m}^{-1}$$

Hence one ampere is that steady current which, flowing in each of two infinitely long parallel conductors of negligible cross-section placed 1 m apart in vacuum, produces a force of 2×10^{-7} N per metre of length between them.

Answer: $F/L = \mu_0 I_1 I_2/2\pi d$; attractive for parallel currents. For 1 A at 1 m, $F/L = 2 \times 10^{-7}$ N m^{-1}.

CHAPTER 5

Magnetism and Matter

The bar magnet and its magnetic dipole moment, magnetic field lines, torque and potential energy of a magnetic dipole, the Earth's magnetism and its elements, and the classification of materials into diamagnetic, paramagnetic and ferromagnetic substances.

KEY FORMULAE AND RESULTS

Magnetic dipole moment	$m = q_m \times 2l$ (bar magnet); $m = NIA$ (current loop)
Field of a bar magnet	Axial: $B = (\mu_0/4\pi) \cdot 2m/r^3$; Equatorial: $B = (\mu_0/4\pi) \cdot m/r^3$
Torque and energy	$\tau = mB \sin\theta$; $U = -mB \cos\theta$ (stable at $\theta = 0°$, unstable at 180°)
Oscillating magnet	$T = 2\pi\sqrt{(I/mB)}$, where I is the moment of inertia
Elements of Earth's magnetism	Declination, dip δ, horizontal component B_H; $B_H = B \cos\delta$, $B_V = B \sin\delta$, $\tan\delta = B_V/B_H$
Magnetisation and susceptibility	$M = m_{net}/V$; $B = \mu_0(H + M)$; $\chi = M/H$; $\mu_r = 1 + \chi$
Curie's law	$\chi \propto 1/T$ for a paramagnetic material

Q1 A bar magnet of magnetic moment 0.32 J T^{-1} is placed in a uniform magnetic field of 0.15 T. (a) What are the potential energies in the stable and unstable orientations? (b) Find the torque when the magnet makes 30° with the field.

SOLUTION

Formula: $U = -mB \cos\theta$, $\tau = mB \sin\theta$.

$mB = 0.32 \times 0.15 = 0.048$ J

(a) Stable equilibrium (θ = 0°, m parallel to B):

$U = -0.048 \times \cos 0° = -4.8 \times 10^{-2}$ J (minimum energy)

Unstable equilibrium (θ = 180°, m antiparallel to B):

$U = -0.048 \times \cos 180° = +4.8 \times 10^{-2}$ J (maximum energy)

In both positions the torque is zero, but a small displacement from θ = 0° produces a restoring torque, while at θ = 180° it produces a torque that drives the magnet further away.

(b) Torque at 30°:

$\tau = 0.048 \times \sin 30° = 0.048 \times 0.5 = 0.024$ N m

Answer: (a) $U = -4.8 \times 10^{-2}$ J (stable) and $+4.8 \times 10^{-2}$ J (unstable). (b) $\tau = 0.024$ N m.

Q2 A short bar magnet has a magnetic moment of 0.48 J T^{-1}. Find the magnitude and direction of the magnetic field it produces at a point 10 cm from its centre on (a) its axis and (b) its equatorial line.

SOLUTION

Given: $m = 0.48$ J T^{-1}, $r = 0.10$ m, $\mu_0/4\pi = 10^{-7}$ T m A^{-1}. $r^3 = 1 \times 10^{-3}$ m^3.

(a) Axial point:

$$B = (\mu_0/4\pi) \cdot 2m/r^3 = 10^{-7} \times 2 \times 0.48 / 10^{-3}$$

$$B = 9.6 \times 10^{-8} / 10^{-3} = 9.6 \times 10^{-5} \text{ T}$$

On the axis the field is directed *along* the magnetic moment, i.e. from S to N inside the magnet, pointing away from the north pole.

(b) Equatorial point:

$$B = (\mu_0/4\pi) \cdot m/r^3 = 10^{-7} \times 0.48 / 10^{-3}$$

$$B = 4.8 \times 10^{-5} \text{ T}$$

The equatorial field is exactly half the axial field at the same distance, and it points *opposite* to the magnetic moment.

Answer: (a) 9.6×10^{-5} T along m. (b) 4.8×10^{-5} T, antiparallel to m.

Q3 At a certain place the angle of dip is 22° and the horizontal component of the Earth's magnetic field is 0.16 G. Find the total magnetic field of the Earth at that place and its vertical component.

SOLUTION

Relations: $B_H = B \cos\delta$ and $B_V = B \sin\delta$.

Total field:

$$B = B_H / \cos\delta = 0.16 / \cos 22°$$

$$B = 0.16 / 0.9272 = 0.173 \text{ G}$$

Vertical component:

$$B_V = B \sin\delta = 0.173 \times \sin 22° = 0.173 \times 0.3746 = 0.065 \text{ G}$$

Check: $\sqrt{(0.16^2 + 0.065^2)} = \sqrt{(0.0256 + 0.0042)} = 0.173$ G. ✓

(1 G = 10^{-4} T, so $B \approx 1.73 \times 10^{-5}$ T.)

Answer: $B \approx 0.17$ G $\approx 1.73 \times 10^{-5}$ T; $B_V \approx 0.065$ G.

Q4 A short bar magnet is placed on a horizontal table with its north pole pointing north. A neutral point is found 14 cm from the centre of the magnet on its equatorial line. If the horizontal component of the Earth's field is 0.36 G, find the magnetic moment of the magnet.

SOLUTION

Meaning of a neutral point: It is a point where the magnet's field exactly cancels the Earth's horizontal field, so their magnitudes are equal.

With the north pole pointing north, the magnet's equatorial field points south and can cancel B_H, so neutral points lie on the equatorial line.

$$(\mu_0/4\pi) \cdot m/r^3 = B_H$$

Working: $r = 0.14$ m, so $r^3 = 2.744 \times 10^{-3}$ m^3; $B_H = 0.36$ G $= 0.36 \times 10^{-4}$ T.

$$m = B_H\, r^3 / (\mu_0/4\pi) = (0.36 \times 10^{-4} \times 2.744 \times 10^{-3}) / 10^{-7}$$

$$m = 9.878 \times 10^{-8} / 10^{-7} = 0.99\ \text{J T}^{-1}$$

Answer: $m \approx 0.99$ J T^{-1}.

Q5 Compare diamagnetic, paramagnetic and ferromagnetic substances on the basis of susceptibility, relative permeability, behaviour in a non-uniform field and temperature dependence.

SOLUTION

Property	Diamagnetic	Paramagnetic	Ferromagnetic
Susceptibility χ	Small and negative ($-1 \le \chi < 0$)	Small and positive ($0 < \chi < \varepsilon$)	Very large and positive ($\chi >> 1$)
Relative permeability μ_r	Slightly less than 1	Slightly greater than 1	Very much greater than 1
In a non-uniform field	Moves from strong to weak field (repelled)	Moves from weak to strong field (weakly attracted)	Strongly attracted towards the stronger field
Field lines	Expelled from the material	Slightly concentrated inside	Strongly concentrated inside
Effect of temperature	Essentially independent of temperature	$\chi \propto 1/T$ (Curie's law)	Becomes paramagnetic above the Curie temperature
Origin	Induced moments opposing the field; no permanent atomic moments	Permanent atomic moments, randomly oriented by thermal motion	Permanent moments aligned in domains
Examples	Bismuth, copper, water, gold	Aluminium, sodium, oxygen, platinum	Iron, cobalt, nickel, gadolinium

Answer: See the comparison table: χ is small and negative for diamagnetic, small and positive for paramagnetic, and very large and positive for ferromagnetic substances.

Q6 Explain why the Earth behaves like a magnet, and why a magnetic needle at the magnetic poles cannot be used to determine direction.

SOLUTION

Why the Earth is magnetic: The most widely accepted explanation is the *dynamo effect*. The Earth's outer core is a shell of molten, electrically conducting iron and nickel. Convection currents in this fluid, combined with the Earth's rotation, set up large circulating electric currents. These currents generate and sustain the geomagnetic field, much as a current loop generates a magnetic dipole field.

To a first approximation the field resembles that of a giant bar magnet tilted at about 11° to the rotation axis, with its magnetic south pole near the geographic north.

Behaviour of a needle at the magnetic poles: At the magnetic poles the angle of dip is 90°, which means the Earth's field is entirely vertical there.

$$B_H = B\cos 90^\circ = 0$$

A compass needle is pivoted so that it can rotate only in a horizontal plane, and it aligns itself with the *horizontal* component. Since $B_H = 0$ at the poles, there is no directive force acting on the needle; it can rest in any direction and gives no information about geographic direction. A dip needle, free to rotate in a vertical plane, would simply point straight down.

Answer: The geodynamo in the molten outer core produces the field. At the magnetic poles $B_H = 0$, so a compass needle has no directive force and is useless for finding direction.

CHAPTER 6

Electromagnetic Induction

Magnetic flux, Faraday's laws of electromagnetic induction, Lenz's law and its link with energy conservation, motional EMF, eddy currents, and self and mutual inductance.

KEY FORMULAE AND RESULTS

Magnetic flux	$\varphi_B = \int B \cdot dA = BA\cos\theta$ (unit: weber, Wb)
Faraday's law	$\varepsilon = -d\varphi_B/dt$; for N turns, $\varepsilon = -N\, d\varphi_B/dt$
Lenz's law	The induced current opposes the change producing it (the minus sign above)
Motional EMF	$\varepsilon = Blv$; rotating rod: $\varepsilon = \frac{1}{2}B\omega l^2$
Self-inductance	$\varphi = LI$; $\varepsilon = -L\, dI/dt$; solenoid: $L = \mu_0 n^2 Al$
Mutual inductance	$\varphi_2 = MI_1$; $\varepsilon_2 = -M\, dI_1/dt$
Energy stored in an inductor	$U = \frac{1}{2}LI^2$
Induced charge	$q = \Delta\varphi/R$ (independent of the time taken)

Q1 State Lenz's law and show that it is a consequence of the law of conservation of energy.

SOLUTION

Statement: The direction of the induced EMF (and hence of the induced current) is always such that it opposes the change in magnetic flux that produced it.

Illustration: Push the north pole of a bar magnet towards a closed coil. The flux through the coil increases, so the induced current flows in a direction that makes the near face of the coil a north pole, which repels the approaching magnet.

Connection with energy conservation: Because the coil repels the magnet, the person moving it must do work against this repulsion. That mechanical work is exactly what appears as electrical energy in the coil, and is finally dissipated as heat in the coil's resistance.

Now suppose the opposite were true — that the induced current *attracted* the magnet. The magnet would accelerate towards the coil on its own, gaining kinetic energy, while the coil simultaneously generated electrical energy. Energy would be created from nothing, giving a perpetual motion machine. Since this is impossible, the induced effect must oppose the change.

Hence Lenz's law is simply the statement of energy conservation applied to electromagnetic induction, and it fixes the minus sign in Faraday's law $\varepsilon = -d\varphi/dt$.

> **Answer:** The induced current opposes the change causing it; any other direction would allow energy to be created from nothing.

Q2 A metallic rod of length 1.0 m is rotated with an angular frequency of 400 rad s^{-1} about an axis passing through one end and perpendicular to the rod. A uniform magnetic field of 0.5 T is directed parallel to the axis. Find the EMF developed between the centre of the rod and the ring at its far end.

SOLUTION

Derivation: Consider a small element dx at distance x from the axis. Its linear speed is $v = \omega x$, so the EMF contributed is $dE = B(\omega x)dx$. Integrating from 0 to l:

$$\varepsilon = \int_0^l B\omega x \, dx = ½B\omega l^2$$

Working:

$$\varepsilon = ½ \times 0.5 \times 400 \times (1.0)^2$$

$$\varepsilon = 0.5 \times 0.5 \times 400 = 100 \text{ V}$$

The centre of the rod (the axis) is at one potential and the outer end at another; the sense is fixed by the direction of rotation and of B.

Answer: ε = 100 V.

Q3 A jet plane with a wingspan of 25 m is flying horizontally at 1800 km h^{-1}. The vertical component of the Earth's magnetic field at that location is 5.0×10^{-4} T. Find the voltage difference developed between the wing tips.

SOLUTION

Convert the speed:

$$v = 1800 \text{ km h}^{-1} = 1800 \times (1000/3600) = 500 \text{ m s}^{-1}$$

Formula: The wings act as a conductor of length l moving with speed v across the vertical component of the field, so $\varepsilon = B_V \, l \, v$.

$$\varepsilon = 5.0 \times 10^{-4} \times 25 \times 500$$

$$\varepsilon = 5.0 \times 10^{-4} \times 1.25 \times 10^4 = 6.25 \text{ V}$$

Only the vertical component matters, because the horizontal component is (broadly) along the direction of flight and contributes no motional EMF across the wingspan.

Answer: ε = 6.25 V.

Q4 A rectangular loop of sides 8 cm and 2 cm with a small cut moves out of a region of uniform magnetic field of 0.3 T directed normal to the loop. The loop moves with a velocity of 1 cm s^{-1} in a direction normal to the longer side. Find the induced EMF and the time for which it lasts.

SOLUTION

Induced EMF: As the loop leaves the field region, the effective length cutting the field lines is the longer side, l = 8 cm = 0.08 m.

$\varepsilon = Blv = 0.3 \times 0.08 \times 0.01$

$\varepsilon = 2.4 \times 10^{-4}$ V = 0.24 mV

Duration: The EMF exists only while the loop is partly in and partly out of the field. The loop must travel a distance equal to its shorter side, 2 cm, to leave completely.

t = distance / speed = 0.02 / 0.01 = 2 s

Note: Because the loop has a cut, no current flows — only an EMF appears across the gap. If instead the loop moved out along the direction of the shorter side, the effective length would be 2 cm and the time 8 s.

Answer: $\varepsilon = 2.4 \times 10^{-4}$ V, lasting for 2 s.

Q5 The current in a coil falls steadily from 20 A to zero in 0.5 s, inducing an EMF of 200 V. Find the self-inductance of the coil.

SOLUTION

Formula: $|\varepsilon| = L\,|dI/dt|$

Rate of change of current:

$|dI/dt| = 20 / 0.5 = 40$ A s^{-1}

Self-inductance:

$L = |\varepsilon| / |dI/dt| = 200 / 40$

L = 5 H

Energy check: The energy that was stored in the coil before the current fell was $U = \frac{1}{2}LI^2 = \frac{1}{2} \times 5 \times 400 = 1000$ J.

Answer: L = 5 H.

Q6 A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A in 0.5 s, what is the change of flux linkage with the other coil, and what EMF is induced in it?

SOLUTION

Change in flux linkage: The flux linked with the second coil is $\varphi_2 = M I_1$, so

$$\Delta\varphi_2 = M\,\Delta I_1 = 1.5 \times (20 - 0)$$

$$\Delta\varphi_2 = 30 \text{ Wb}$$

Induced EMF:

$$|\varepsilon_2| = M\,|dI_1/dt| = 1.5 \times (20/0.5) = 1.5 \times 40$$

$$|\varepsilon_2| = 60 \text{ V}$$

Answer: $\Delta\varphi = 30$ Wb; $|\varepsilon| = 60$ V.

Q7 What are eddy currents? Give two useful applications and two ways of minimising the losses they cause.

SOLUTION

Definition: When the magnetic flux through the body of a bulk conductor changes, induced currents are set up in closed loops within the volume of the metal. These circulating currents are called *eddy currents* (or Foucault currents). Their direction, by Lenz's law, is such as to oppose the change in flux, and they dissipate energy as heat.

Useful applications:

• **Electromagnetic braking** in trains and some vehicles: a strong electromagnet near the rotating metal drum or rail induces eddy currents that oppose the motion and bring it smoothly to rest without mechanical contact.

• **Induction furnaces and induction cooktops:** a high-frequency alternating field induces large eddy currents in the metal, and the resulting heating melts the charge or cooks the food.

• (Also: electromagnetic damping in moving-coil galvanometers, and speedometers.)

Minimising eddy-current losses:

• **Lamination:** the cores of transformers and motors are built from thin sheets insulated from one another by varnish. This confines the eddy currents to thin loops of high resistance, sharply reducing the heat produced.

• **Using high-resistivity core material** such as silicon steel or ferrite, which lowers the eddy currents for the same induced EMF.

Answer: Eddy currents are induced circulating currents in bulk conductors; used in magnetic braking and induction heating, and reduced by laminating the core and using high-resistivity materials.

CHAPTER 7

Alternating Current

Alternating voltage and current, RMS and peak values, AC through a resistor, an inductor and a capacitor, the series LCR circuit and phasor analysis, resonance and the quality factor, power in AC circuits and the power factor, LC oscillations, and the transformer.

KEY FORMULAE AND RESULTS

RMS values	$I_{rms} = I_0/\sqrt{2} = 0.707\ I_0$; $V_{rms} = V_0/\sqrt{2}$
Reactances	$X_L = \omega L = 2\pi fL$; $X_C = 1/\omega C = 1/2\pi fC$
Impedance of a series LCR circuit	$Z = \sqrt{[R^2 + (X_L - X_C)^2]}$; $\tan\varphi = (X_L - X_C)/R$
Resonance	$X_L = X_C \Rightarrow \omega_0 = 1/\sqrt{(LC)}$; Z = R (minimum), I is maximum
Quality factor	$Q = \omega_0 L/R = 1/(\omega_0 CR) = (1/R)\sqrt{(L/C)}$
Average power	$P = V_{rms}\ I_{rms}\cos\varphi$; power factor = $\cos\varphi = R/Z$
Wattless current	In a pure L or pure C, $\varphi = 90°$, so $\cos\varphi = 0$ and $P = 0$
Ideal transformer	$V_s/V_p = N_s/N_p = I_p/I_s$

Q1 A 100 Ω resistor is connected to a 220 V, 50 Hz AC supply. (a) Find the RMS current in the circuit. (b) Find the net power consumed over a full cycle.

SOLUTION

(a) RMS current: For a purely resistive circuit, Ohm's law applies directly to RMS values.

$$I_{rms} = V_{rms}/R = 220/100 = 2.2\ \text{A}$$

(b) Power: In a pure resistor the current and voltage are in phase, so $\varphi = 0$ and $\cos\varphi = 1$.

$$P = V_{rms}I_{rms}\cos\varphi = 220 \times 2.2 \times 1$$

$$P = 484\ \text{W}$$

Check: $P = V_{rms}^2/R = (220)^2/100 = 48400/100 = 484$ W. ✓

Note that the instantaneous power fluctuates, but the average over a complete cycle is 484 W.

Answer: (a) $I_{rms} = 2.2$ A. (b) P = 484 W.

Q2 (a) The peak voltage of an AC supply is 300 V. What is its RMS value? (b) The RMS current in a circuit is 10 A. What is the peak current?

SOLUTION

(a) RMS from peak voltage:

$V_{rms} = V_0/\sqrt{2} = 300/1.414$

$V_{rms} = 212.1\text{ V} \approx 212\text{ V}$

(b) Peak from RMS current:

$I_0 = \sqrt{2} \times I_{rms} = 1.414 \times 10$

$I_0 = 14.14\text{ A} \approx 14.1\text{ A}$

The RMS value is the DC value that would dissipate the same average power in a given resistance, which is why AC meters are calibrated to read RMS.

Answer: (a) $V_{rms} \approx 212$ V. (b) $I_0 \approx 14.1$ A.

Q3 A 44 mH inductor is connected to a 220 V, 50 Hz AC supply. Find the inductive reactance and the RMS value of the current in the circuit.

SOLUTION

Inductive reactance:

$X_L = 2\pi fL = 2 \times 3.1416 \times 50 \times 44 \times 10^{-3}$

$X_L = 314.16 \times 0.044 = 13.82\ \Omega$

RMS current:

$I_{rms} = V_{rms}/X_L = 220/13.82$

$I_{rms} = 15.9\text{ A}$

The current lags the voltage by 90°, and the average power consumed over a cycle is zero for an ideal inductor.

Answer: $X_L \approx 13.82\ \Omega$; $I_{rms} \approx 15.9$ A.

Q4 A 60 μF capacitor is connected to a 110 V, 60 Hz AC supply. Find the capacitive reactance and the RMS current. What is the average power consumed?

SOLUTION

Capacitive reactance:

$X_C = 1/(2\pi fC) = 1/(2 \times 3.1416 \times 60 \times 60 \times 10^{-6})$

$X_C = 1/(376.99 \times 6.0 \times 10^{-5}) = 1/0.022619$

$X_C = 44.2\ \Omega$

RMS current:

$I_{rms} = 110/44.2 = 2.49$ A

Average power: In a pure capacitor the current leads the voltage by 90°, so $\varphi = 90°$ and $\cos\varphi = 0$.

$P = V_{rms} I_{rms} \cos 90° = 0$

Energy is alternately stored in the electric field and returned to the source, giving a *wattless* current.

Answer: $X_C \approx 44.2\ \Omega$; $I_{rms} \approx 2.49$ A; average power = 0.

Q5 A series LCR circuit has L = 5.0 H, C = 80 μF and R = 40 Ω, connected to a 230 V variable-frequency supply. Find (a) the resonant angular frequency, (b) the current amplitude at resonance and (c) the average power at resonance.

SOLUTION

(a) Resonant angular frequency: At resonance $X_L = X_C$, giving $\omega_0 = 1/\sqrt{(LC)}$.

$LC = 5.0 \times 80 \times 10^{-6} = 4.0 \times 10^{-4}$

$\omega_0 = 1/\sqrt{(4.0 \times 10^{-4})} = 1/0.02 = 50$ rad s^{-1}

$f_0 = \omega_0/2\pi = 50/6.283 = 7.96$ Hz

(b) Current amplitude: At resonance the impedance is minimum and equals R.

$V_0 = \sqrt{2} \times 230 = 325.3$ V

$I_0 = V_0/R = 325.3/40 = 8.13$ A

(c) Average power: At resonance $\varphi = 0$, so the power factor is 1.

$P = V_{rms}^2/R = (230)^2/40 = 52900/40$

$P = 1322.5$ W

Answer: (a) $\omega_0 = 50$ rad s^{-1} ($f_0 \approx 7.96$ Hz). (b) $I_0 \approx 8.13$ A. (c) P = 1322.5 W.

Q6 For the circuit of the previous question, find the quality factor. State how Q can be increased without changing the resonant frequency.

SOLUTION

Formula: $Q = \omega_0 L / R$

Working:

$Q = (50 \times 5.0) / 40 = 250/40$

$Q = 6.25$

Meaning: Q measures the sharpness of resonance. A high Q means a narrow bandwidth $\Delta\omega = \omega_0/Q$, so the circuit responds strongly only to frequencies very close to ω_0 — the basis of tuning in a radio receiver.

How to increase Q at the same ω_0: Since $\omega_0 = 1/\sqrt{LC}$ must stay fixed, the product LC cannot change. Q can then be raised by

• reducing R (using thicker wire or a better core), since $Q \propto 1/R$; and

• increasing L while decreasing C in the same proportion, because $Q = (1/R)\sqrt{L/C}$ leaves ω_0 unchanged but raises Q.

For example, halving R to 20 Ω would double Q to 12.5.

Answer: Q = 6.25. It can be increased by lowering R, or by increasing L and decreasing C so that LC (and hence ω_0) stays the same.

Q7 Explain the principle of a transformer. A step-down transformer converts 2300 V to 230 V. If the primary has 4000 turns and the output current is 100 A, find the number of secondary turns and the primary current for an ideal transformer.

SOLUTION

Principle: A transformer works on *mutual induction*. An alternating current in the primary coil produces a continuously changing magnetic flux, which is guided by a soft-iron core so that (ideally) the same flux links every turn of the secondary. The changing flux induces an EMF in the secondary given by Faraday's law. Since each turn links the same flux,

$V_s/V_p = N_s/N_p$

A transformer works only with AC, because a steady DC produces no change of flux and hence no induced EMF.

Secondary turns:

$N_s = N_p \times (V_s/V_p) = 4000 \times (230/2300)$

$N_s = 4000 \times 0.1 = 400$ turns

Primary current: For an ideal transformer, input power = output power.

$V_pI_p = V_sI_s$

$I_p = (230 \times 100) / 2300 = 23000/2300 = 10$ A

The voltage steps down by a factor of 10 while the current steps up by the same factor, keeping the power constant.

Answer: $N_s = 400$ turns; $I_p = 10$ A.

Q8 Why is a choke coil preferred over a resistor for controlling current in an AC circuit?

SOLUTION

The problem with a resistor: A resistance limits current by converting electrical energy into heat, at a rate $P = I^2R$. All the energy removed from the circuit is wasted, and the component gets hot.

The choke coil: A choke is an inductor of large inductance and very small resistance. It limits current through its reactance:

$I_{rms} = V_{rms}/Z$, where $Z = \sqrt{(R^2 + \omega^2L^2)} \approx \omega L$ when $R \to 0$

Why it wastes almost no power: The power factor of the choke is

$\cos\varphi = R/Z \approx 0 \Rightarrow P = V_{rms}I_{rms} \cos\varphi \approx 0$

Energy is alternately stored in the magnetic field of the coil and returned to the source, rather than being dissipated. This is why fluorescent tubes and similar loads use a choke rather than a series resistor.

Limitation: A choke works only on AC. In a DC circuit $\omega = 0$, so $X_L = 0$ and the choke offers only its small ohmic resistance.

Answer: A choke limits current through reactance rather than resistance, so its power factor is nearly zero and almost no energy is dissipated as heat.

CHAPTER 8

Electromagnetic Waves

The need for displacement current, Maxwell's contribution to Ampère's law, the nature and properties of electromagnetic waves, energy carried by them, and the electromagnetic spectrum from radio waves to gamma rays together with the uses of each band.

KEY FORMULAE AND RESULTS

Displacement current	$I_d = \varepsilon_0\, d\varphi_E/dt$
Ampère–Maxwell law	$\oint B \cdot dl = \mu_0(I_c + I_d)$
Speed of light	$c = 1/\sqrt{\mu_0\varepsilon_0} = 3 \times 10^8$ m s^{-1}; in a medium $v = 1/\sqrt{\mu\varepsilon}$
Field amplitudes	$E_0/B_0 = c$; E, B and the direction of propagation are mutually perpendicular
Energy density	$u_E = ½\varepsilon_0 E^2$, $u_B = B^2/2\mu_0$; average total $u = ½\varepsilon_0 E_0^2$
Intensity and momentum	$I = u_{av}\, c$; momentum delivered $p = U/c$ (complete absorption)
Wave relation	$c = f\lambda$

Q1 What is displacement current, and why did Maxwell find it necessary to introduce it? Show that for a charging capacitor the displacement current between the plates equals the conduction current in the wires.

SOLUTION

The inconsistency: Ampère's circuital law in its original form states $\oint B\cdot dl = \mu_0 I_c$. Consider a capacitor being charged and take a closed loop around the connecting wire. Two different surfaces can be bounded by the same loop:

- a flat surface cutting the wire, which the conduction current I_c passes through;
- a bulging surface passing between the capacitor plates, through which no conduction current flows at all.

The same loop integral would then give two different answers, so the law as stated is incomplete.

Maxwell's resolution: Between the plates the electric field is changing, and Maxwell proposed that a changing electric flux is itself a source of magnetic field. He defined the displacement current

$$I_d = \varepsilon_0\, d\varphi_E/dt$$

Proof of equality: For a parallel plate capacitor with charge q on plates of area A,

$$E = \sigma/\varepsilon_0 = q/(\varepsilon_0 A) \;\Rightarrow\; \varphi_E = EA = q/\varepsilon_0$$

$$I_d = \varepsilon_0\, d\varphi_E/dt = \varepsilon_0 \cdot (1/\varepsilon_0)\, dq/dt = dq/dt = I_c$$

The total current is therefore continuous throughout the circuit, and the modified law $\oint B \cdot dl = \mu_0(I_c + I_d)$ gives the same result for both surfaces.

> **Answer:** $I_d = \varepsilon_0\, d\varphi_E/dt = dq/dt = I_c$, restoring the continuity of current and making Ampère's law consistent.

Q2 A plane electromagnetic wave travelling in vacuum has a magnetic field amplitude of 510 nT. Find the amplitude of the electric field.

SOLUTION

Formula: In an electromagnetic wave in vacuum, the field amplitudes are related by $E_0 = cB_0$.

Working:

$$E_0 = 3 \times 10^8 \times 510 \times 10^{-9}$$

$$E_0 = 3 \times 510 \times 10^{-1} = 153 \text{ V m}^{-1}$$

Note how much larger the electric field is numerically. This is why the electric vector is called the "light vector" — it is the component that produces most optical effects.

> **Answer:** $E_0 = 153 \text{ V m}^{-1}$.

Q3 A radio transmitter operates at a frequency of 30 MHz. What is the wavelength of the electromagnetic waves it produces? To which part of the spectrum do they belong?

SOLUTION

Formula: $c = f\lambda$, so $\lambda = c/f$.

$$\lambda = 3 \times 10^8 / 30 \times 10^6$$

$$\lambda = 10 \text{ m}$$

Classification: Wavelengths of the order of metres correspond to the **radio wave** region (specifically the short-wave / VHF border). Such waves are produced by accelerating charges in aerials and are used in broadcasting and communication.

> **Answer:** $\lambda = 10$ m — radio waves.

Q4 Radiation of wavelength 21 cm is emitted by atomic hydrogen in interstellar space. Find the frequency of this radiation and name the region of the spectrum it belongs to.

SOLUTION

Formula: $f = c/\lambda$

$f = 3 \times 10^8 / 0.21$

$f = 1.43 \times 10^9$ Hz = 1.43 GHz

Classification: A frequency of about 1.4 GHz lies in the **microwave / short radio wave** region. This famous 21 cm line arises from a hyperfine transition in neutral hydrogen and is the principal tool of radio astronomy for mapping the structure of our galaxy.

Answer: $f \approx 1.43 \times 10^9$ Hz (1.43 GHz) — microwave/short radio region.

Q5 In a plane electromagnetic wave the electric field oscillates sinusoidally at a frequency of 2.0 × 10^{10} Hz with an amplitude of 48 V m^{-1}. Find (a) the wavelength, (b) the amplitude of the magnetic field and (c) the average energy density of the wave.

SOLUTION

(a) Wavelength:

$\lambda = c/f = 3 \times 10^8 / 2.0 \times 10^{10}$

$\lambda = 1.5 \times 10^{-2}$ m = 1.5 cm

(b) Magnetic field amplitude:

$B_0 = E_0/c = 48 / 3 \times 10^8$

$B_0 = 1.6 \times 10^{-7}$ T = 160 nT

(c) Average energy density: The electric and magnetic contributions are equal, each averaging $\frac{1}{4}\varepsilon_0 E_0^2$, so the total average is

$u_{av} = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2} \times 8.854 \times 10^{-12} \times (48)^2$

$u_{av} = \frac{1}{2} \times 8.854 \times 10^{-12} \times 2304$

$u_{av} = 1.02 \times 10^{-8}$ J m^{-3}

Answer: (a) λ = 1.5 cm. (b) $B_0 = 1.6 \times 10^{-7}$ T. (c) $u_{av} \approx 1.02 \times 10^{-8}$ J m^{-3}.

Q6 List the parts of the electromagnetic spectrum in order of increasing frequency, with their approximate wavelength ranges, method of production and one use of each.

SOLUTION

Region	Approx. wavelength	Production	Typical use
Radio waves	> 0.1 m	Accelerated charges in aerials and oscillating LC circuits	Radio and television broadcasting, mobile communication
Microwaves	0.1 m – 1 mm	Klystrons, magnetrons, Gunn diodes	Radar, microwave ovens, satellite links
Infrared	1 mm – 700 nm	Vibration and rotation of molecules in hot bodies	Thermal imaging, remote controls, physiotherapy
Visible light	700 nm – 400 nm	Outer-electron transitions in atoms; incandescent bodies	Vision, photography, optical instruments
Ultraviolet	400 nm – 1 nm	Very hot bodies, mercury and arc lamps, the Sun	Sterilisation of water and instruments, LASIK surgery
X-rays	1 nm – 10^{-3} nm	Rapid deceleration of high-energy electrons on a metal target	Medical radiography, crystal structure analysis
Gamma rays	< 10^{-3} nm	Radioactive decay of atomic nuclei	Cancer radiotherapy, sterilisation of medical equipment

All these waves are identical in nature — transverse, requiring no medium and travelling at 3×10^8 m s^{-1} in vacuum. They differ only in frequency and wavelength, and the boundaries between the bands overlap.

Answer: Radio → microwave → infrared → visible → ultraviolet → X-rays → gamma rays, in order of increasing frequency.

Q7 Show that in an electromagnetic wave the average energy stored in the electric field equals that stored in the magnetic field.

SOLUTION

Energy densities: At any instant,

$$u_E = ½\varepsilon_0 E^2 \quad \text{and} \quad u_B = B^2/2\mu_0$$

Substituting B = E/c:

$$u_B = (E/c)^2 / 2\mu_0 = E^2 / (2\mu_0 c^2)$$

Using $c^2 = 1/\mu_0\varepsilon_0$:

$$u_B = E^2\ \mu_0\varepsilon_0 / 2\mu_0 = ½\varepsilon_0 E^2 = u_E$$

The two energy densities are therefore equal at every instant, and hence their time averages are equal as well:

$\langle u_E \rangle = \langle u_B \rangle = \frac{1}{4}\varepsilon_0 E_0^2$

$\langle u_{total} \rangle = \frac{1}{2}\varepsilon_0 E_0^2 = B_0^2/2\mu_0$

The energy of an electromagnetic wave is thus shared equally between its electric and magnetic components.

Answer: Using $B = E/c$ and $c^2 = 1/\mu_0\varepsilon_0$, $u_B = \frac{1}{2}\varepsilon_0 E^2 = u_E$ — the energy is shared equally.

CHAPTER 9

Ray Optics and Optical Instruments

Reflection at spherical mirrors, refraction at plane and spherical surfaces, total internal reflection and optical fibres, refraction through a prism and dispersion, the lens maker's formula and lens combinations, and the working of the eye, the microscope and the telescope.

KEY FORMULAE AND RESULTS

Mirror formula	$1/v + 1/u = 1/f$, $f = R/2$; magnification $m = -v/u = h'/h$
Refraction at a plane surface	n = real depth / apparent depth; $n_1 \sin i = n_2 \sin r$
Total internal reflection	$\sin C = 1/n$ (light must travel from denser to rarer medium)
Refraction at a spherical surface	$n_2/v - n_1/u = (n_2 - n_1)/R$
Lens maker's formula	$1/f = (n - 1)(1/R_1 - 1/R_2)$
Thin lens formula	$1/v - 1/u = 1/f$; $m = v/u$; power $P = 1/f$ (in metres), unit dioptre
Lenses in contact	$1/f = 1/f_1 + 1/f_2 + \ldots$; $P = P_1 + P_2 + \ldots$
Prism	$A + D = i + e$; $n = \sin[(A + D_m)/2] / \sin(A/2)$
Compound microscope	$m = (v_o/u_o)(1 + D/f_e)$ at the near point
Astronomical telescope	$m = f_o/f_e$; tube length $L = f_o + f_e$ (normal adjustment)

Q1 A candle 2.5 cm tall is placed 27 cm in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed to obtain a sharp image? Describe the nature and size of the image.

SOLUTION

Given (using the sign convention, distances measured from the pole, with the incident direction positive):

$$u = -27 \text{ cm}, \quad R = -36 \text{ cm} \Rightarrow f = R/2 = -18 \text{ cm}, \quad h = 2.5 \text{ cm}$$

Mirror formula: $1/v + 1/u = 1/f$

$$1/v = 1/f - 1/u = (-1/18) - (-1/27) = -1/18 + 1/27$$

$$1/v = (-3 + 2)/54 = -1/54$$

$$v = -54 \text{ cm}$$

Magnification:

$m = -v/u = -(-54)/(-27) = -2$

$h' = m\,h = -2 \times 2.5 = -5.0$ cm

Interpretation: v is negative, so the image is formed 54 cm in front of the mirror — on the same side as the object — and is therefore **real**. The negative magnification shows it is **inverted**, and $|m| = 2$ means it is magnified to twice the object size.

Answer: The screen must be 54 cm from the mirror. The image is real, inverted and 5.0 cm tall.

Q2 A needle 4.5 cm long is placed 12 cm in front of a convex mirror of focal length 15 cm. Find the position and size of the image. What happens to the image as the needle is moved further from the mirror?

SOLUTION

Given: $u = -12$ cm, $f = +15$ cm (convex mirror), $h = 4.5$ cm.

Mirror formula:

$1/v = 1/f - 1/u = 1/15 - (-1/12) = 1/15 + 1/12$

$1/v = (4 + 5)/60 = 9/60$

$v = 60/9 = +6.67$ cm

Magnification:

$m = -v/u = -6.67/(-12) = +0.556$

$h' = 0.556 \times 4.5 = 2.5$ cm

Nature: v is positive, so the image is behind the mirror — **virtual**. The positive magnification means it is **erect**, and it is diminished.

As the needle moves away: The image moves towards the focus of the mirror (6.67 cm → 15 cm behind it is the limiting position for a very distant object, but the image approaches F at 15 cm) and steadily shrinks in size, remaining virtual and erect throughout. This wide field of view is why convex mirrors are used as rear-view mirrors.

Answer: Image is 6.67 cm behind the mirror, virtual, erect and 2.5 cm long. As the object recedes, the image moves towards the focus and shrinks.

Q3 A tank is filled with water to a height of 12.5 cm. The apparent depth of a needle lying at the bottom, viewed from directly above, is 9.4 cm. Find the refractive index of water. If water is replaced by a liquid of refractive index 1.63, by how much must the microscope be moved to focus on the needle again?

SOLUTION

Refractive index of water:

n = real depth / apparent depth = 12.5 / 9.4

n = 1.33

With the new liquid: The real depth is still 12.5 cm.

apparent depth = real depth / n′ = 12.5 / 1.63 = 7.67 cm

Displacement of the microscope:

Δx = 9.4 − 7.67 = 1.73 cm

The needle appears to rise further, so the microscope must be moved **up** by about 1.73 cm to refocus.

Answer: n_{water} ≈ 1.33. The microscope must be raised by about 1.73 cm.

Q4 A ray of light passing through an equilateral glass prism suffers a minimum deviation of 40°. Find the refractive index of the glass and hence its critical angle.

SOLUTION

Given: A = 60° (equilateral prism), D_m = 40°.

Prism formula:

$n = \sin[(A + D_m)/2] / \sin(A/2)$

n = sin[(60 + 40)/2] / sin(60/2) = sin 50° / sin 30°

n = 0.766 / 0.5 = 1.53

Critical angle:

sin C = 1/n = 1/1.53 = 0.653

$C = \sin^{-1}(0.653) = 40.8°$

At minimum deviation the ray passes symmetrically through the prism, travelling parallel to the base inside the glass.

Answer: n ≈ 1.53; critical angle C ≈ 40.8°.

Q5 Explain total internal reflection and state the conditions for it. Describe how it is used in an optical fibre.

SOLUTION

Definition: When light travels from an optically denser medium into a rarer one, the refracted ray bends away from the normal. Beyond a certain angle of incidence, refraction becomes impossible and the light is reflected entirely back into the denser medium. This is total internal reflection.

Conditions:

- Light must travel from the denser medium towards the rarer medium.
- The angle of incidence must exceed the critical angle C for the pair of media, where

$$\sin C = n_{rarer} / n_{denser} \quad (= 1/n \text{ for a medium in contact with air})$$

Why it is "total": Unlike ordinary reflection, no light energy is transmitted into the second medium, so there is no loss due to partial refraction. The reflected intensity is essentially 100 per cent.

Optical fibre: A fibre consists of a very thin cylindrical *core* of high refractive index surrounded by a *cladding* of slightly lower refractive index. Light entering one end at a suitable angle strikes the core–cladding boundary at an angle greater than the critical angle and is totally internally reflected. It undergoes thousands of such reflections and is guided along the fibre — even around bends — emerging at the far end with very little loss.

Uses: High-speed data and telephone communication, endoscopy in medicine, and decorative and sensing applications.

> **Answer:** TIR occurs when light goes from denser to rarer medium at an angle greater than the critical angle ($\sin C = 1/n$). Optical fibres guide light by repeated total internal reflection at the core–cladding boundary.

Q6 A double convex lens is made of glass of refractive index 1.55, with both faces having the same radius of curvature of 20 cm. Find the focal length of the lens and its power.

SOLUTION

Sign convention: For a double convex lens with light incident from the left, $R_1 = +20$ cm and $R_2 = -20$ cm.

Lens maker's formula:

$$1/f = (n - 1)(1/R_1 - 1/R_2)$$

$$1/f = (1.55 - 1)[1/20 - (-1/20)]$$

$$1/f = 0.55 \times (2/20) = 0.55 \times 0.1 = 0.055$$

$$f = 1/0.055 = 18.18 \text{ cm} \approx 18.2 \text{ cm}$$

Power: f must be expressed in metres.

$P = 1/f = 1/0.1818 = +5.5$ D

The positive sign confirms that the lens is converging.

Answer: f ≈ 18.2 cm (converging); P ≈ +5.5 D.

Q7 A convex lens of focal length 30 cm is placed in contact with a concave lens of focal length 20 cm. Find the focal length and power of the combination, and state whether it converges or diverges light.

SOLUTION

Given: $f_1 = +30$ cm (convex), $f_2 = -20$ cm (concave).

Formula for thin lenses in contact:

$1/f = 1/f_1 + 1/f_2 = 1/30 + (-1/20)$

$1/f = (2 - 3)/60 = -1/60$

$f = -60$ cm

Power:

$P = 1/f$ (in m) $= 1/(-0.60) = -1.67$ D

Nature: The negative focal length and negative power show that the combination behaves as a **diverging** lens. This happens because the concave lens is the stronger of the two (its focal length is shorter, so its power −5 D outweighs the +3.33 D of the convex lens).

Answer: f = −60 cm, P = −1.67 D — the combination is diverging.

Q8 A compound microscope has an objective of focal length 2.0 cm and an eyepiece of focal length 6.25 cm, separated by 15 cm. Where should the object be placed so that the final image is formed at the near point (25 cm)? Find the total magnifying power.

SOLUTION

Step 1 — work backwards from the eyepiece. The final image is at the near point, so $v_e = -25$ cm and $f_e = 6.25$ cm.

$1/u_e = 1/v_e - 1/f_e = -1/25 - 1/6.25$

$1/u_e = -0.04 - 0.16 = -0.20$

$u_e = -5.0$ cm

So the intermediate image lies 5.0 cm in front of the eyepiece.

Step 2 — locate the image formed by the objective. The lenses are 15 cm apart, so

$v_o = 15 - 5.0 = 10$ cm

Step 3 — find the object position. With $f_o = 2.0$ cm:

$1/u_o = 1/v_o - 1/f_o = 1/10 - 1/2 = 0.1 - 0.5 = -0.4$

$u_o = -2.5$ cm

Step 4 — magnifying power.

$m = (v_o/|u_o|) \times (1 + D/f_e) = (10/2.5) \times (1 + 25/6.25)$

$m = 4 \times (1 + 4) = 4 \times 5 = 20$

Answer: The object must be placed 2.5 cm from the objective; the magnifying power is 20.

Q9 An astronomical telescope has an objective of focal length 144 cm and an eyepiece of focal length 6.0 cm. Find its magnifying power and the separation between the two lenses in normal adjustment. Why is the objective made of large aperture?

SOLUTION

Magnifying power (normal adjustment, final image at infinity):

$m = f_o/f_e = 144/6.0 = 24$

Tube length: In normal adjustment the intermediate image lies at the common focus of the two lenses, so

$L = f_o + f_e = 144 + 6.0 = 150$ cm

Why a large aperture objective:

• **Light gathering power:** The amount of light collected is proportional to the area of the objective, i.e. to the square of its diameter. A large aperture makes faint stars visible.

• **Resolving power:** The smallest resolvable angular separation is $\theta \approx 1.22\lambda/D$. A larger diameter D gives a smaller θ, so closely spaced objects can be distinguished.

The image formed is inverted, which is acceptable for astronomical objects but not for terrestrial viewing.

Answer: $m = 24$ and $L = 150$ cm. A large objective aperture increases both the light gathering power and the resolving power.

CHAPTER 10

Wave Optics

Huygens' principle and the wave explanation of reflection and refraction, coherent sources and interference, Young's double slit experiment, diffraction at a single slit, resolving power, and polarisation including Brewster's law and Malus's law.

KEY FORMULAE AND RESULTS

Huygens' principle	Every point on a wavefront acts as a secondary source; the envelope of the secondary wavelets gives the new wavefront
Interference conditions	Constructive: path difference = $n\lambda$; Destructive: path difference = $(2n + 1)\lambda/2$
Resultant intensity	$I = I_1 + I_2 + 2\sqrt{(I_1 I_2)}\cos\varphi$; $I_{max}/I_{min} = (a_1 + a_2)^2/(a_1 - a_2)^2$
Young's double slit	Fringe width $\beta = \lambda D/d$; bright fringes at $y = n\lambda D/d$
Single slit diffraction	Minima at $a \sin\theta = n\lambda$; width of central maximum = $2\lambda D/a$
Fresnel distance	$z_F = a^2/\lambda$
Resolving power	Telescope: $\theta = 1.22\lambda/D$; Microscope: $d_{min} = 1.22\lambda/(2n \sin\beta)$
Brewster's law	$n = \tan i_B$; the reflected and refracted rays are perpendicular at i_B
Malus's law	$I = I_0 \cos^2\theta$

Q1 Monochromatic light of wavelength 589 nm travelling in air is incident on a water surface (refractive index 1.33). Find the wavelength, frequency and speed of (a) the reflected light and (b) the refracted light.

SOLUTION

(a) Reflected light: Reflection takes place back into the same medium (air), so nothing changes.

$\lambda = 589$ nm, $v = 3 \times 10^8$ m s^{-1}

$f = c/\lambda = 3 \times 10^8 / 589 \times 10^{-9} = 5.09 \times 10^{14}$ Hz

(b) Refracted light: The frequency is fixed by the source and never changes on refraction.

$f = 5.09 \times 10^{14}$ Hz (unchanged)

$v = c/n = 3 \times 10^8 / 1.33 = 2.26 \times 10^8$ m s^{-1}

$\lambda' = \lambda/n = 589/1.33 = 443$ nm

Key idea: Frequency is a property of the source; speed and wavelength are properties of the medium, and they change together so that $v = f\lambda$ remains valid.

Answer: (a) 589 nm, 5.09×10^{14} Hz, 3×10^{8} m s^{-1}. (b) 443 nm, 5.09×10^{14} Hz, 2.26×10^{8} m s^{-1}.

Q2 In a Young's double slit experiment the slits are 0.28 mm apart and the screen is 1.4 m away. The distance between the central bright fringe and the fourth bright fringe is measured to be 1.2 cm. Find the wavelength of the light used.

SOLUTION

Fringe width: The fourth bright fringe is four fringe widths from the centre.

$$\beta = 1.2 \times 10^{-2} / 4 = 3.0 \times 10^{-3} \text{ m}$$

Formula: $\beta = \lambda D/d$, so $\lambda = \beta d/D$.

$$\lambda = (3.0 \times 10^{-3} \times 0.28 \times 10^{-3}) / 1.4$$

$$\lambda = 8.4 \times 10^{-7} / 1.4 = 6.0 \times 10^{-7} \text{ m}$$

$$\lambda = 600 \text{ nm}$$

This lies in the orange region of the visible spectrum.

Answer: $\lambda = 6.0 \times 10^{-7}$ m = 600 nm.

Q3 What are coherent sources? Explain why two independent sodium lamps placed side by side cannot produce a sustained interference pattern, and how Young's experiment overcomes this.

SOLUTION

Coherent sources: Two sources are coherent if they emit waves of the same frequency with a phase difference between them that remains constant in time.

Why two independent lamps fail: Light from an ordinary source is emitted in short, independent bursts by billions of atoms, each burst lasting only about 10^{-8} s. The phase of the light from one lamp therefore changes randomly and extremely rapidly, and quite independently of the other lamp.

As a result the phase difference φ between the two beams varies randomly millions of times per second. The interference pattern shifts about far faster than the eye (or any detector) can follow, so what is observed is the *average* of $\cos\varphi$, which is zero. The intensities then simply add:

$$I = I_1 + I_2 + 2\sqrt{(I_1 I_2)}\ \langle\cos\varphi\rangle = I_1 + I_2$$

Uniform illumination is seen, with no fringes.

How Young solved it: He used a *single* narrow slit illuminated by monochromatic light, and let the light from it fall on two closely spaced slits. Both secondary slits are then lit by the same original wavefront, so any random phase change at the source occurs in both simultaneously. The phase difference between them stays constant, the sources are coherent, and stable fringes are obtained. This technique is known as division of wavefront.

Answer: Coherent sources have a constant phase relationship. Independent lamps have randomly fluctuating phases, so fringes average out. Young derived both slits from one source, keeping the phase difference constant.

Q4 Light of wavelength 500 nm is incident on a single slit of width 1.0 mm. A screen is placed 1.0 m away. Find the distance of the first minimum from the centre and the width of the central maximum.

SOLUTION

Condition for the first minimum: $a \sin\theta = \lambda$. For small angles, $\sin\theta \approx \theta \approx y/D$, so

$$y_1 = \lambda D/a$$

Working:

$$y_1 = (500 \times 10^{-9} \times 1.0) / (1.0 \times 10^{-3})$$

$$y_1 = 5.0 \times 10^{-4} \text{ m} = 0.5 \text{ mm}$$

Width of the central maximum: It extends from the first minimum on one side to the first minimum on the other.

$$\text{Width} = 2y_1 = 2\lambda D/a = 1.0 \times 10^{-3} \text{ m} = 1.0 \text{ mm}$$

Note the contrast with interference: in diffraction the central maximum is twice as wide as the other maxima, and the intensity of the secondary maxima falls off rapidly.

Answer: First minimum at 0.5 mm from the centre; the central maximum is 1.0 mm wide.

Q5 Distinguish between interference and diffraction of light on at least four counts.

SOLUTION

Interference	Diffraction
Arises from the superposition of waves from two (or more) separate coherent sources	Arises from the superposition of secondary wavelets from different parts of the *same* wavefront
All bright fringes have the same intensity	The central maximum is much brighter; the intensity of successive maxima falls off rapidly
Fringes are of equal width	The central maximum is twice as wide as the secondary maxima
Regions of complete darkness are obtained at the minima (for equal amplitudes)	Minima are not perfectly dark
Fringe width $\beta = \lambda D/d$ depends on the slit separation d	The spread depends on the slit width a; a narrower slit gives greater spreading

Both phenomena confirm the wave nature of light, and in a real double slit experiment the interference fringes are always modulated by the diffraction envelope of each individual slit.

Answer: Interference involves separate coherent sources with equal-intensity, equal-width fringes; diffraction involves a single wavefront with a dominant, wider central maximum.

Q6 Light is incident on a glass surface of refractive index 1.5. At what angle of incidence will the reflected light be completely plane polarised? What is the angle of refraction at this incidence?

SOLUTION

Brewster's law: $n = \tan i_B$

$$\tan i_B = 1.5$$

$$i_B = \tan^{-1}(1.5) = 56.3°$$

Angle of refraction: At the Brewster angle the reflected and refracted rays are exactly perpendicular to each other, so

$$i_B + r = 90°$$

$$r = 90° - 56.3° = 33.7°$$

Check using Snell's law: sin 56.3°/sin 33.7° = 0.832/0.555 = 1.50. ✓

At this angle the reflected beam is completely plane polarised, with its vibrations perpendicular to the plane of incidence.

Answer: $i_B \approx 56.3°$; the angle of refraction is 33.7°.

Q7 A beam of unpolarised light of intensity I_0 passes through a polariser and then through an analyser whose axis makes 60° with that of the polariser. Find the intensity of the emergent light.

SOLUTION

After the polariser: An unpolarised beam contains vibrations in all directions perpendicular to propagation. A polariser transmits only one component, and averaging $\cos^2\theta$ over all orientations gives ½.

$$I_1 = I_0/2$$

After the analyser (Malus's law):

$$I_2 = I_1 \cos^2\theta = (I_0/2) \times \cos^2 60°$$

$$I_2 = (I_0/2) \times (0.5)^2 = (I_0/2) \times 0.25$$

$$I_2 = I_0/8 = 0.125\ I_0$$

If the analyser were rotated to 90° the transmitted intensity would fall to zero — the crossed-polaroid position.

Answer: $I = I_0/8 = 0.125\ I_0$.

Q8 For what distance is ray optics a good approximation when the aperture is 4 mm wide and the wavelength of light is 400 nm?

SOLUTION

Concept: A beam passing through an aperture of width a stays approximately parallel over the Fresnel distance, beyond which diffraction spreading becomes comparable to the aperture size.

$$z_F = a^2/\lambda$$

Working:

$$z_F = (4 \times 10^{-3})^2 / (400 \times 10^{-9})$$

$$z_F = 1.6 \times 10^{-5} / 4 \times 10^{-7}$$

$$z_F = 40\ \text{m}$$

Interpretation: Up to about 40 m the beam behaves essentially as a ray travelling in a straight line, and ray optics gives good results. Beyond that distance the diffraction spread becomes significant and the wave description must be used.

Answer: $z_F = 40$ m — ray optics is a good approximation up to about 40 m.

CHAPTER 11

Dual Nature of Radiation and Matter

Electron emission and work function, the photoelectric effect and its experimental laws, Einstein's photoelectric equation and the particle nature of light, and the wave nature of matter through de Broglie's hypothesis and the Davisson–Germer experiment.

KEY FORMULAE AND RESULTS

Photon energy and momentum	$E = h\nu = hc/\lambda$; $p = h/\lambda = E/c$; $h = 6.626 \times 10^{-34}$ J s
Work function and threshold	$\varphi_0 = h\nu_0 = hc/\lambda_0$
Einstein's photoelectric equation	$K_{max} = h\nu - \varphi_0 = ½mv_{max}^2$
Stopping potential	$eV_0 = K_{max} \Rightarrow V_0 = (h/e)\nu - \varphi_0/e$
de Broglie wavelength	$\lambda = h/p = h/mv$; for an electron accelerated through V: $\lambda = 1.227/\sqrt{V}$ nm
Useful shortcut	E (in eV) = 1240 / λ (in nm)

Q1 Light of frequency 6.0×10^{14} Hz falls on a metal surface of work function 2.14 eV. Find (a) the energy of each photon, (b) the maximum kinetic energy of the emitted electrons and (c) their maximum speed.

SOLUTION

(a) Photon energy:

$E = h\nu = 6.626 \times 10^{-34} \times 6.0 \times 10^{14}$

$E = 3.98 \times 10^{-19}$ J

$E = 3.98 \times 10^{-19} / 1.6 \times 10^{-19} = 2.49$ eV

(b) Maximum kinetic energy:

$K_{max} = h\nu - \varphi_0 = 2.49 - 2.14 = 0.35$ eV

$K_{max} = 0.35 \times 1.6 \times 10^{-19} = 5.6 \times 10^{-20}$ J

(c) Maximum speed:

$½mv^2 = 5.6 \times 10^{-20}$

$v^2 = 2 \times 5.6 \times 10^{-20} / 9.11 \times 10^{-31} = 1.21 \times 10^{11}$

$v = 3.5 \times 10^5$ m s^{-1}

Answer: (a) E ≈ 2.49 eV. (b) K_{max} ≈ 0.35 eV. (c) v_{max} ≈ 3.5×10^5 m s^{-1}.

Q2 The threshold frequency for a certain metal is 3.3×10^{14} Hz. Find its work function in electron volts and its threshold wavelength.

SOLUTION

Work function:

$\varphi_0 = h\nu_0 = 6.626 \times 10^{-34} \times 3.3 \times 10^{14}$

$\varphi_0 = 2.19 \times 10^{-19}$ J

$\varphi_0 = 2.19 \times 10^{-19} / 1.6 \times 10^{-19} = 1.37$ eV

Threshold wavelength:

$\lambda_0 = c/\nu_0 = 3 \times 10^8 / 3.3 \times 10^{14}$

$\lambda_0 = 9.1 \times 10^{-7}$ m = 910 nm

Check with the shortcut: λ_0 = 1240/1.37 = 905 nm ≈ 910 nm. ✓

This lies in the infrared, so even infrared light can eject electrons from this metal.

Answer: φ_0 ≈ 1.37 eV; λ_0 ≈ 910 nm.

Q3 A monochromatic source emits light of wavelength 632.8 nm with a power of 9.42 mW. Find (a) the energy of each photon and (b) the number of photons emitted per second.

SOLUTION

(a) Photon energy:

$E = hc/\lambda = (6.626 \times 10^{-34} \times 3 \times 10^8) / (632.8 \times 10^{-9})$

$E = 1.988 \times 10^{-25} / 6.328 \times 10^{-7}$

$E = 3.14 \times 10^{-19}$ J = 1.96 eV

(b) Number of photons per second: Power is the energy delivered per second, so

$N = P/E = 9.42 \times 10^{-3} / 3.14 \times 10^{-19}$

> $N = 3.0 \times 10^{16}$ photons per second

The number is so enormous that the granular, photon-by-photon nature of the beam is completely undetectable in ordinary experiments — the light appears perfectly continuous.

> **Answer:** (a) $E \approx 3.14 \times 10^{-19}$ J (1.96 eV). (b) About 3.0×10^{16} photons per second.

Q4 State the experimental laws of the photoelectric effect and explain why the wave theory of light fails to account for them.

SOLUTION

Experimental laws:

1. For a given metal there exists a **threshold frequency** ν_0. Below this frequency no photoelectrons are emitted, no matter how intense the light or how long it is applied.

2. The **maximum kinetic energy** of the emitted electrons depends only on the frequency of the incident light, and increases linearly with it. It is completely independent of the intensity.

3. The **number of photoelectrons** per second (the saturation current) is proportional to the intensity, provided the frequency exceeds ν_0.

4. The emission is **instantaneous**, with no measurable time lag (less than 10^{-9} s), even for very feeble light.

Failure of the wave theory:

• On the wave picture, energy is spread continuously over the wavefront, so a sufficiently intense beam of *any* frequency should eventually supply the required energy. There should be no threshold frequency at all.

• A more intense wave carries a larger amplitude and hence more energy, so the electrons should emerge faster. Experiment shows intensity has no effect on their maximum kinetic energy.

• With very weak light, an electron would need to accumulate energy from the wave over minutes or even hours before escaping. Emission is in fact immediate.

Einstein's resolution: Light is absorbed as discrete quanta of energy $h\nu$. One photon is absorbed by one electron in a single event, giving $K_{max} = h\nu - \varphi_0$. Every observed law follows at once.

> **Answer:** Threshold frequency, intensity-independent K_{max}, intensity-dependent current and instantaneous emission cannot be explained by continuous wave energy; the photon picture explains all four.

Q5 Find the de Broglie wavelength of (a) an electron accelerated from rest through a potential difference of 100 V and (b) a ball of mass 0.040 kg moving at 1.0 m s^{-1}. Comment on the result.

SOLUTION

(a) Electron: The kinetic energy gained is eV, so $p = \sqrt{2meV}$ and

> $\lambda = h/\sqrt{2meV} = 1.227/\sqrt{V}$ nm (standard result)

$\lambda = 1.227/\sqrt{100} = 1.227/10$

$\lambda = 0.1227 \text{ nm} = 1.227 \text{ Å} = 1.23 \times 10^{-10} \text{ m}$

This is comparable to atomic spacings in crystals, which is exactly why electron diffraction from a crystal lattice (the Davisson–Germer experiment) is observable.

(b) Ball:

$\lambda = h/mv = 6.626 \times 10^{-34} / (0.040 \times 1.0)$

$\lambda = 1.66 \times 10^{-32} \text{ m}$

Comment: The wavelength of the ball is smaller than a nucleus by some seventeen orders of magnitude. No aperture or obstacle of that size exists, so its wave nature can never be detected. This is why matter waves matter only for microscopic particles.

Answer: (a) $\lambda \approx 1.23 \times 10^{-10}$ m (1.23 Å). (b) $\lambda \approx 1.66 \times 10^{-32}$ m — far too small to observe.

Q6 The work function of a metal is 4.2 eV. Find its threshold wavelength and state whether visible light of wavelength 550 nm can eject photoelectrons from it.

SOLUTION

Threshold wavelength: Using E (eV) = 1240/λ (nm),

$\lambda_0 = 1240 / \varphi_0 = 1240 / 4.2$

$\lambda_0 = 295 \text{ nm}$

Testing the visible light: The incident wavelength is 550 nm, whose photon energy is

$E = 1240/550 = 2.25 \text{ eV}$

Since 2.25 eV < 4.2 eV (equivalently 550 nm > 295 nm), each photon carries less energy than is needed to free an electron.

Conclusion: No photoemission occurs, however intense the light. Increasing the intensity only sends more photons, each still too weak individually — ultraviolet light of wavelength shorter than 295 nm would be required.

Answer: $\lambda_0 \approx 295$ nm. Light of 550 nm (2.25 eV) cannot eject photoelectrons since its photon energy is below the work function.

CHAPTER 12

Atoms

Rutherford's alpha-scattering experiment and the nuclear model of the atom, the distance of closest approach, the limitations of the classical model, Bohr's postulates, energy levels of the hydrogen atom, and the origin of the hydrogen spectral series.

KEY FORMULAE AND RESULTS

Bohr's quantisation condition	$mvr = nh/2\pi$, $n = 1, 2, 3, \ldots$
Radius of the n-th orbit	$r_n = (n^2h^2\varepsilon_0)/(\pi me^2) = 0.529\ n^2$ Å (hydrogen)
Speed in the n-th orbit	$v_n = e^2/(2\varepsilon_0 nh) = 2.19 \times 10^6/n$ m s^{-1}
Energy of the n-th level	$E_n = -13.6/n^2$ eV; $KE = -E_n$, $PE = 2E_n$
Transition frequency	$h\nu = E_i - E_f$
Rydberg formula	$1/\lambda = R(1/n_f^2 - 1/n_i^2)$, $R = 1.097 \times 10^7$ m^{-1}
Spectral series	Lyman ($n_f = 1$, UV) • Balmer ($n_f = 2$, visible) • Paschen (3) • Brackett (4) • Pfund (5), all infrared
Distance of closest approach	$d = (1/4\pi\varepsilon_0) \cdot 2Ze^2/K$

Q1 State Bohr's postulates for the hydrogen atom and explain how they resolve the instability of Rutherford's model.

SOLUTION

The problem with Rutherford's model: An electron revolving around the nucleus is continuously accelerated (centripetal acceleration). Classical electromagnetic theory requires an accelerated charge to radiate energy continuously. The electron would therefore spiral into the nucleus within about 10^{-8} s, and would emit a continuous spectrum on the way. Atoms are in fact stable and emit sharp line spectra.

Bohr's postulates:

1. Stationary orbits. An electron can revolve only in certain permitted circular orbits, called stationary states, without radiating energy. In these orbits the electrostatic attraction supplies the centripetal force:

$$mv^2/r = (1/4\pi\varepsilon_0)\ e^2/r^2$$

2. Quantisation of angular momentum. Only those orbits are allowed for which the angular momentum is an integral multiple of $h/2\pi$:

$$mvr = nh/2\pi, \quad n = 1, 2, 3, \ldots$$

3. Frequency condition. Radiation is emitted or absorbed only when the electron jumps between two stationary states, the photon energy being the difference in the energies of the levels:

$$h\nu = E_i - E_f$$

How this resolves the difficulty: The first postulate simply declares that the classical radiation law does not apply to stationary states, which secures the atom's stability. The second restricts the electron to a discrete set of orbits with discrete energies. The third then explains why the emitted spectrum consists of sharp lines rather than a continuum — only certain energy differences are possible.

Answer: Stationary non-radiating orbits, quantised angular momentum ($mvr = nh/2\pi$), and emission only on transitions ($h\nu = E_i - E_f$) together explain atomic stability and line spectra.

Q2 Calculate the radius, orbital speed and period of revolution of the electron in the first, second and third Bohr orbits of hydrogen.

SOLUTION

Formulae: $r_n = 0.529\, n^2$ Å, $v_n = 2.19 \times 10^6/n$ m s^{-1}, $T_n = 2\pi r_n/v_n$.

n = 1:

$$r_1 = 0.529 \times 1 = 0.529 \text{ Å} = 5.29 \times 10^{-11} \text{ m}; \quad v_1 = 2.19 \times 10^6 \text{ m s}^{-1}$$

$$T_1 = 2\pi \times 5.29 \times 10^{-11} / 2.19 \times 10^6 = 1.52 \times 10^{-16} \text{ s}$$

n = 2:

$$r_2 = 0.529 \times 4 = 2.12 \text{ Å}; \quad v_2 = 1.095 \times 10^6 \text{ m s}^{-1}$$

$$T_2 = 2\pi \times 2.116 \times 10^{-10} / 1.095 \times 10^6 = 1.21 \times 10^{-15} \text{ s}$$

n = 3:

$$r_3 = 0.529 \times 9 = 4.76 \text{ Å}; \quad v_3 = 0.73 \times 10^6 \text{ m s}^{-1}$$

$$T_3 = 2\pi \times 4.761 \times 10^{-10} / 7.3 \times 10^5 = 4.10 \times 10^{-15} \text{ s}$$

Pattern: $r \propto n^2$, $v \propto 1/n$ and $T \propto n^3$. The electron in the ground state completes roughly 6.6×10^{15} revolutions per second.

Answer: r = 0.53, 2.12, 4.76 Å; v = 2.19, 1.10, 0.73 ($\times 10^6$ m s^{-1}); T = 1.52×10^{-16}, 1.21×10^{-15}, 4.10×10^{-15} s.

Q3 The ground state energy of the hydrogen atom is −13.6 eV. What are the kinetic and potential energies of the electron in this state? What is the ionisation energy of hydrogen?

SOLUTION

Relations between the energies: For an electron bound by the Coulomb force,

$$KE = -E_n \quad \text{and} \quad PE = 2E_n$$

Working for the ground state (E_1 = −13.6 eV):

$$KE = -(-13.6) = +13.6 \text{ eV}$$

$$PE = 2 \times (-13.6) = -27.2 \text{ eV}$$

Check: Total E = KE + PE = 13.6 − 27.2 = −13.6 eV. ✓

Ionisation energy: This is the energy needed to take the electron from the ground state to n = ∞, where E = 0.

$$E_{\text{ionisation}} = 0 - (-13.6) = 13.6 \text{ eV}$$

The negative total energy indicates a bound system — energy must be supplied to free the electron.

Answer: KE = +13.6 eV, PE = −27.2 eV, and the ionisation energy is 13.6 eV.

Q4 Find the wavelength of the photon emitted when the electron in a hydrogen atom makes a transition from the n = 4 level to the n = 2 level. To which spectral series does this line belong?

SOLUTION

Energy of the transition:

$$E_4 = -13.6/16 = -0.85 \text{ eV}; \quad E_2 = -13.6/4 = -3.40 \text{ eV}$$

$$\Delta E = E_4 - E_2 = -0.85 - (-3.40) = 2.55 \text{ eV}$$

Wavelength: Using λ (nm) = 1240 / E (eV),

$$\lambda = 1240 / 2.55 = 486 \text{ nm}$$

Series: The final level is n_f = 2, so this line belongs to the **Balmer series**, which lies in the visible region. Specifically it is the H_β line, seen as blue-green.

Answer: λ ≈ 486 nm — the H_β line of the Balmer series (visible, blue-green).

Q5 A hydrogen atom in its ground state is bombarded by electrons of energy 12.5 eV. To which levels can the atom be excited, and what wavelengths will be emitted as it returns to the ground state?

SOLUTION

Energy levels: $E_1 = -13.6$ eV, $E_2 = -3.40$ eV, $E_3 = -1.51$ eV, $E_4 = -0.85$ eV.

Excitation energies from the ground state:

$E_2 - E_1 = 10.2$ eV; $E_3 - E_1 = 12.09$ eV; $E_4 - E_1 = 12.75$ eV

The bombarding electrons supply 12.5 eV, which is enough to reach n = 3 but not n = 4. So the atom can be excited to **n = 2 or n = 3**, and the leftover energy stays with the electron.

Possible downward transitions and wavelengths (using $\lambda = 1240/\Delta E$):

3 → 1: $\Delta E = 12.09$ eV ⇒ $\lambda = 1240/12.09 = 102.6$ nm (Lyman, UV)

3 → 2: $\Delta E = 1.89$ eV ⇒ $\lambda = 1240/1.89 = 656$ nm (Balmer H_α, red)

2 → 1: $\Delta E = 10.2$ eV ⇒ $\lambda = 1240/10.2 = 121.6$ nm (Lyman, UV)

Three spectral lines are therefore observed.

Answer: The atom can be excited up to n = 3. Three lines appear: 102.6 nm, 121.6 nm (Lyman series) and 656 nm (Balmer H_α).

Q6 An alpha particle of energy 5.5 MeV is fired head-on at a gold nucleus (Z = 79). Find the distance of closest approach.

SOLUTION

Principle: At the closest point the alpha particle has momentarily stopped, so all its kinetic energy has been converted into electrostatic potential energy.

$K = (1/4\pi\varepsilon_0) \cdot (2e)(Ze)/d \Rightarrow d = (1/4\pi\varepsilon_0) \cdot 2Ze^2/K$

Working: $K = 5.5\text{ MeV} = 5.5 \times 10^6 \times 1.6 \times 10^{-19} = 8.8 \times 10^{-13}$ J.

Numerator $= 9 \times 10^9 \times 2 \times 79 \times (1.6 \times 10^{-19})^2$

$= 9 \times 10^9 \times 158 \times 2.56 \times 10^{-38} = 3.64 \times 10^{-26}$

$d = 3.64 \times 10^{-26} / 8.8 \times 10^{-13} = 4.14 \times 10^{-14}$ m

Significance: This distance, about 41 fm, sets an upper limit on the size of the gold nucleus. Rutherford's discovery that such close approaches occur — and that a few alpha particles were scattered through more than 90° — proved that the positive charge and nearly all the mass of the atom are concentrated in a tiny central nucleus.

Answer: $d \approx 4.1 \times 10^{-14}$ m (about 41 fm).

CHAPTER 13

Nuclei

Composition and size of the nucleus, atomic masses and the unified mass unit, mass–energy equivalence, nuclear binding energy and the binding energy per nucleon curve, nuclear forces, radioactivity and the law of radioactive decay, and nuclear fission and fusion.

KEY FORMULAE AND RESULTS

Nuclear radius	$R = R_0A^{1/3}$, $R_0 = 1.2 \times 10^{-15}$ m; nuclear density is independent of A
Mass–energy equivalence	$E = mc^2$; 1 u = 931.5 MeV/c^2
Mass defect and binding energy	$\Delta m = [Zm_p + (A - Z)m_n] - M$; BE = $\Delta m \times 931.5$ MeV
Law of radioactive decay	$N = N_0e^{-\lambda t}$; activity $R = \lambda N = R_0e^{-\lambda t}$
Half-life and mean life	$T_{1/2} = 0.693/\lambda$; $\tau = 1/\lambda = T_{1/2}/0.693$
Decay processes	α: A decreases by 4, Z by 2 • β^-: Z increases by 1 • β^+: Z decreases by 1 • γ: no change in A or Z
Units of activity	1 becquerel = 1 decay s^{-1}; 1 curie = 3.7×10^{10} Bq

Q1 Show that the density of a nucleus is independent of its mass number, and estimate its value.

SOLUTION

Radius–mass number relation: Experiment shows that

$$R = R_0A^{1/3}, \quad \text{with } R_0 = 1.2 \times 10^{-15}\ \text{m}$$

Volume:

$$V = (4/3)\pi R^3 = (4/3)\pi R_0^3A$$

Mass: A nucleus of mass number A has a mass of approximately $A \times m_n$, where $m_n \approx 1.66 \times 10^{-27}$ kg.

Density:

$$\rho = M/V = (A\ m_n) / [(4/3)\pi R_0^3A] = 3m_n / (4\pi R_0^3)$$

The mass number A cancels completely, so the density is the same for every nucleus.

Numerical estimate:

$$R_0^3 = (1.2 \times 10^{-15})^3 = 1.728 \times 10^{-45}\ \text{m}^3$$

$$\rho = (3 \times 1.66 \times 10^{-27}) / (4\pi \times 1.728 \times 10^{-45})$$

$\rho = 4.98 \times 10^{-27} / 2.171 \times 10^{-44} \approx 2.3 \times 10^{17}$ kg m^{-3}

This is about 10^{14} times the density of ordinary matter, confirming that the atom is mostly empty space.

Answer: $\rho = 3m_n/4\pi R_0^3 \approx 2.3 \times 10^{17}$ kg m^{-3}, independent of A.

Q2 Calculate the binding energy and the binding energy per nucleon of the iron-56 nucleus. Take m_p = 1.007825 u (hydrogen atom), m_n = 1.008665 u and the atomic mass of Fe-56 = 55.934939 u.

SOLUTION

Composition: Iron-56 has Z = 26 protons and A − Z = 30 neutrons.

Mass of the constituents:

$26 \times 1.007825 = 26.203450$ u

$30 \times 1.008665 = 30.259950$ u

Total = 56.463400 u

Mass defect:

$\Delta m = 56.463400 - 55.934939 = 0.528461$ u

Binding energy:

$BE = \Delta m \times 931.5 = 0.528461 \times 931.5$

BE = 492.3 MeV

Binding energy per nucleon:

BE/A = 492.3 / 56 = 8.79 MeV per nucleon

Significance: This is very close to the maximum of the binding energy curve, which peaks at about 8.8 MeV per nucleon near A = 56. Iron is therefore among the most tightly bound (most stable) of all nuclei, and neither fission nor fusion of iron releases energy.

Answer: Δm = 0.5285 u, BE ≈ 492.3 MeV, and BE/A ≈ 8.79 MeV per nucleon.

Q3 Explain the shape of the binding energy per nucleon curve and use it to show why both fission of heavy nuclei and fusion of light nuclei release energy.

SOLUTION

Shape of the curve: Plotting BE/A against mass number A gives a curve that

• rises steeply for light nuclei ($A < 20$), with sharp peaks at helium-4, carbon-12 and oxygen-16;

• reaches a broad maximum of about 8.8 MeV per nucleon near $A = 56$ (iron and nickel);

• falls slowly for heavier nuclei, dropping to about 7.6 MeV per nucleon at uranium ($A = 238$).

Why it is nearly flat in the middle: The nuclear force is short-ranged and saturating — each nucleon interacts only with its immediate neighbours, not with all the others. Adding more nucleons therefore adds roughly the same binding energy per nucleon. The slow decline at large A comes from the long-range Coulomb repulsion between all pairs of protons, which grows as Z^2.

Energy release in fission: A heavy nucleus (BE/A ≈ 7.6 MeV) splits into two medium nuclei (BE/A ≈ 8.5 MeV). The products are more tightly bound, so the gain of about 0.9 MeV per nucleon over roughly 236 nucleons is released — approximately 200 MeV per fission.

Energy release in fusion: Two very light nuclei with low BE/A combine into a heavier one further up the steep part of the curve. The increase in BE/A is large, so energy is released — this powers the Sun and the stars.

General rule: Energy is released in any nuclear process that moves the products *towards* the peak of the curve.

> **Answer:** BE/A peaks near A = 56. Fission of heavy nuclei and fusion of light nuclei both move products towards this peak, increasing binding energy and releasing the difference.

Q4 The half-life of radium-226 is 1600 years. Calculate the activity of 1.0 g of pure radium-226 in becquerel and in curie. (Avogadro's number = 6.022×10^{23} mol^{-1}.)

SOLUTION

Number of nuclei in 1.0 g:

$$N = (\text{mass / molar mass}) \times N_A = (1.0/226) \times 6.022 \times 10^{23}$$

$$N = 2.665 \times 10^{21} \text{ nuclei}$$

Decay constant: Convert the half-life to seconds using 1 year ≈ 3.156×10^7 s.

$$T_{1/2} = 1600 \times 3.156 \times 10^7 = 5.05 \times 10^{10} \text{ s}$$

$$\lambda = 0.693 / 5.05 \times 10^{10} = 1.372 \times 10^{-11} \text{ s}^{-1}$$

Activity:

$$R = \lambda N = 1.372 \times 10^{-11} \times 2.665 \times 10^{21}$$

$$R = 3.66 \times 10^{10} \text{ Bq}$$

$$R = 3.66 \times 10^{10} / 3.7 \times 10^{10} = 0.99 \text{ Ci} \approx 1 \text{ Ci}$$

This is no coincidence — the curie was originally defined as the activity of one gram of radium.

Answer: R ≈ 3.66 × 10^{10} Bq ≈ 1.0 curie.

Q5 A radioactive sample has a half-life of 20 minutes. What fraction of the sample remains undecayed after 1 hour? Derive the relation between half-life and decay constant.

SOLUTION

Number of half-lives:

$$n = t / T_{1/2} = 60 / 20 = 3$$

Fraction remaining: After each half-life the number of undecayed nuclei halves.

$$N/N_0 = (1/2)^n = (1/2)^3 = 1/8 = 0.125$$

So 12.5 per cent remains undecayed and 87.5 per cent has decayed.

Relation between $T_{1/2}$ and λ: Starting from the decay law $N = N_0e^{-\lambda t}$, put $N = N_0/2$ when $t = T_{1/2}$:

$$N_0/2 = N_0e^{-\lambda T_{1/2}}$$

$$e^{-\lambda T_{1/2}} = 1/2 \;\Rightarrow\; \lambda T_{1/2} = \ln 2$$

$$T_{1/2} = \ln 2 / \lambda = 0.693/\lambda$$

The mean life is $\tau = 1/\lambda = T_{1/2}/0.693 = 1.44\ T_{1/2}$.

Answer: One-eighth (12.5 per cent) remains after 1 hour; $T_{1/2} = 0.693/\lambda$.

Q6 Assuming that 200 MeV of energy is released per fission of uranium-235, calculate the energy released by the complete fission of 1.0 kg of U-235.

SOLUTION

Number of nuclei in 1.0 kg:

$$N = (1000\ g / 235\ g\ mol^{-1}) \times 6.022 \times 10^{23}$$

$$N = 4.255 \times 6.022 \times 10^{23} = 2.56 \times 10^{24} \text{ nuclei}$$

Total energy in MeV:

$$E = 2.56 \times 10^{24} \times 200 = 5.12 \times 10^{26} \text{ MeV}$$

Converting to joules (1 MeV = 1.6 × 10^{-13} J):

$$E = 5.12 \times 10^{26} \times 1.6 \times 10^{-13}$$

$E = 8.2 \times 10^{13}$ J

Perspective: Burning 1 kg of coal releases roughly 3×10^{7} J, so 1 kg of U-235 yields about 2.7 million times as much energy as 1 kg of coal.

Answer: $E \approx 8.2 \times 10^{13}$ J (about 5.1×10^{26} MeV).

Q7 Distinguish between nuclear fission and nuclear fusion. Why is it so difficult to achieve controlled fusion on Earth?

SOLUTION

Nuclear fission	Nuclear fusion
A heavy nucleus splits into two lighter nuclei of comparable mass	Two very light nuclei combine to form a heavier nucleus
Triggered by bombardment with a slow neutron	Requires extremely high temperature and pressure
About 200 MeV released per event	Less energy per event, but far more per unit mass of fuel
Produces highly radioactive fission fragments	Products are largely non-radioactive (mainly helium)
Can be controlled — the basis of nuclear power reactors	Not yet controlled for sustained power generation
Sustained by a chain reaction of neutrons	Sustained by the thermal energy of the plasma

Why controlled fusion is so difficult:

• **The Coulomb barrier.** Both nuclei are positively charged and repel each other strongly. To get within range of the short-range attractive nuclear force, they must approach to about 10^{-15} m, which requires kinetic energies corresponding to temperatures of the order of 10^{8} K.

• **Containment.** At such temperatures matter exists as a plasma that would vaporise any material container instantly. Magnetic confinement (as in a tokamak) or inertial confinement must be used instead, and keeping the plasma stable is extremely demanding.

• **Break-even.** The plasma must be held at sufficient density for long enough that the energy released exceeds the energy spent heating and confining it — a condition that has proved very hard to sustain.

Answer: Fission splits heavy nuclei; fusion joins light ones. Controlled fusion is hard because overcoming the Coulomb barrier needs temperatures around 10^{8} K, and no material can contain such a plasma.

CHAPTER 14

Semiconductor Electronics: Materials, Devices and Simple Circuits

Energy bands in solids, intrinsic and extrinsic semiconductors, p-type and n-type doping, the p-n junction and its characteristics, the junction diode as a rectifier, special purpose diodes such as the Zener diode, LED, photodiode and solar cell, and logic gates.

KEY FORMULAE AND RESULTS

Mass action law	$n_e n_h = n_i^2$ at a given temperature
Charge neutrality	n-type: $n_e \approx N_D$; p-type: $n_h \approx N_A$
Conductivity	$\sigma = e(n_e\mu_e + n_h\mu_h)$
Band gaps (approximate)	Conductors: no gap (overlap) • Semiconductors: $E_g < 3$ eV • Insulators: $E_g > 3$ eV
Photodiode / LED condition	$h\nu > E_g$, $\lambda_{max} = hc/E_g = 1240/E_g$ nm
Rectifier output frequency	Half wave: same as input; Full wave: twice the input frequency
Logic gates	AND: $Y = A \cdot B$ • OR: $Y = A + B$ • NOT: $Y = \bar{A}$ • NAND and NOR are the universal gates

Q1 A silicon specimen has an intrinsic carrier concentration of 1.5×10^{16} m^{-3} at room temperature. It is doped with a pentavalent impurity at a concentration of 5×10^{22} m^{-3}. Find the electron and hole concentrations in the doped sample and identify the majority and minority carriers.

SOLUTION

Type of semiconductor: A pentavalent impurity (such as arsenic or phosphorus) donates one free electron per atom, so the material becomes **n-type**.

Electron concentration: Each donor contributes one electron, and the donor concentration far exceeds n_i, so

$$n_e \approx N_D = 5 \times 10^{22}\ m^{-3}$$

Hole concentration: By the law of mass action, $n_e n_h = n_i^2$.

$$n_h = n_i^2/n_e = (1.5 \times 10^{16})^2 / 5 \times 10^{22}$$

$$n_h = 2.25 \times 10^{32} / 5 \times 10^{22} = 4.5 \times 10^9\ m^{-3}$$

Carriers: Electrons are the majority carriers and holes the minority carriers. Doping has multiplied the electron concentration enormously while suppressing the hole concentration far below its intrinsic value; the crystal as a whole remains electrically neutral.

> **Answer:** $n_e \approx 5 \times 10^{22}$ m^{-3} (majority) and $n_h = 4.5 \times 10^{9}$ m^{-3} (minority); the sample is n-type.

Q2 Explain the formation of the depletion region and the barrier potential in a p-n junction. How are they affected by forward and reverse biasing?

SOLUTION

Formation of the depletion region: When a p-type and an n-type semiconductor are joined, there is a steep concentration gradient across the junction. Electrons diffuse from the n-side to the p-side and holes diffuse from the p-side to the n-side, and each pair recombines near the junction.

This leaves behind immobile *ionised* impurity atoms — negative acceptor ions on the p-side and positive donor ions on the n-side. The thin region around the junction is therefore depleted of free charge carriers and is called the **depletion region** (typically about 10^{-6} m wide).

Barrier potential: The exposed ions set up an electric field directed from the n-side to the p-side. This field opposes further diffusion, and the corresponding potential difference is the **barrier potential** V_b (about 0.3 V for germanium and 0.7 V for silicon). Equilibrium is reached when the diffusion current is exactly balanced by the drift current of minority carriers.

Forward bias (p-side to positive terminal):

• The applied field opposes the barrier field, so the barrier potential is reduced.

• The depletion region becomes **narrower**.

• Majority carriers cross the junction easily, giving a large current (of the order of milliamperes) that rises sharply once the applied voltage exceeds the knee voltage.

Reverse bias (p-side to negative terminal):

• The applied field adds to the barrier field, so the barrier potential increases.

• The depletion region becomes **wider**.

• Majority carriers cannot cross. Only a very small current (of the order of microamperes) flows, due to minority carriers, and it is almost independent of the applied voltage until breakdown occurs.

> **Answer:** Diffusion and recombination leave immobile ions, creating a depletion region and barrier potential. Forward bias narrows the region and lowers the barrier; reverse bias widens it and raises the barrier.

Q3 With the help of the working principle, explain how a p-n junction diode acts as a half-wave rectifier and as a full-wave rectifier. State the output frequency in each case for a 50 Hz input.

SOLUTION

Basic principle: A junction diode conducts freely when forward biased and almost not at all when reverse biased. It therefore allows current in one direction only, which converts alternating current into unidirectional current.

Half-wave rectifier: A single diode is connected in series with the load across the secondary of a transformer.

• During the half cycle in which the diode is forward biased, current flows through the load and an output voltage appears across it.

• During the next half cycle the diode is reverse biased, no current flows, and the output is zero.

The output is a series of pulses corresponding to alternate half cycles, so **only half the input power is used**.

> Output frequency = input frequency = 50 Hz

Full-wave rectifier: A centre-tapped transformer feeds two diodes whose outputs are joined across a common load (a bridge arrangement with four diodes achieves the same result without a centre tap).

• In the first half cycle diode D_1 is forward biased and D_2 is reverse biased; current flows through the load in one direction.

• In the second half cycle the roles are reversed, but the current through the load flows in the *same* direction as before.

Both halves of the input are therefore used, giving a more efficient and less pulsating output.

> Output frequency = 2 × input frequency = 100 Hz

Filtering: In both cases a capacitor connected across the load smooths the pulsating output by charging at the peaks and discharging slowly through the load.

> **Answer:** A diode conducts only when forward biased. Half-wave output is 50 Hz; full-wave output is 100 Hz for a 50 Hz input.

Q4 A silicon diode with a forward voltage drop of 0.7 V is connected in series with a 1.0 kΩ resistor across a 12 V supply. Find the current in the circuit and the power dissipated in the resistor.

SOLUTION

Circuit analysis: Applying Kirchhoff's loop rule, the supply voltage is shared between the diode drop and the resistor.

> $V_{supply} = V_{diode} + IR$

> $12 = 0.7 + I \times 1000$

Current:

> $I = (12 - 0.7)/1000 = 11.3/1000$

> $I = 1.13 \times 10^{-2}$ A = 11.3 mA

Power in the resistor:

> $P = I^2R = (1.13 \times 10^{-2})^2 \times 1000$

$P = 1.277 \times 10^{-4} \times 1000 = 0.128$ W

Note: If the diode were connected the other way round it would be reverse biased, and the current would fall to a negligible leakage value of a few microamperes.

Answer: I = 11.3 mA;   P ≈ 0.128 W dissipated in the resistor.

Q5 Explain how a Zener diode is used as a voltage regulator. Why is it always operated in reverse bias, and why is a series resistance essential?

SOLUTION

Construction: A Zener diode is a heavily doped p-n junction. The heavy doping makes the depletion region very thin, so a modest reverse voltage produces a very intense electric field across it.

Breakdown behaviour: When the reverse voltage reaches the Zener voltage V_Z, the field is strong enough to break covalent bonds directly and release a large number of carriers. The reverse current then rises steeply while the voltage across the diode remains almost exactly constant at V_Z. This breakdown is non-destructive as long as the current is limited.

Why reverse bias: The sharp, near-vertical breakdown region — the flat voltage characteristic that makes regulation possible — occurs only in reverse bias. In forward bias a Zener behaves like an ordinary diode with a drop of about 0.7 V and offers no regulation.

Working as a regulator: The unregulated DC input is applied through a series resistance R_s, with the Zener connected in parallel with the load.

• If the input voltage rises, the current through the Zener increases while the load current stays the same. The extra drop appears across R_s, so the load voltage is unchanged.

• If the load draws more current, the Zener current falls by the same amount, keeping the total current and hence the drop across R_s constant.

In both cases the output stays fixed at V_Z.

Why the series resistance is essential: Without R_s the reverse current in breakdown would be limited only by the source, and the diode would be destroyed by excessive power dissipation. R_s absorbs the surplus voltage and keeps the Zener current within its safe rating.

Answer: A Zener held in reverse breakdown maintains a constant V_Z across the load, absorbing changes in input or load current. The series resistance drops the surplus voltage and limits the current to a safe value.

Q6 What is a light emitting diode? Explain its working and state why the semiconductor used must have a suitable band gap. Find the band gap needed to emit red light of wavelength 650 nm.

SOLUTION

Definition: An LED is a heavily doped p-n junction diode, forward biased, that converts electrical energy directly into light.

Working: Under forward bias, electrons are injected from the n-side into the p-side and holes from the p-side into the n-side. Near the junction these excess minority carriers recombine with the majority carriers. Each recombination event releases an amount of energy approximately equal to the band gap, and in certain materials this energy is emitted as a photon:

> $h\nu \approx E_g$

Why the band gap matters: The wavelength of the emitted light is fixed by the band gap:

> $\lambda = hc/E_g = 1240/E_g$ nm   (E_g in eV)

For visible emission E_g must lie roughly between 1.8 eV and 3 eV. Silicon and germanium have gaps that are too small (1.1 eV and 0.7 eV), and they are indirect band gap materials in which recombination gives up its energy mostly as heat. Compound semiconductors such as gallium arsenide phosphide are used instead.

Band gap for red light at 650 nm:

> $E_g = 1240/650 = 1.91$ eV

Advantages of LEDs: low operating voltage and power, very long life, fast switching, no warm-up time, and near-monochromatic output.

> **Answer:** An LED emits light when injected carriers recombine across the junction. For 650 nm, $E_g \approx 1.91$ eV.

Q7 Draw the truth tables of the AND, OR, NOT, NAND and NOR gates, and explain why NAND and NOR are called universal gates.

SOLUTION

A	B	AND (A·B)	OR (A+B)	NAND	NOR
0	0	0	0	1	1
0	1	0	1	1	0
1	0	0	1	1	0
1	1	1	1	0	0

NOT gate (single input): $Y = \bar{A}$. If A = 0 then Y = 1; if A = 1 then Y = 0.

Why NAND and NOR are universal: A gate is called universal if the three basic gates — NOT, AND and OR — can all be built from it alone. Any logic circuit whatsoever can then be constructed using only that one type of gate.

Using NAND gates:

- **NOT:** tie both inputs of a NAND together. $Y = \overline{(A \cdot A)} = \bar{A}$.
- **AND:** follow a NAND with a NAND-based NOT. $Y = \overline{\overline{(A \cdot B)}} = A \cdot B$.
- **OR:** invert both inputs with NAND-NOTs and feed them into a NAND. By De Morgan's theorem, $\overline{(\bar{A} \cdot \bar{B})} = A + B$.

A similar set of constructions works for NOR gates.

Practical importance: Manufacturing a single gate type in large numbers is cheaper and more reliable, so most integrated circuits are built predominantly from NAND or NOR gates.

> **Answer:** See the truth table. NAND and NOR are universal because NOT, AND and OR can each be constructed using only NAND gates, or only NOR gates.

Q8 On the basis of energy bands, distinguish between conductors, semiconductors and insulators. Explain why the resistance of a semiconductor decreases with rising temperature while that of a metal increases.

SOLUTION

Property	Conductor	Semiconductor	Insulator
Energy gap E_g	No gap; valence and conduction bands overlap	Small, $E_g < 3$ eV (Si: 1.1 eV, Ge: 0.7 eV)	Large, $E_g > 3$ eV (diamond: about 6 eV)
Conduction band at room temperature	Partly filled	Almost empty, with a few thermally excited electrons	Completely empty
Resistivity	10^{-8} – 10^{-6} Ω m	10^{-5} – 10^{6} Ω m	10^{11} – 10^{19} Ω m
Temperature coefficient	Positive	Negative	Negative (but effectively non-conducting)

Semiconductor — resistance falls with temperature: At absolute zero a pure semiconductor behaves as a perfect insulator, since its conduction band is empty. As the temperature rises, thermal energy excites electrons across the small gap into the conduction band, leaving holes behind. The number of charge carriers n grows roughly exponentially with temperature. Since $\sigma = ne\mu$, the sharp increase in n far outweighs the modest fall in mobility μ, so conductivity rises and resistance falls.

Metal — resistance rises with temperature: In a metal the number of free electrons is fixed and does not depend on temperature. Heating merely increases the amplitude of the lattice vibrations, so electrons collide more frequently and the relaxation time τ falls. Since

$$\rho = m/(ne^2\tau)$$

a smaller τ with unchanged n means a larger resistivity.

> **Answer:** Conductors have overlapping bands, semiconductors a small gap (< 3 eV), insulators a large gap (> 3 eV). In semiconductors carrier number rises steeply with temperature, lowering resistance; in metals n is fixed and increased lattice scattering raises resistance.

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